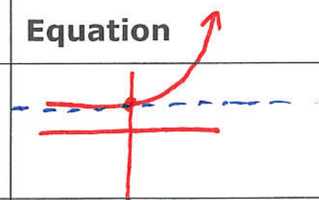
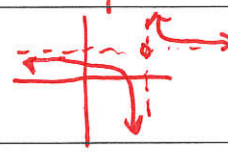
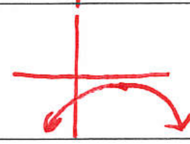
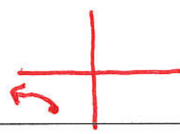
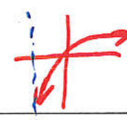
Name ______________________________ Date ________________ Period _____

REVIEW for Algebra II FINAL EXAM

1 For each equation, identify the function type, give the parent equation for that function type, and describe how the graph of this equation is different from the parent graph.

	Equation	**Function Type**	**Sketch the Equation**	**Describe parameter changes**
a.	$y = 2^x + 3$	exponential		up 3
b.	$y = \frac{1}{x-5} + 2$	Rational		Rt 5 up 2
c.	$y = -\left[\frac{1}{2}(x-3)\right]^2 - 1$	Quadratics		Rt 3 Down 1 flip y horiz str $\frac{1}{4}$
d.	$g(x) = \sqrt{-(x+2)} - 7$	Square Root		flip y Rt 2 Down 7
e.	$y = \log(x+1)$	logarithm		Left 1

2 Solve each equation. Check your solutions to make sure they work.

a. $3\sqrt{x+4} - 8 = 7$

$3\sqrt{x+4} = 15$

$\sqrt{x+4} = 5$

$x+4 = 25$

$\boxed{x = 21}$

b. $\sqrt{4x+12} = 13$

$4x+12 = 169$

$\frac{4x}{4} = \frac{157}{4}$

$\boxed{x = \frac{157}{4}}$

c. $-3 = \sqrt{x-1}$

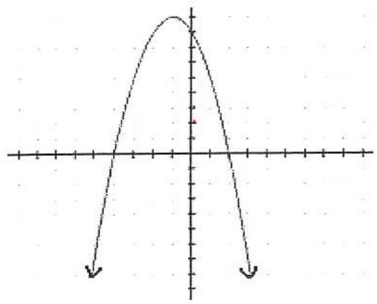
No solution

Square root cannot $= -3$

3 Tell whether each is a function or not. State the domain and range of each graph or table below.

a. D: $(-\infty, \infty)$

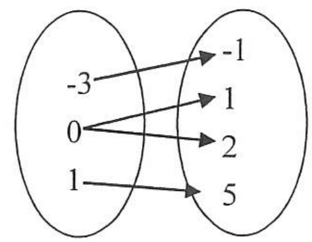
R: $(-\infty, 9]$



Function? yes

b. D: $\{-3, 0, 1\}$

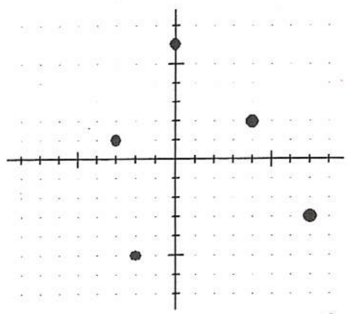
R: $\{-1, 1, 2, 5\}$



Function? No (repeated x)

c. D: $\{-3, -2, 0, 4, 7\}$

R: $\{-5, -3, 1, 2, 6\}$



Function? yes (discrete)

4 Consider the graph of $y = a(x-h)^2 + k$, describe the change in the graph for each of the following parameter changes

a. if the "a" value is changed from 1 to 4

vertical stretch 4

b. if the "h" value is changed from 1 to 4

Rt 3

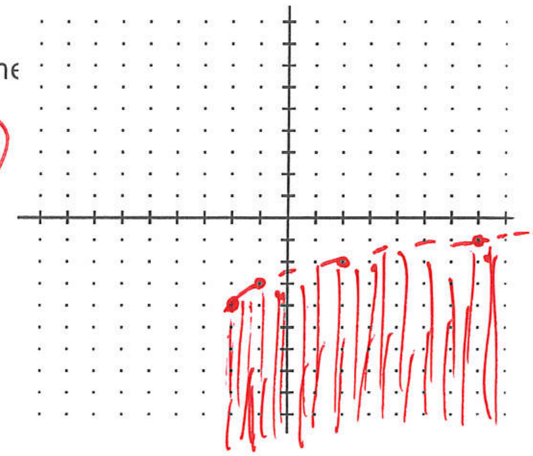
c. if the "k" value is changed from 1 to 4

