

Rewrite the equation as a logarithm or exponent, or simplify the following using your logarithm properties.

$$1. \quad 2\log_4 6 + (2\log_4 5 - \log_4 5)$$

$$\log_4 6^{2^4} + \log_4 5^{2^4} - \log_4 5$$

$$\log_4 \frac{36 \cdot 25^5}{5} = \boxed{\log_4 180}$$

$$2. \quad \log_2 x + \log_2(x+8) - \frac{1}{3}\log_3 27$$

$$\log_2 x(x+8) - \log_3 (3^3)^{1/3}$$

$$\log_2(x^2+8x) - \log_3 3$$

$$\log_2(x^2+8x) - 1$$

$$3. \quad \frac{1}{5}\log_4 32 - \log_4 16 + 2\log_4 7$$

$$\log_4 (2^5)^{1/5} - \log_4 16 + \log_4 7^2$$

$$\log_4 \frac{2 \cdot 49}{16} = \boxed{\log_4 \frac{49}{8}}$$

$$4. \quad \log_9 z + \log_9(z-1) + \frac{2}{3}\log_9 27 \quad 3 \cdot \frac{2}{3} = 2$$

$$\log_9 z(z-1) + \log_9 (3^3)^{2/3}$$

$$\log_9(z^2-z) + \log_9 3^2$$

$$\log_9 9(z^2-z) = \boxed{\log_9(9z^2-9z)}$$

Expand the following

$$5. \quad \frac{\ln x(x-4)}{y^4} \ln \frac{x(x-4)}{y^4}$$

$$\ln x + \ln(x-4) - \ln y^4$$

$$\boxed{\ln x + \ln(x-4) - 4\ln y}$$

$$6. \quad \log_4 \frac{5x}{z^6}$$

$$\boxed{\log_4 5 + \log_4 x - 6\log_4 z}$$

Put in exponential form

Put in Log form

$$7. \quad \log_{\frac{9}{2}} r = 12x$$

$$r = \frac{9}{2}^{12x}$$

$$8. \quad a^{x-7} = \frac{1}{3}$$

$$\log_a a^{x-7} = \log_a \frac{1}{3}$$

$$\boxed{x-7 = \log_a \frac{1}{3} \quad \text{or} \quad x = \log_a \frac{1}{3} + 7}$$

Solve each equation or evaluate each expression. Show your work.
Round all answers three decimal places.

9. $\log_4 x + \log_4 (x - 15) = \log_4 36$

$4^{\log_4 x(x-15)} = 4^{\log_4 36}$

$x^2 - 15x = 36$

$x^2 - 15x - 36 = 0$

$(x + 1)(x - 16) = 0$

$x = -1$
 $x = 16$

10. $\log_3 3 + \log_3 x = \log_3 24$

$3^{\log_3 3x} = 3^{\log_3 24}$

$3x = 24$

$x = 8$

11. $\log_3 (13x - 36) = 2 \log_3 x^2$

$13x - 36 = x^2$

$0 = x^2 - 13x + 36$

$0 = (x - 9)(x - 4)$

$x = 9, x = 4$

12. $2 \log x = 2 \log 7 - \log 3 + \log 12$

$10^{2 \log x} = 10^{2 \log 7 - \log 3 + \log 12}$

$\sqrt{x^2} = \sqrt{196}$

$x = \pm 14$

$x = 14$

$\frac{7^2 \cdot 12}{3} = \frac{49 \cdot 12}{3}$

\downarrow
 $\frac{49 \cdot 4}{3} = \frac{196}{3}$

13. $\log_x 4 = \frac{1}{3}$

$4 = x^{1/3}$

$(4)^3 = (x^{1/3})^3$

$64 = x$

14. $\log_7 x + \log_7 (x - 12) = 2 \log_7 8$

$7^{\log_7 x(x-12)} = 7^{2 \log_7 8}$

$x^2 - 12x = 64$

$x^2 - 12x - 64 = 0$

$(x - 16)(x + 4) = 0$

$x = 16, -4$

$x = 16$

15. $\log_3 x + \log_3 (x + 9) = 3 \log_3 3$

$3^{\log_3 x(x+9)} = 3^{\log_3 3^3}$

$x^2 + 9x = 27$

$x^2 + 9x - 27 = 0$

$x = \frac{-9 \pm \sqrt{81 - 4(1)(-27)}}{2(1)} = \frac{-9 \pm \sqrt{189}}{2}$
 $x = \frac{-9 \pm 13.75}{2}$

