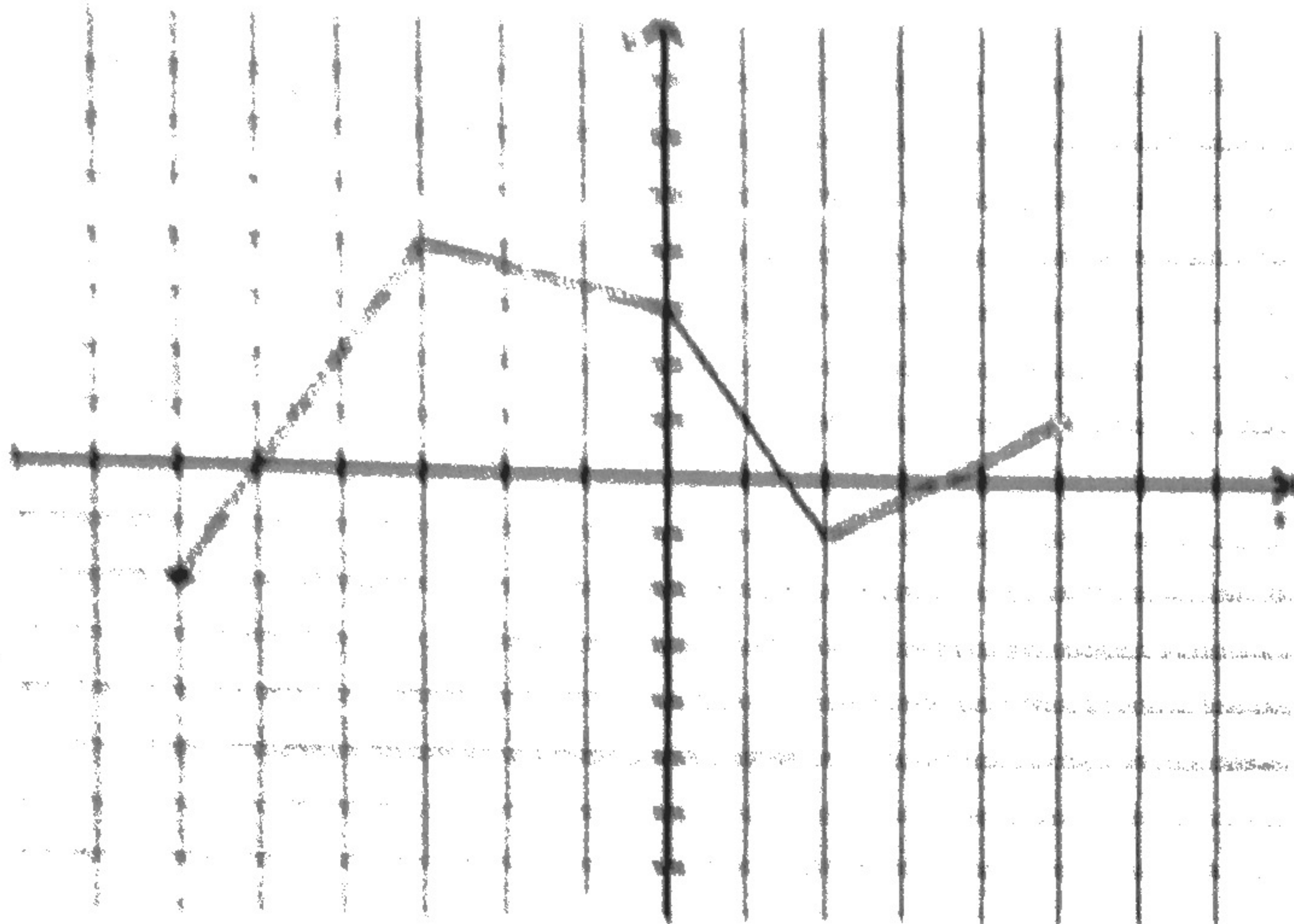


10 The graph of $g(x)$ is shown. Find the domain and range for the functions with the indicated parameter changes



(a) $g(x)$
 Domain: $[-4, 6]$
 Range: $[-2, 6]$

(b) $h(x) = 2g(x+1) - 2$
 Domain: $[-5, 5] = [-5, 5]$
 Range: $[-2(2) - 2, 2(6) - 2] = [-6, 6]$

(c) $f(x) = -g(x-2)$
 Domain: $[-2, 8] = [-2, 8]$
 Range: $[-(-2), -4] = [2, -4] = [-4, 2]$

(d) $p(x) = \frac{1}{2}g(x) + 4$
 Domain: $[-4, 6]$
 Range: $[-2(\frac{1}{2}) + 4, 4(\frac{1}{2}) + 4] = [3, 6]$

11. A function $f(x)$ has a domain $x \in (0, 4]$ and a range of $f(x) \in [-1, 4)$. Find the domain and range for the given transformations of $f(x)$.

