Algebra II

Review – Polynomials

Name Key

Period ____ Date _____

Lesson 14

1. The volume of a rectangular box is given by the expression $V = (120 \quad 6w)w^2$, where w is measured in inches.

a. What is a reasonable domain for the function in this situation? Express the domain as an inequality, in interval notation, and in set notation.

Set factors = to $\emptyset$

$w^2 = 0$ $\qquad$ $120 - 6w = 0$

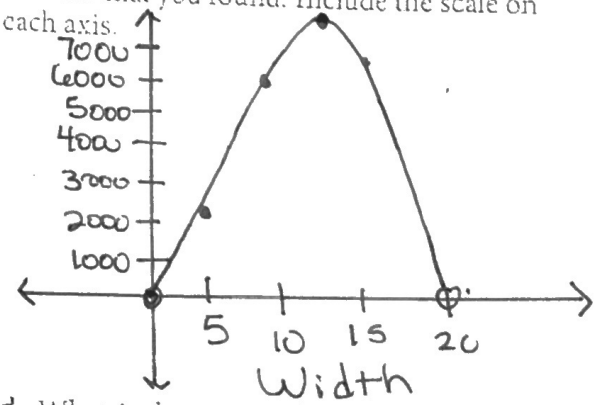
$w = 0$ $\qquad$ $120 = 6w$

$w = 20$

$(0, 20)$

Domain/Window for x

b. Sketch a graph of the function over the domain that you found. Include the scale on each axis.



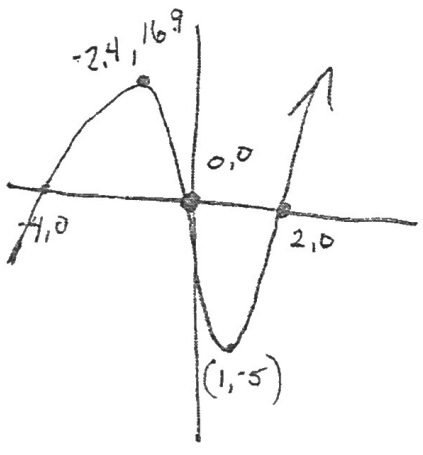
c. Use a graphing calculator to find the coordinates of the maximum point of the function.

$7,111.111 \text{ in}^3$

d. What is the w .h of the box, in inches, that produces the maximum volume?

13.333 in

2. Sketch the graph of the polynomial function $f(x) = x^3 + 2x^2 - 8x$



3. Name any x- or y- intercepts of the function in Item #2

x int → (-4,0) (0,0) (2,0)

y int → (0,0)

4. Name any relative maximum and/or minimum values of the function in item 2.

R-max ≈ (-2.4, 16.9)

R min ≈ (1, -5)

For Items 5-9, decide if each function is a polynomial. If it is, write the function in standard form, and then state the degree and leading coefficient.

5. $f(x)=7x^2-9x^3+3x^7-2$

yes

$3x^7-9x^3+7x^2-2$

deg 7

L.C. 3

6. $f(x)=2x^3+x-5^{(x)}+9$ NO

7. $f(x)=x^4+x+5-\frac{1}{4}x^3$

yes

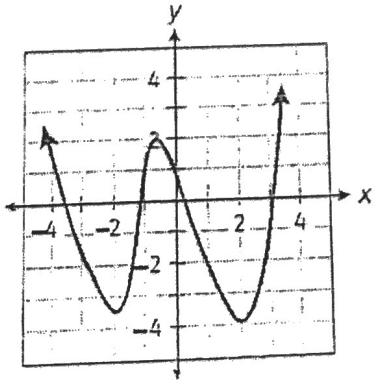
$x^4-\frac{1}{4}x^3+x+5$

deg 4

L.C. 1

8. $f(x)=-0.32x^3+0.08x^4+5x^{(x-1)}-3$ NO

9. Examine the graph below.

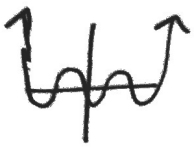


Which of the following statements is NOT true regarding the polynomial whose graph is shown?