$3^3 = 27$
 $3^2 = 9$
 $3^1 = 3$
 $3^0 = 1$

16. $\ln x + \ln(x + 7) = \frac{1}{2} \ln 100$

$\ln x(x + 7) = \ln 100^{1/2}$

$e^{\ln x(x+7)} = e^{\ln 10}$

$x^2 + 7x = 10$

$x^2 + 7x - 10 = 0$

$(x + 5)(x - 2) = 0$

$x = 5$
 $x = 2$

$100^{1/2}$

$\sqrt{100} = 10$

17. $\log_5 (x + 6) + \log_5 (x - 6) = \log_4 16$

$5^{\log_5 (x+6)(x-6)} = 5^2$

$x^2 - 36 = 25$

$x^2 - 61 = 0$

$x^2 - 61 = 0$

$\sqrt{x^2} = \sqrt{61}$

$x = \pm \sqrt{61}$

18. $\log(2x) + \log(x + \frac{35}{2}) = 2$

Do on separate paper.

$x = \frac{5}{2}$

19. Given the logarithmic function $\frac{1}{3}\log_2 x + 5\log_6 y + \sqrt{7}\log_6 w - 9\log_2 z$, a student simplified it down to $\log_{12} \frac{x^3 y^5 w^{\sqrt{7}}}{z^9}$. Please explain the error made in simplifying (if any) and provide the correct simplified answer.

The bases must match to use the log rules.

20. Use the graph of the parent function $y = \log_2(x)$ to answer the following questions.

- a) State the domain of this function. $(0, \infty)$
 b) State the range of this function. $(-\infty, \infty)$
 c) Write the equation of the asymptote. $x = 0$