(a) $y = 2f(x) - 3$
 Domain: $(0, 4]$
 Range: $[-1(2) - 3, 4(2) - 3] = [-5, 5]$

(b) $y = -0.5f(x-1)$
 Domain: $(0+1, 4+1] = (1, 5]$
 Range: $[-1(-0.5), 4(-0.5)] = [0.5, -2] = [-2, 0.5]$

(c) $y = f(x-3) + 1$
 Domain: $[0+3, 4+3] = [3, 7]$
 Range: $[-1+1, 4+1] = [0, 5]$

(d) $y = 3f(x-1) - 2$
 Domain: $(0+1, 4+1] = (1, 5]$
 Range: $[-1(3) - 2, 4(3) - 2] = [-5, 10]$

(e) $y = f(-x) - 3$
 Domain: $(0(-1), 4(-1)] = (0, -4] = [-4, 0]$
 Range: $[-1-3, 4-3] = [-4, 1]$

(f) $y = -4(x+2) - 2$
 Domain: $(0-2, 4-2] = (-2, 2]$
 Range: $[-4(-4) - 2, 4(-4) - 2] = [2, -18] = [-18, 2]$

12. Write the equation for a quadratic function in standard form: $y = ax^2 + bx + c$

13. Write the equations in vertex form for the quadratics that have the given vertex and pass through the given point.

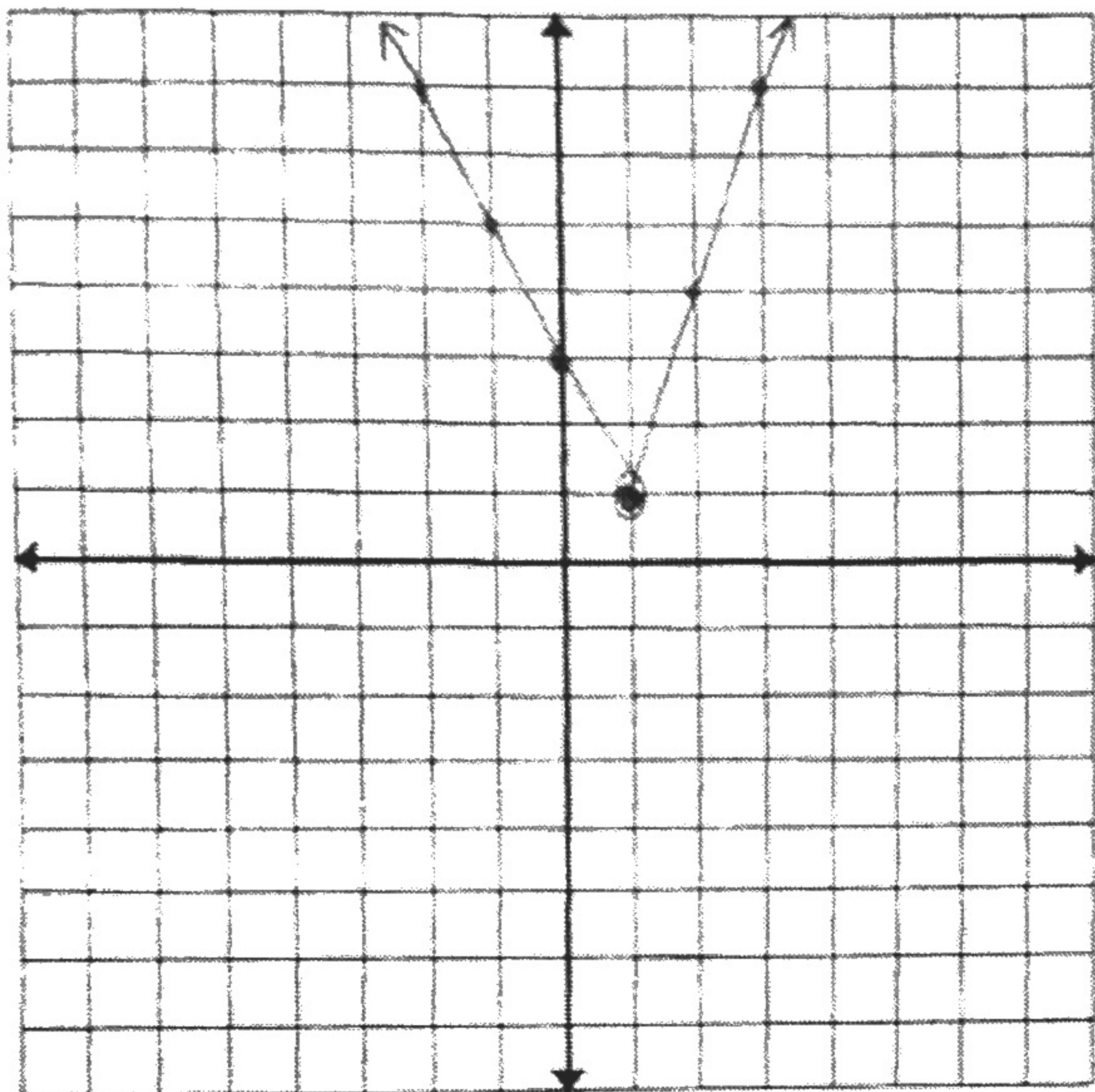
| | |
|---|---|
| <p>(a) Vertex: (-2, 5); point: (0, 9)</p> $y = a(x+2)^2 + 5$ $9 = a(0+2)^2 + 5$ $4 = 4a$ $a = 1$ $y = (x+2)^2 + 5$ | <p>(b) Vertex: (4, -1); point: (2, 3)</p> $y = a(x-4)^2 - 1$ $3 = a(2-4)^2 - 1$ $4 = 4a$ $a = 1$ $y = (x-4)^2 - 1$ |
