L 4

Name _____ Date _____ Pd_____

Review for Polynomials Unit 2 Test 3-4

LESSON 17-1

41. Factor each polynomial.

a. $x^2 - 7x + 12$

$(x-3)(x-4)$

b. $3x^2 + x - 10$

$(3x-5)(x+2)$

c. $3x^4 + 2x^2 - 5$

$(3x^2+5)(x-1)(x+1)$

d. $x^2 + 5x - 36$

$(x+9)(x-4)$

42. Factor by grouping.

a. $2x^3 - 6x^2 + 5x - 15$

$2x(x-3)+5(x-3)$

$(2x+5)(x-3)$

b. $3x^4 - x^3 + 6x - 2$

$x^3(3x-1)+2(3x-1)$

$(x^3+2)(3x-1)$

c. $x^3 + 5x^2 - 9x - 45$

$x^2(x+5)-9(x+5)$

$(x^2-9)(x+5) = (x+3)(x-3)(x+$

d. $x^3 - 5x^2 - 3x + 15$

$x^2(x-5)-3(x-5)$

$(x^2-3)(x-5)$

43. Factor each sum or difference of cubes.

a. $x^3 + 125$

$(x+5)(x^2-5x+25)$

c. $8x^3 + 216$

$(2x+6)(4x^2-12x+36)$

d. $64x^3 - 27$

$(4x-3)(16x^2+12x+9)$

44. Use the formulas for factoring quadratic binomials and trinomials to factor each expression.

a. $25x^4 - 169$

$(5x^2-13)(5x^2+13)$

c. $x^4 - 10x^2 + 25$

$(x^2-5)^2$

d. $4x^{10} - 81$

$(2x^5-9)(2x^5+9)$

LESSON 17-2

46. Find the zeros of the functions. Show that the Fundamental Theorem of Algebra is true for each function by counting the number of complex zeros.

a. $f(x) = x^3 + 4x$

$f(x) = x^2(x+4)$

$\sqrt{0} = \sqrt{x^2}$ $\quad x+4=0$

$\boxed{x=0}$ $\quad \boxed{x=-4}$

b. $g(x) = x^4 - 81$

$(x^2-9)(x^2+9)$

$(x+3)(x-3)(x^2+9)$

$\boxed{x=-3 \quad x=3}$ $\quad x^2+9=0$, $\sqrt{x^2}=\sqrt{-9}$, $\boxed{x=\pm 3i}$

c. $h(x) = 2x^4 - 16x^3 + 32x^2$

$2x^2(x^2-8x+16)$

$2x^2(x-4)^2$

$\frac{2x^2}{2}=\frac{0}{2}$, $x^2=0$ $\quad \sqrt{(x-4)^2}=\sqrt{0}$, $x-4=0$ $\quad \boxed{x=4 \quad x=0}$