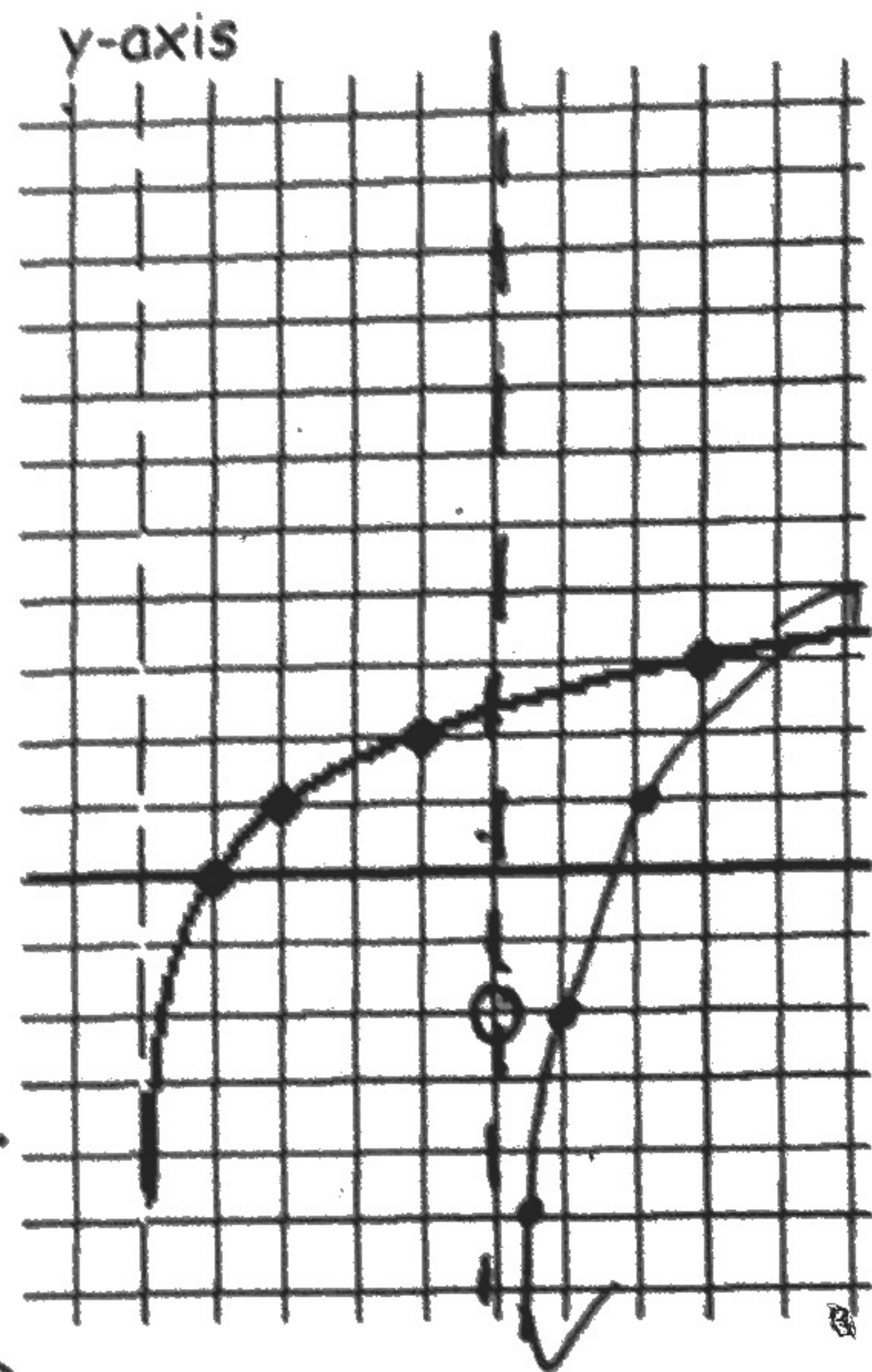
If the parent function changed by shifting down 2 and left 5 and stretching by 3:
 right 5

d) Graph the new function. Use the same coordinate axes.