up 3

5 Graph the inequality $y < \sqrt{x+2} - 4$.

Which point(s) listed below are solutions of the ine

~~(2, 5)~~, ~~(7, 0)~~, (2, -8), ~~(7, -1)~~, and/or (7, -6)



x	y
~~-4~~	
0	0
1	1
4	2
9	3

6 An inverse of a function takes the x and y-values of the original function and switches them; the x-values become y-values and the y-values become x-values. Using this definition, for part a, write the set of points for the inverse. For parts b-d, write the equation of the inverse.

a. { (-1, 3), (0, -4), (2, 6), (7, -5) }

{(3,-1), (-4,0), (6,2), (-5,7)}

b. $y = (x - 2)^2$

$\sqrt{x} = \sqrt{(y-2)^2}$

$\sqrt{x} = y-2$

$\boxed{\sqrt{x} + 2 = y}$

c. $y = \sqrt{x-4}$

$x^2 = \sqrt{y-4}^{\,2}$

$x^2 = y-4$

$\boxed{x^2 + 4 = y}$

d. $y = 2^{x-1} + 5$

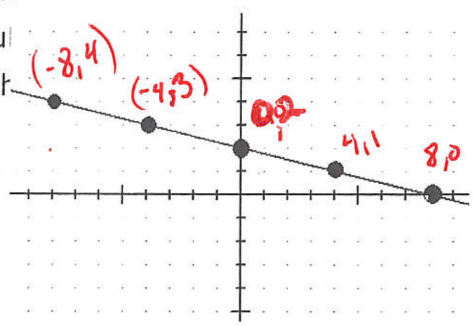
$x = 2^{y-1} + 5$

$\log_2(x-5 = 2^{y-1})$

$\log_2(x-5) = y-1$

$\boxed{\log_2(x-5) + 1 = y}$

7 Consider the graph on the right. Make a table of points that woul lie on the INVERSE of this graph

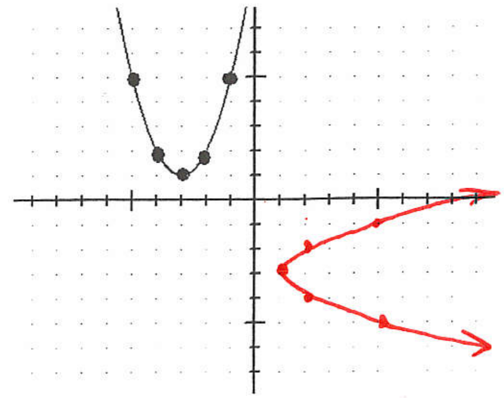


x	y
4	-8
3	-4
2	0
1	4
0	8

8 Use the points on the graph to the right, to graph the inverse of the quadratic on the same graph.

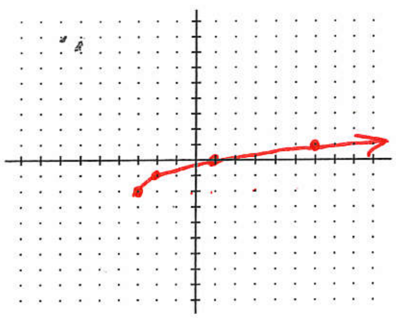
Then, complete the statement:

The inverse of a quadratic is two square roots functions.



9 Sketch a graph of the function. Then, write the domain & range for each equation.

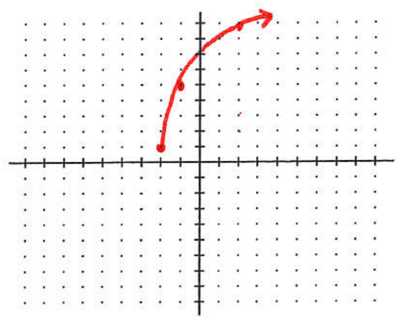
a. $y=\sqrt{x+3}-2$



D: $[-3,\infty)$

R: $[-2,\infty)$

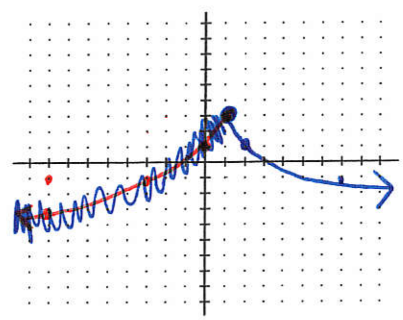
b. $y=4\sqrt{x+2}+1$



D: $[-2,\infty)$

R: $[1,\infty)$

c. $y=-2\sqrt{x-1}+3$



D: $[1,\infty)$

R: $(-\infty,3]$

10 The rate of population decline of Baytown is modeled by the equation $P(t)=15000e^{-0.0081t}$, where t is the time in years and 15,000 is the current population. In approximately how many years will the population of Baytown be 12,000?