9. Which is the degree of the polynomial function with the roots $x = -3$, $x = \frac{2}{3}$, $x = i$, and $x = 1 - i$?

A. 4 B. 5 C. 6 D. 8

47. Write a polynomial in factored form of the n-th degree and has the given roots.

a. $n = 3$; zeros: $-1, 0, 2$

$\boxed{y = x(x+1)(x-2)}$ factored form

$x(x^2-x-2)$

$\boxed{y = x^3 - x^2 - 2x}$ Expanded form

b. $n = 4$; zeros: $-3, 2, \pm 1$

$\boxed{y = (x+3)(x-2)(x+1)(x-1)}$ factored

$(x^2+x-6)(x^2-1)$

	x^2	x	-6
x^2	x^4	x^3	$-6x^2$
-1	$-x^2$	$-x$	$+6$

$\boxed{y = x^4 + x^3 - 7x^2 - x + 6}$ Expanded

c. $n = 3$; $x = -1$, and $x = 3$ is a double root

$\boxed{y = (x+1)(x-3)^2}$

$(x+1)(x^2-6x+9)$

	x^3	$-6x^2$	$9x$
x	x^3	$-6x^2$	$9x$
$+1$	x^2	$-6x$	$+9$

$\boxed{y = x^3 - 5x^2 + 9x + 9}$

50. Write a polynomial in factored and expanded (standard) Form that goes through the given roots and the given points.

a. $n = 3$; $x = 2, x = 3i$ goes through $(3,-10)$

$y = a(x-2)(x-3i)(x+3i)$

$(x-2)(x^2-9i^2)$

$y = a(x-2)(x^2+9)$

$-10 = a(1)(18)$, $\frac{-10}{18} = \frac{a(18)}{18}$, $-\frac{5}{9} = a$

$\boxed{f(x) = -\frac{5}{9}(x-2)(x^2+9)}$

b. $n = 4$; $x = -i, x = 1 + 3i$ goes through $(3,20)$

$y = a(x-i)(x+i)(x-1+3i)(x-1-3i)$

$y = a(x^2-i^2)$

$y = a(x^2-(-1))$

$y = a(x^2+1)(x^2-2x+10)$

$20 = a(9+1)(9-6+10)$

$20 = a(10)(13)$, $\frac{20}{130} = \frac{a(130)}{130}$, $\frac{2}{13} = a$

x	x^2	$-x$	2
-1	$-x$	$+1$	2
$-3i$			$-9i^2 = -9(-1) = 9$

50a Expanded

$f(x) = -\frac{5}{9}(x-2)(x^2+9)$

$f(x) = -\frac{5}{9}(x^3-2x^2+9x-18)$

$\boxed{f(x) = -\frac{5}{9}x^3 + \frac{10}{9}x^2 - 5x + 10}$

50b Factored form

$\boxed{y = \frac{2}{13}(x^2+1)(x^2-2x+10)}$

$y = \frac{2}{13}(x^4-2x^3+11x+10)$

$\boxed{y = \frac{2}{13}x^4 - \frac{4}{13}x^3 + \frac{22}{13}x + \frac{20}{13}}$

	x^4	$-2x^3$	$10x^2$
x^2	x^4	$-2x^3$	$10x^2$
$+1$	x^2	$-2x$	10

LESSON 18-1

51. *Match each equation to its graph.*

a. $f(x) = \frac{1}{3}x - 2$ I

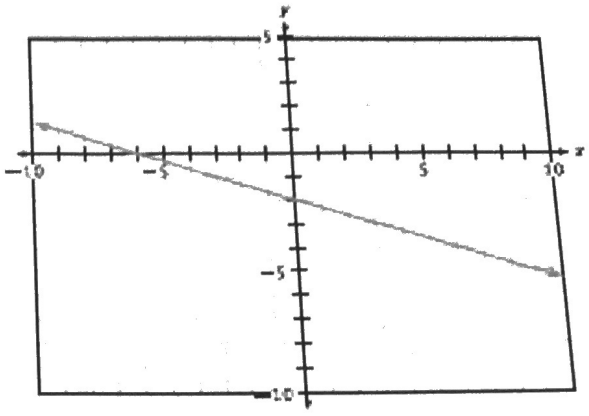
b. $g(x) = x^2 - 3$ V

c. $h(x) = 2x^3 - 3x^2 + 1$ III

d. $j(x) = \frac{1}{2}x^4 - 3x^2 + 4$ II

e. $k(x) = \frac{1}{2}x^5 - x^4 - x^3 - x^2 - x$ IV

I.