e) Write the equation of the new function. $y = 3\log_2(x-5) - 2$

f) State the new domain & range. D: $(5, \infty)$ R: $(-\infty, \infty)$

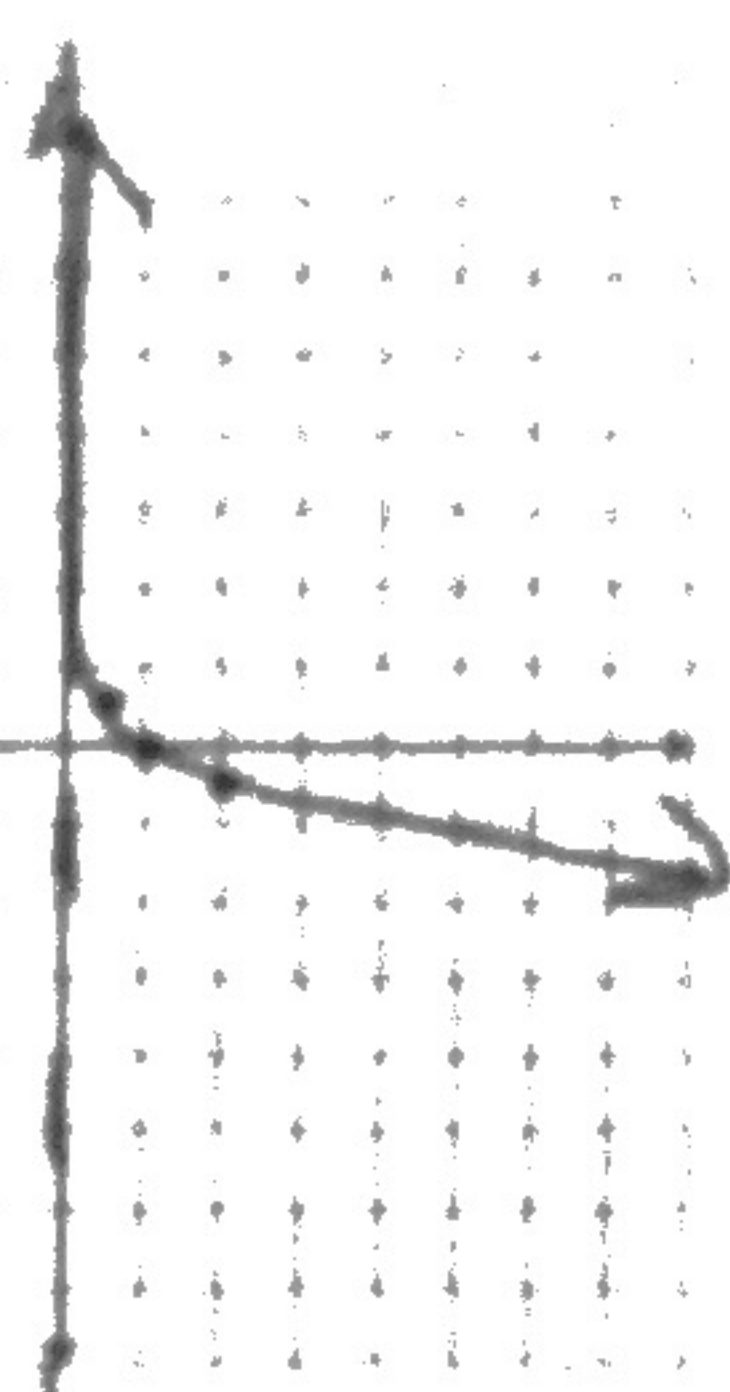
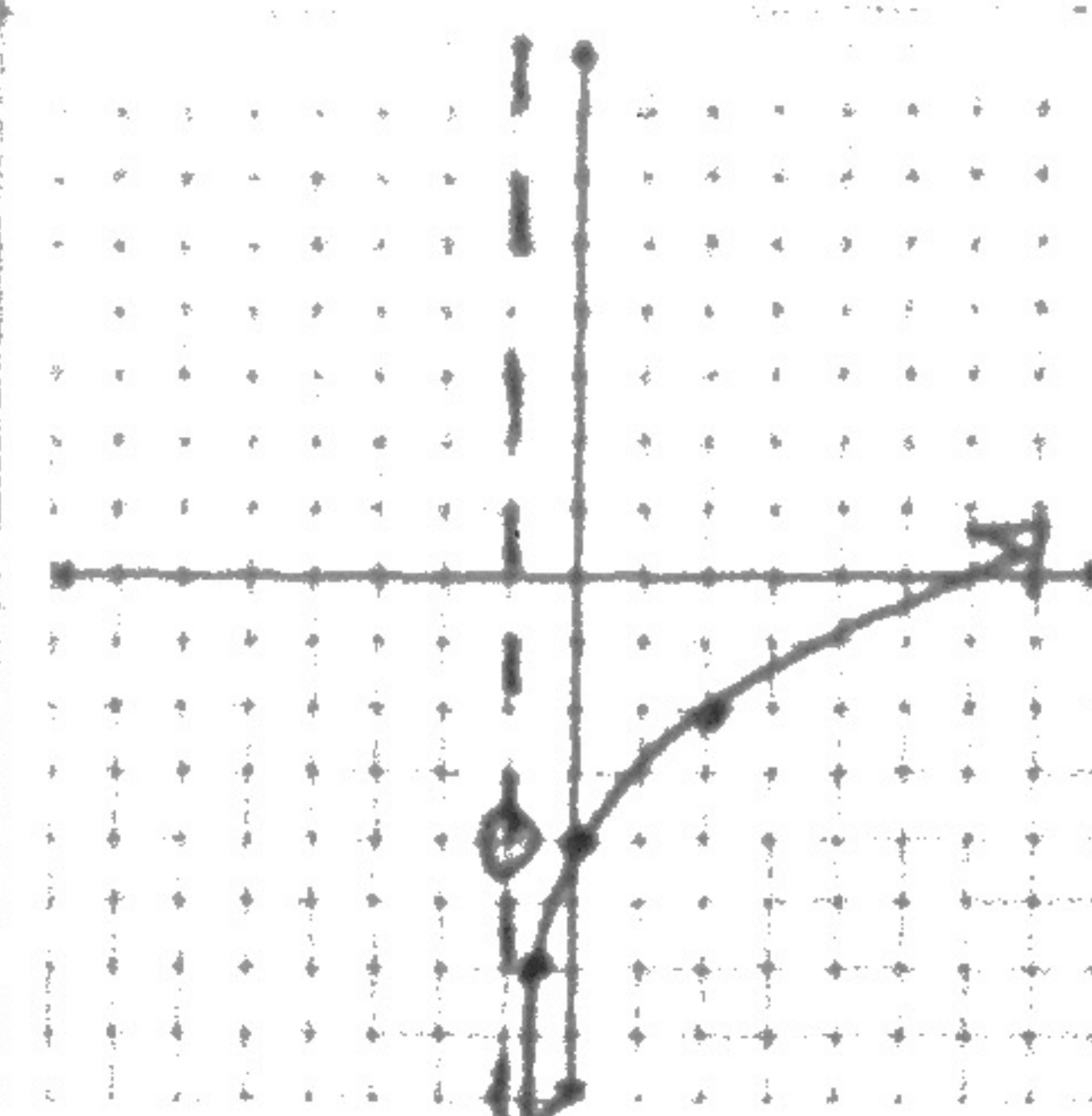
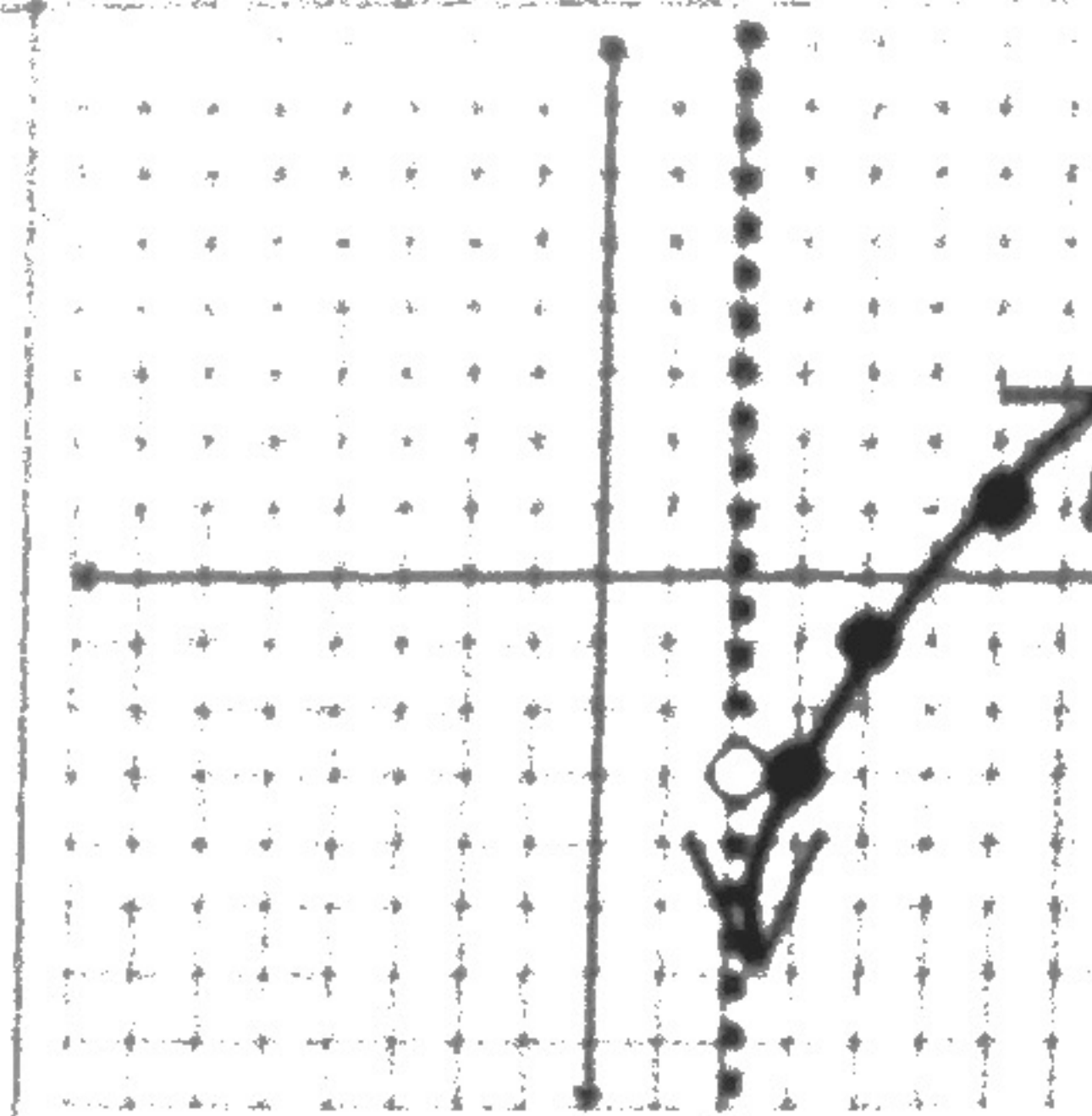
g) Write the equation of the new asymptote. $x = 5$