$$\frac{12000}{15000}=\frac{15000\,e^{-.0081t}}{15000}$$

$$\ln\left(\frac{4}{5}\right)=\cancel{\ln e}^{-.0081t}$$

$$-.2231=-.0081t$$

$$\boxed{27.5486\approx t}$$

11 Consider the graph of the exponential function $y = e^x$.
Write an equation that would result in the following transformations:

a. Shift the graph 5 units right

$\boxed{y = e^{x-5}}$

b. Shift the graph 2 units left

$\boxed{y = e^{x+2}}$

c. Shift the graph down 4 units

$\boxed{y = e^{x} - 4}$

d. Shift the graph up 6 units

$\boxed{y = e^{x} + 6}$

e. Vertically stretch the graph by a scale factor of 2

$\boxed{y = 2e^{x}}$

f. Reflect the graph over the x-axis.

$\boxed{y = -e^{x}}$

12 Use the interest formulas $A = P\left(1+\frac{r}{n}\right)^{nt}$ and $A = Pe^{rt}$ to answer the following questions.

a. If you invest \$3000 in an account that offers 4.5% interest compounded monthly, how much money will you have in 12 years?

$A = 3000\left(1 + \frac{.045}{12}\right)^{12 \cdot 12}$

$\boxed{A = 5142.82}$

b. If you invest \$450 in an account that offers 3.8% compounded continuously, how long will it take to double your money?

$900 = 450\, e^{.038t}$

$\ln 2 = \cancel{\ln} e^{.038t}$

$\boxed{18.24 \approx t}$

c. John wishes to save \$5000 to use for a down payment on a car that he plans to buy in 5 years. About how much money should he deposit into a Money Market account now, if the account pays 6.25% interest compounded continuously, to earn enough money for his car down payment?

$5000 = P \cdot e^{.0625 \cdot 5}$

$\frac{5000}{1.3668} = \frac{1.3668 \cdot P}{1.3668}$

$\boxed{P = 3658.08}$

13 When you swim underwater, the pressure in your ears varies directly with the depth at which you swim. At 10 feet below the surface, the pressure is about 4.3 pounds per square inch (psi).

y

x

a. Write an equation that relates your ear pressure, p, to your swimming depth, d.

$$\frac{4.3}{10} = \frac{a \cdot 10}{10}$$

$a = .43$

$$\boxed{y = .43x}$$

b. Predict the pressure in your ears if you swim at 50 feet.

$y = .43 \cdot 50$

$$\boxed{y = 21.5}$$

c. It is unsafe for amateur divers to swim where the pressure is more than 65 psi.

How deep can an amateur diver safely swim?

$$\frac{65}{.43} = \frac{.43x}{.43}$$

$$\boxed{x = 151.16}$$

14 The height, h of a 1-liter orange juice box with a square base is a function of the width of the base, $h = \frac{1000}{w^2}$ where w is the width of the base in centimeters. In order to optimize cost a manufacturer needs to make a box that is between 20 and 30 centimeters tall. Determine the smallest and largest possible widths of the box.