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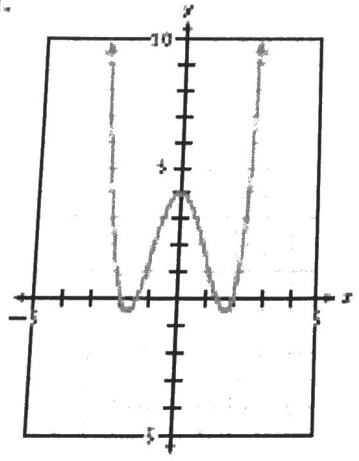

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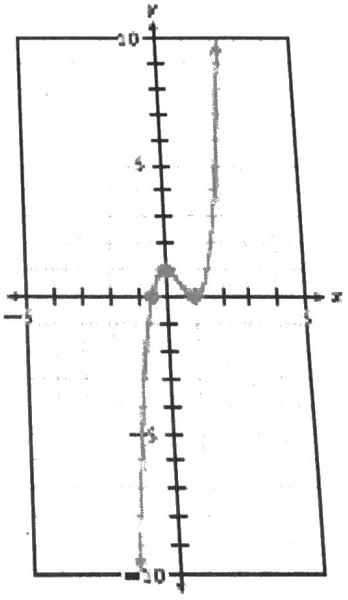

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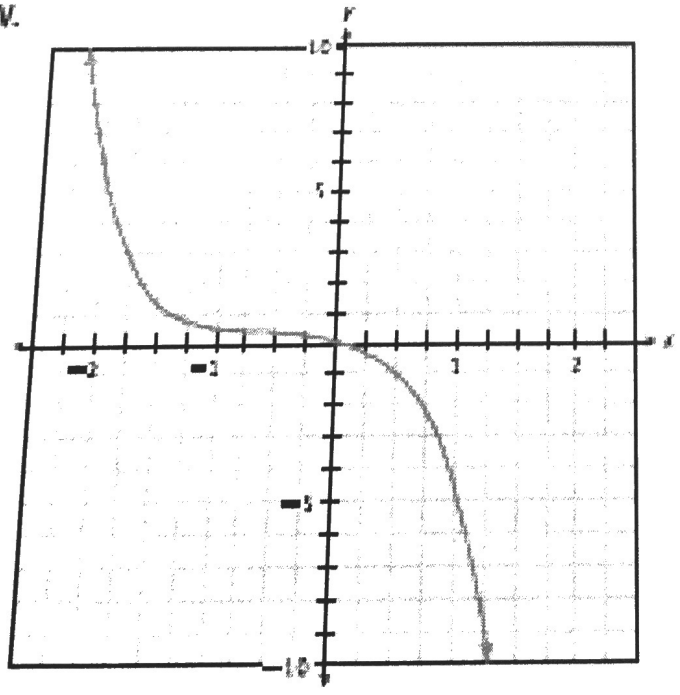
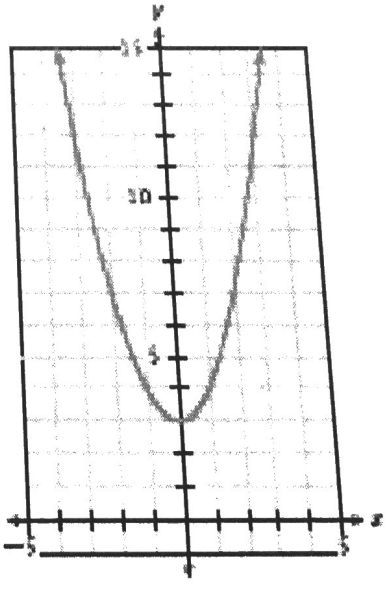


V.



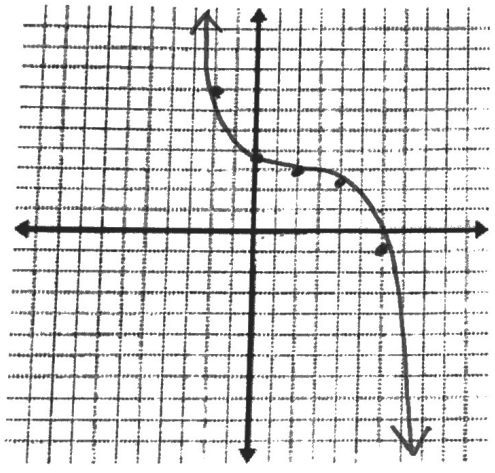
55.