x	y
1/2	-1(3) = -3
1	0(3) = 0
2	1(3) = 3

* Changed the question!

21. Fill in the boxes with the correct equations, graphs, domain, range and equation of asymptote given the provided information.

Equation	$y = -\frac{1}{2} \log_2 x$	$y = 2 \log_3 (x+1) - 4$	$y = 2 \log_2 (x-2) - 3$
Equation 2			
Graph			
Domain & Range	D: $(0, \infty)$ R: $(-\infty, \infty)$	D: $(-1, \infty)$ R: $(-\infty, \infty)$ base = 3 With a b ² of 9, a vertical stretch of 2 and a vertical shift down 4.	D: $(2, \infty)$ R: $(-\infty, \infty)$
Equation of Asymptote	$x = 0$	$x = -1$	$x = 2$

x	y(2)
1/3	-1(2) = -2
1	0(2) = 0
3	1(2) = 2

Find the inverse

22. $y = -3 \log_4 (x+8) - 9$

$x = -3 \log_4 (y+8) - 9$

$4^{\frac{x+9}{-3}} = \log_4 (y+8)$
 $4^{-\frac{x+9}{3}} = y+8$
 $y = 4^{-\frac{x+9}{3}} - 8$

23. $y = 7^{3x+8}$
 $x = 7^{3y+8}$

$\log_7 x = \log_7 7^{3y+8}$
 $\log_7 x = 3y+8$
 $\frac{\log_7(x) - 8}{3} = y$
 $y = \frac{\log_7(x) - 8}{3}$

24. $y = \frac{4}{7} \ln(x+2) - 8$

$x = \frac{4}{7} \ln(y+2) - 8$

$e^{\frac{7}{4}(x+8)} = \ln(y+2)$
 $e^{\frac{7}{4}(x+8)} = y+2$
 $y = e^{\frac{7}{4}(x+8)} - 2$

25. $y = e^{7x-2}$
 $x = e^{7y-2}$

$\ln x = \ln e^{7y-2}$
 $\ln x = 7y-2$
 $\frac{\ln x + 2}{7} = y$
 $y = \frac{\ln x + 2}{7}$

Solve each equation or evaluate each expression. Show your work.
Round all answers three decimal places.