h

$30 = \frac{1000}{w^2}$ $\boxed{w = 5.77}$

$\frac{30w^2}{30} = \frac{1000}{30}$

$w^2 = \frac{1000}{30}$

$20 = \frac{1000}{w^2}$

$20w^2 = 1000$

$w^2 = 50$

$$\boxed{w = 7.07}$$

15 Use log properties to simplify:

a. $3\log_4(x) + \log_4(2)$

$\log_4 x^3 + \log_4 2$

$$\boxed{\log_4 2x^3}$$

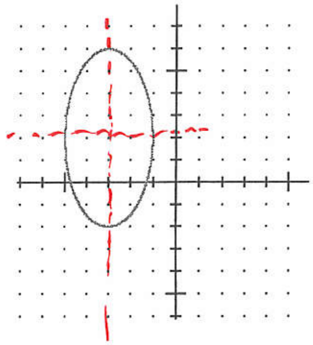
b. $2\log_3(4) - \frac{1}{2}\log_3(y)$

$\log_3 4^2 - \log_3 (y)^{\frac{1}{2}}$

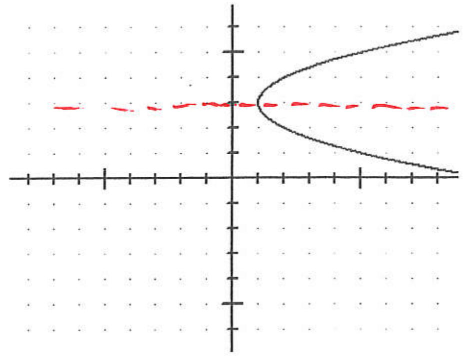
$$\boxed{\log_3 \frac{16}{y^{\frac{1}{2}}}}$$

16 State the equation(s) of all lines of symmetry of the conic sections below.

a. $y = -3$ $x = 2$



b. $y = 3$



17 The total cost to take x number of students on a field trip to Sea World is given by the function $C(x) = 25x + 300$.

a. Write an equation to represents the cost per student?

$$\boxed{\frac{25x+300}{x}}$$

b. What is the cost per student if 40 students go on the field trip?

$$\frac{25(40)+300}{40} = \boxed{32.50}$$

c. How many students need to go to keep the cost per student under \$30?

$$\frac{25x+300}{x} < 30$$

$$25x + 300 < 30x$$
$$300 < 5x$$
$$60 < x$$

$$\boxed{x > 60}$$

18 From 1990 to 2001, the number d (in thousands) of doctors in the United States can be modeled by the function $d = \dfrac{950t^2 + 52{,}300}{t^2 + 80.1}$ where t is the number of years since 1990. During which years were there fewer than 800,000 doctors?

$$\frac{800}{1} = \frac{950t^2 + 52{,}300}{t^2 + 80.1}$$

$$800t^2 + 64080 = 950t^2 + 52.300$$
$$-800t^2 - 52300 \quad -800t^2 - 52{,}300$$

$$\frac{11780}{150} = \frac{150t^2}{150}$$

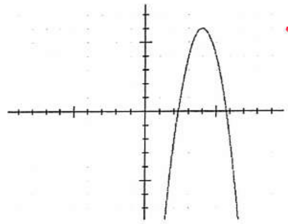
$$78.533 = t^2$$

$$\boxed{t = 8.86}$$ ✓

19 Name the type of function graphed, state the parent equation, dash in and write the equation(s) for any asymptotes.

a.

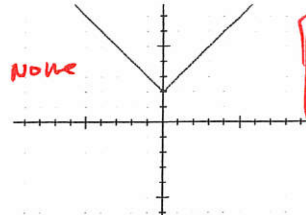
None



$\boxed{y = x^2}$

b.

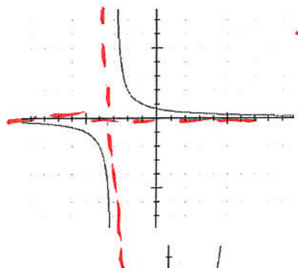
None



$\boxed{y = |x|}$

c.

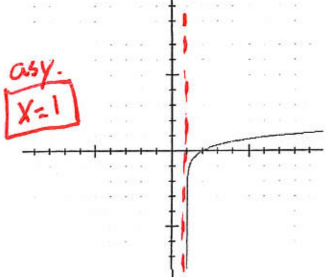
asym $\boxed{x = -3 \quad y = 0}$



$\boxed{y = \frac{1}{x}}$

d.

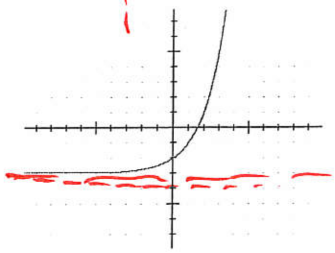
asy. $\boxed{x = 1}$



$\boxed{y = \log_2 x}$

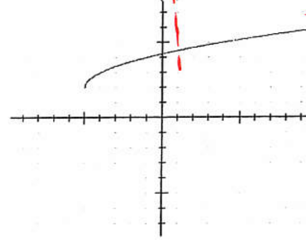
e.