A. The degree of the polynomial is even.
B. The leading coefficient is positive.
C. The function is a second-degree polynomial.
D. As $x \to \pm\infty, y \to \infty$.

For Items 10 & 11, describe the end behavior of each function using arrow notation.

10. $f(x) = x^6 - 2x^3 + 3x^2 - 2$



$x \to -\infty \quad y \to \infty$

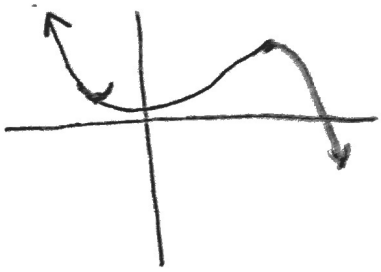
$x \to \infty \quad y \to \infty$

11. $f(x) = -x^3 + 7x^2 - 11$

$x \to -\infty \quad y \to \infty$

$x \to -\infty \quad y \to -\infty$

12. Sketch a graph of any third-degree polynomial function that has exactly one x-intercept, a relative minimum at $(-2, 1)$, and a relative maximum at $(4, 3)$.



13. Use the concept of end behavior to explain why a third-degree polynomial function must have at least one x-intercept.

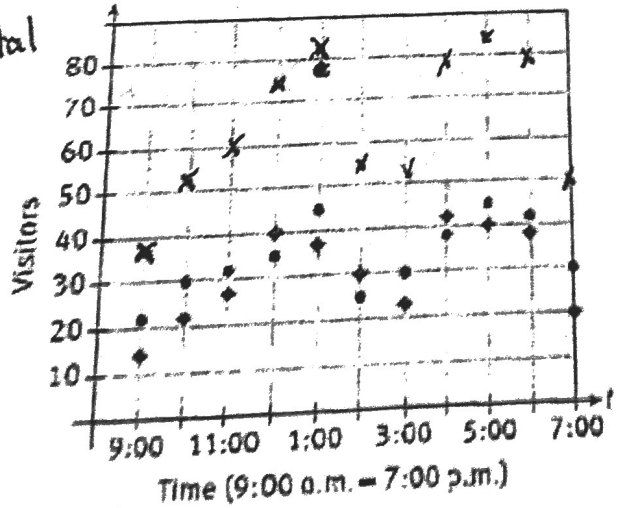
one side must go down infinitely

Lesson 15

1. The graph below shows the number of visitors at a public library one day between the hours of 9:00 a.m. and 7:00 p.m. The round dots represent $A(t)$, the number of adult visitors, and the diamonds represent $C(t)$, the number of children and teenage visitors. Graph $V(t)$, the total number of visitors, and explain how you used the graph to find the values of $V(t)$.

• $A(t)$
• $C(t)$

$V(t)$ → total



2. Examine the functions graphed in Item 1. W
of the statements is true over the given dom
the functions?

A. $A(t) > C(t)$
B. $C(t) > A(t)$
C. $A(t) - C(t) > 0$
D. $V(t) > C(t)$

Student | Mo | Tu | We | Th | Fr | Mo | Tu | We | Th | Fr | Mo | Tu | We | Th | Fr

3. The polynomial expressions $5x + 7$, $3x^2 + 9$, and $3x^2 - 2x$ represent the lengths of the sides of a triangle for all whole-number values of $x > 1$. Write an expression for the perimeter of the triangle.

$5x+7+3x^2+9+3x^2-2x$

$6x^2+3x+16$

$5x+7$
$3x^2+9$
$3x^2-2x$
$6x^2+3x+16$

4. In Item 3, what kind of expression is the perimeter expression?

quadratic / polynomial

5. An open box will be made by cutting four squares of equal size from the corners of a 10-inch-by-12-inch rectangular piece of cardboard and then folding up the sides. The expression $V(x) = x(10 - 2x)(12 - 2x)$ can be used to represent the volume of the box. Write this expression as a polynomial in standard form.

$x(10-2x)(12-2x)$

$x(120-44x+4x^2)$

$120x-44x^2+4x^3$

$V(x) = 4x^3-44x^2+120x$

6. Write an expression for the volume of a box that is constructed in the same way as in Item 5, but from a rectangular piece of cardboard that measures 8 inches by 14 inches. Write your expression in factored form, and then as a polynomial in standard form.