1. $\log_{16} x = 2.5$

$$x = 16^{2.5}$$

$$x = 1,024$$

2. $\log_5 125 = x$

$$125 = 5^x$$

$$5^3 = 5^x$$

$$3 = x$$

3. $\frac{3(4)^{x+5}}{3} = \frac{210}{3}$

$$4^{x+5} = 70$$

$$\log_4 4^{x+5} = \log_4 70$$

$$x+5 = \frac{3.064}{-5}$$

$$x = -1.935$$

4. $\frac{2.6^{4a-1}}{-5} + \frac{5}{-5} = \frac{11.37}{-5}$

$$2.6^{4a-1} = 6.37$$

$$\log_{2.6} 2.6^{4a-1} = \log_{2.6} 6.37$$

$$\frac{4a-1}{+1} = \frac{1.938}{+1}$$

$$\frac{4a}{4} = \frac{2.938}{4} \quad a = .735$$

5. $\frac{4e^{2x}}{4} = \frac{232}{4}$

$$e^{2x} = 58$$

$$\ln e^{2x} = \ln 58$$

$$\frac{2x}{2} = \frac{4.060}{2}$$

$$x = 2.030$$

6. $\log_5(4x - 24) - 2\log_5 3 = \log_5(6x) - \log_5 6$

$$5^{\log_5 \frac{4x-24}{3^2}} = 5^{\log_5 \frac{6x}{6}}$$

$$9 \cdot \frac{4x-24}{9} = x \cdot 9$$

$$\begin{array}{r} 4x - 24 = 9x \\ -4x \quad -4x \\ \hline -24 = 5x \end{array}$$

No Solution

$$\frac{-24}{5} = \frac{5x}{5} \quad x = -\frac{24}{5}$$

7. A bowl in Mrs. Jackson's kitchen contains common yeast. The yeast has been found to double every hour. Mrs. Jackson starts with 15 yeast in the bowl.
- Find the formula.
 - How much yeast will she have after 5.5 hours
 - How long until she has 300 yeast? Solve using logs. Show all work, as illustrated in class.
 - How long will it take to triple the amount of yeast?

a) $A(t) = 15 \cdot 2^t$

b) $A(5.5) = 15 \cdot 2^{5.5}$
1678.823 yeast

c) $\frac{300}{15} = \frac{15 \cdot 2^t}{15}$ $\rightarrow 20 = 2^t$
 $\log_2 20 = \log_2 2^t$
4.322 = t
4.322 hours

d) $\frac{45}{15} = \frac{15 \cdot 2^t}{15}$
 $3 = 2^t$
 $\log_2 3 = \log_2 2^t$
 $1.585 = t$
1.585 hours

8. The equation $y = 580e^{0.495x}$ represents the number of wood ants in a rotting tree. The independent variable, x , represents the number of years since 2005. Using the provided equation, approximately when will there be 5000 wood ants in the rotten tree? Use **LOGS TO SOLVE**. Show all work.

$$\frac{5000}{580} = \frac{580e^{.495x}}{580}$$

$$8.621 = e^{.495x}$$

$$\ln 8.621 = \ln e^{.495x}$$

$$2.154 = .495x$$

$$4.352 = x$$

$$2009.352$$

In the year 2009 there will be 5000 ants.

9. If you decide to invest \$300 in your savings account. What interest rate will you need to accumulate \$500 in the next 2 years if your savings account compounds monthly?

$$\frac{500}{300} = \frac{300(1 + \frac{r}{12})^{12 \cdot 2}}{300}$$

$$(1.667)^{\frac{1}{24}} = (1 + \frac{r}{12})^{24 \cdot \frac{1}{24}}$$

$$1.022 = 1 + \frac{r}{12}$$

$$\frac{-1}{12 \cdot .022} = \frac{r}{12 \cdot 12}$$

$$r = .25815$$

The interest rate will need to be 25.815%.

10. How much time will it take for your money to double if you put \$420 into an account with an annual interest rate of 2.7%?

$$\frac{840}{420} = \frac{420(1 + .027)^t}{420}$$

$$2 = 1.027^t$$

$$\log_{1.027} 2 = \log_{1.027} 1.027^t$$

$$26.017 = t$$

It will take 26.017 years to double