$\boxed{\text{asym } y = -3}$



$\boxed{x = 2^x}$

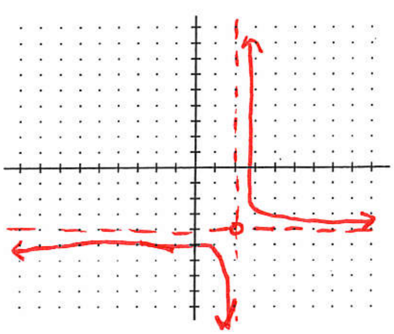
f.



$\boxed{y = \sqrt{x}}$

20 Graph each rational function. Then, state the equations of the asymptotes for each.

a. $y = \frac{1}{(x-2)} - 4$



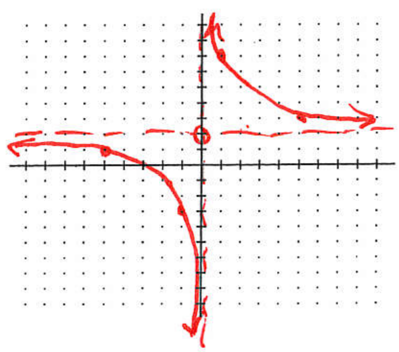
vertical: $x = 2$

horizontal: $y = -4$

Domain: $(-\infty, 2) \cup (2, \infty)$

Range: $(-\infty, -4) \cup (-4, \infty)$

b. $f(x) = 2 + \frac{5}{x}$



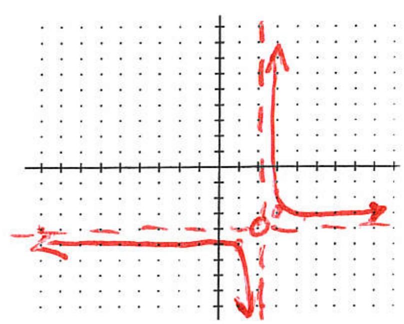
vertical: $x = 0$

horizontal: $y = 2$

Domain: $(-\infty, 0) \cup (0, \infty)$

Range: $(-\infty, 2) \cup (2, \infty)$

c. $y = \frac{1}{(x-2)} - 4$



vertical: $x = 2$

horizontal: $y = -4$

Domain: $(-\infty, 2) \cup (2, \infty)$

Range: $(-\infty, -4) \cup (-4, \infty)$

21 Use exponent properties to simplify the expressions.
Your final answer should not have any negative or zero exponents.

a. $\left(a^3b^2c^{-2}\right)^4$

$a^{12}b^{8}c^{-8}$

$\boxed{\dfrac{a^{12}b^{8}}{c^{8}}}$

b. $\dfrac{\left(5xy^{-7}\right)^3 \cdot -4x^2y^3}{2x^{-7}y^{12}}$

$\dfrac{125x^3y^{-21} \cdot -4x^2y^3}{2x^{-7}y^{12}}$

$\dfrac{-500x^{12}}{2y^{30}} = \boxed{\dfrac{-250x^{12}}{y^{30}}}$

c. $\left(\dfrac{10x^{12}y^{-17}}{35x^5y^{-4}}\right)^2$

$\dfrac{100x^{24}y^{-34}}{1225x^{10}y^{-8}}$

$\boxed{\dfrac{4x^{14}}{49y^{26}}}$

d. $\left(5x^2\right)^0\left(x^3y^4\right)^{-2}$

$\boxed{\dfrac{1}{x^6y^8}}$

e. $\dfrac{24p^3q^{-2}}{8p^{-5}q^8}$

$\boxed{\dfrac{3p^8}{q^{10}}}$

22 Cell phone technology expanded rapidly in the United States toward the end of the 20th century. The approximate number of cell phone subscribers can be modeled by $y = 0.01(e^{0.46x})$ where x is the number of years since January 1980 and y is the number of subscribers in millions.

a. What is the domain of the equation $y = 0.01(e^{0.46x})$?

$(-\infty, \infty)$

b. What is the domain of the problem situation described above?

$(0, 20)$ 1980 - 2000
20th century

c. Are the domains the same or different? Explain why.

different
years since 1980

23 Write a log base 3 equation that has a vertical stretch of 5 and has an asymptote at x = 4

$\boxed{5\log_3 (x-4)}$

24 The number of hulu subscribers since 2010 can be modeled by the equation $H(x) = .2e^{.5x}$ where x is the number of years since 2010. The number of subscribers to cable grew during the same time period by the equation C(x) = 1.2x + 86. Based on these two models, when will the number of Hulu subscribers surpass the number of cable subscribers?