$x(8-2x)(14-2x)$

$x(112-44x+4x^2)$

$112x-44x^2+4x^3$

$4x^3-44x^2+112x$

7. Write an expression to represent the combined volume of the two boxes described in Items 5 and 6.

$4x^3-44x^2+120x$
$+\ 4x^3-44x^2+112x$
$8x^3-88x^2+232x$

For Items 8-18, find each sum, difference or product.

8. $(3x - 4) + (5x + 1)$

$8x-3$

9. $(x^2 - 6x + 5) - (2x^2 + x + 1) =$

$-x^2-7x+4$

$x^2-6x+5-$

10. $(4x^2 - 12x + 9) - (3x - 11)$

$4x^2-9x-2$

11. $(6x^2 - 13x + 4) - (8x^2 - 7x + 25)$

$-2x^2 - 6x - 21$

12. $(4x^3 + 14) - (5x^2 + x)$

$4x^3 + 5x^2 + x + 14$

13. $(2x^2 - x + 1) - (x^2 - 5x + 9)$

$x^2 - 6x - 8$

14. $5x^2(4x^2 - 3x - 9)$

$20x^4 + 15x^3 - 45x^2$

15. $(2x - 5)^3$

$4x^2 - 20x + 25$

16. $(x^3 + y^3)^2$

$x^6 + 2(x^3y^3) + y^6$

17. $(x - 2)(3x^3 - 8x^2 + 2x - 7)$

$3x^4 - 2x^3 - 14x^2 - 3x - 14$

	$3x^3$	$-8x^2$	$+2x$	-7
x	$3x^4$	$-8x^3$	$+2x^2$	$-7x$
$+2$	$6x^3$	$-16x^2$	$+4x$	-14

18. $(x - 3)(2x^3 - 9x^2 + x - 6)$

$2x^4 - 15x^3 + 28x^2 - 9x + 18$

	$2x^3$	$-9x^2$	$+x$	-6
x	$2x^4$	$-9x^3$	$+x^2$	$-6x$
-3	$-6x^3$	$+27x^2$	$-3x$	$+18$

19. Which of the following quotients CANNOT be found using synthetic division?

A. $\dfrac{x^3 + 4x^2 + 5}{x^2 + 1}$

B. $\dfrac{-x^2 - x^2 + 1}{x - 1}$

C. $\dfrac{x^7 - 10}{x - 50}$

D. $\dfrac{2x^3}{x + 1}$

For Items 20-22, find each quotient using long division.

20. $(2x^3 - 3x^2 - 4x - 7) \div (x - 2)$

$2x^2 + x + 6 + \frac{5}{x-2}$

$x-2 \overline{)2x^3 - 3x^2 + 4x - 7}$

$-(2x^3 - 4x^2)$

$x^2 + 4x$

$-(x^2 - 2x)$

$6x - 7$

$-(6x - 12)$

5

$2x^2 + x + 6 + \frac{5}{x-2}$

21. $(5x^4 + 14x^3 + 9x) \div (x^2 + 3x + 1)$

$5x^2 - x - 2 + \frac{16x+2}{x^2+3x+1}$

$x^2+3x+1 \overline{)5x^4 + 14x^3 + 0x^2 + 9x + 0}$

$-(5x^4 + 15x^3 + 5x^2)$

$-x^3 - 5x^2 + 9x$

$-(-x^3 - 3x^2 - x)$

$-2x^2 + 10x + 0$

$(-2x^2 - 6x - 2)$

$16x + 2$

$5x^2 - x - 2 + \frac{16x+2}{x^2+3x+1}$

Use synthetic division

22. $(x^2 + 4) \div (x + 4)$

20

23. $\frac{3x^3 - 10x^2 + 12x - 22}{x - 4}$

4 | 1 0 −4

↓ −4 16

1 −4 | 12

$x - 4 + \frac{12}{x+4}$

4 | 3 −10 +12 −22

↓ 12 8 80

3 2 20 58

$3x^2 + 2x + 20 + \frac{58}{x-4}$