$f(x) = -\frac{1}{2}\left[\frac{1}{2}(x-2)\right]^3 + 3$ Start pt (2, 3)

	2x	$-\frac{1}{2}y$	
-2	-4	4	-8
-1	-2	$\frac{1}{2}$	-1
0	0	0	0
1	2	$-\frac{1}{2}$	1
2	4	-4	8

Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$



58. Make use of structure. Find the number of positive and negative real roots of each equation. Explain.

a. $h(x) = x^3 - x^2 - 3x - 5$

b. $j(x) = 5x^4 - 2x^3 + 3x^2 + 10x - 5$

59. Given $k(x) = x^3 - 2x^2 - 5x + 6$:

a. Find the real zeros of $k(x)$.

Possible roots: $1, 2, 3, 6 \Rightarrow \pm 1, \pm 3, \pm 3, \pm 6$

1	1	-2	-5	6
	↓	1	-1	-6
	1	-1	-6	0

$x^2 - 1x - 6$ ← factor → $(x-3)(x+2)$

$f(x) = (x-1)(x-3)(x+2)$

$x = 1 \quad x = 3 \quad x = -2$

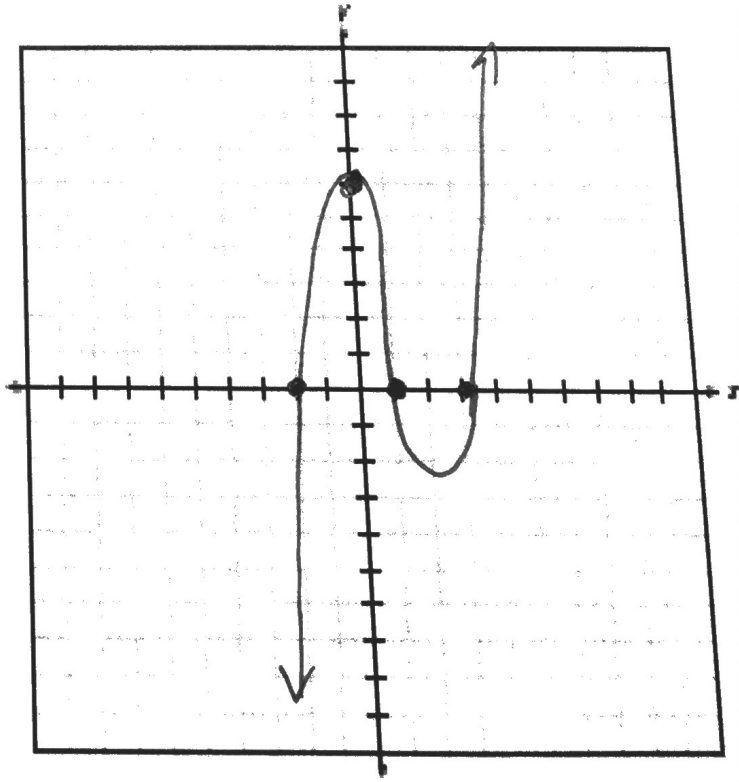
roots

b. What is the y-intercept?

$y = 6$

c. Find the relative maximum and minimum to the nearest integer.

d. Graph $k(x)$ by hand.



LESSON 18-2

56. Find all possible rational roots of each equation.

a. $j(x) = 3x^3 - 5x^2 - 4x + 5$

factors of last / factors of first: $\frac{1, 5}{1, 3} \Rightarrow \pm 1, \pm 5, \pm \frac{1}{3}, \frac{5}{3}$

b. $g(x) = 2x^4 + 7x^3 - 3x^2 + 5x - 6$

1,

57. Given $p(x) = -x^5 + 6x^4 - 3x^3 - 5x^2 + 3x - 7$:

a. How many sign changes are there?

b. How many possible positive real roots are there?

c. Find $p(-x)$.

d. How many negative real roots are there?