$.2e^{.5x} = 1.2x + 86$

$y_1 \qquad y_2$

$x = 12.45$

$\boxed{2022}$

Simplify

25 $$\frac{(x^2+10x+25)}{2x^2+10x} * \frac{40x^3}{8x+40}$$

$$\frac{\cancel{(x+5)}\cancel{(x+5)}}{2x\cancel{(x+5)}} \cdot \frac{40x^3}{8\cancel{(x+5)}} = \boxed{\frac{5x^2}{2}}$$

26 $$\frac{(11a-2)}{a^2-4a-12} - \frac{8\,(a+2)}{a-6}$$

$(a-6)(a+2)$

$$\frac{(11a-2)-(8a+16)}{(a-6)(a+2)} = \frac{3a-18}{(a-6)(a+2)} = \frac{3\cancel{(a-6)}}{\cancel{(a-6)}(a+2)} = \frac{3}{(a+2)}$$

27 Solve.

$$\frac{4(2x+4)}{9} + 1 = 3(X-5) - 4$$

$\frac{8x+16}{9} + 1 = 3x - 15 - 4$

$\frac{8x+16}{9} + 1 = 3x - 19$

$\frac{8x+16}{9} = 3x - 20$

$27x - 180 = 8x + 16$

$19x = 196$

$\boxed{x = \frac{196}{19}}$ ✓

Solve by factoring

28 $y = -4x^2 + 5x - 3$

$a = -4$
$b = 5$
$c = -3$

$$\frac{-5 \pm \sqrt{5^2 - 4 \cdot -4 \cdot -3}}{2(-4)}$$

$$\frac{-5 \pm \sqrt{-23}}{-8} = \boxed{\frac{-5 \pm i\sqrt{23}}{-8}}$$

29 $x^2 - 5x + 10 = 4$

$x^2 - 5x + 6$

$(x-2)(x-3) = 0$

$x-2=0 \quad x-3=0$

$\boxed{x=2 \quad x=3}$

30 $3x^2 + 7x - 24 = 13x$

$3x^2 - 6x - 24 = 0$

$3(x^2 - 2x - 8) = 0$

$3(x- \quad)(x+2) = 0$

$x-4=0 \quad x+2=0$

$\boxed{x=4} \boxed{x=-2}$

Factor

31 $36m^5n^2 - 54m^3n$

$\boxed{18m^3n(2m^2n - 3)}$

32 $4x^2yz + 6xy^2z^5$

$2xyz(2x + 3yz^4)$

$\log_5(15) - \log_5(15) = 0$

32 Solve: $\log_5(6x+3) - \log_5(4x^2-1) = 0$

$\cancel{\log_5} \dfrac{6x+3}{4x^2-1} = \cancel{5}^0$

$\dfrac{6x+3}{4x^2-1} = 1$

$\dfrac{3\cancel{(2x+1)}}{\cancel{(2x+1)}(2x-1)} = 1$

$\dfrac{3}{2x-1} = \dfrac{1}{1}$

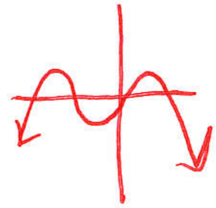
$3 = 2x - 1$

$4 = 2x$

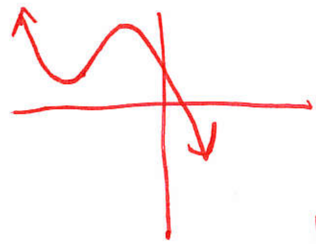
$\boxed{x=2}$

Sketch a graph of the following:

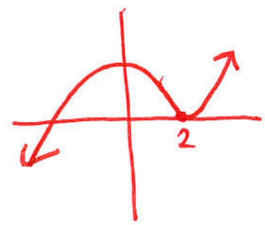
33 4^{th} degree polynomial with 4 real roots



34 Polynomial with 2 imaginary root



35 Polynomial with a double root at x=2



36 A polynomial has zeros at x=-3 and x=5 and a double root at x=6. What is the factored form of the polynomial?

A) (x+3) (x-5) $(x-6)^2$

B) (x-3) $(x+5)^2$ (x-6)

C) $(x+3)^2$ $(x-5)^2$ $(x-6)^2$

D) (x+3) + (x-5) + $(x-6)^2$

37 Which set of factors corresponds to a third-degree polynomial with rational coefficients that has zeros at x=2 and x=5i

A) (x-2) (x^2+25)

B) (x+2) (x^2-25)

C) (x-2) (x^2+5i)

D) (x-2) $(x-5i)$