| <p>(c) Vertex: (3, 4); point: (1, 2)</p> $y = a(x-3)^2 + 4$ $2 = a(1-3)^2 + 4$ $-2 = 4a$ $a = -\frac{1}{2}$ $y = -\frac{1}{2}(x-3)^2 + 4$ | <p>(d) Vertex: (2, 3); point: (0, 2)</p> $y = a(x-2)^2 + 3$ $2 = a(0-2)^2 + 3$ $-1 = 4a$ $a = -\frac{1}{4}$ $y = -\frac{1}{4}(x-2)^2 + 3$ |

14. Find the x- and y-intercepts for each of the quadratics in #13.

| | |
|---|---|
| <p>(a) y-intercept: $y = (0+2)^2 + 5$ $y = 9$ $(0, 9)$</p> <p>x-intercepts: $0 = (x+2)^2 + 5$ $-5 = (x+2)^2$ $x+2 = \pm\sqrt{-5}$ $x+2 = \pm i\sqrt{5}$ $x = -2 \pm i\sqrt{5}$ $(-2 \pm i\sqrt{5}, 0)$</p> | <p>(b) y-intercept: $y = (0-4)^2 - 1$ $y = 15$ $(0, 15)$</p> <p>x-intercepts: $0 = (x-4)^2 - 1$ $1 = (x-4)^2$ $x-4 = \pm 1$ $x = 1+4$ $x = -1+4$ $x = 5$ $x = 3$ $(5, 0), (3, 0)$</p> |
| <p>(c) y-intercept: $y = -\frac{1}{2}(0-3)^2 + 4$ $y = -\frac{1}{2}$ $(0, -\frac{1}{2})$</p> <p>x-intercepts: $0 = -\frac{1}{2}(x-3)^2 + 4$ $-4 = -\frac{1}{2}(x-3)^2$ $8 = (x-3)^2$ $x-3 = \pm\sqrt{8}$ $x-3 = \pm 2\sqrt{2}$ $x = 3 \pm 2\sqrt{2}$ $(3 \pm 2\sqrt{2}, 0)$</p> | <p>(d) y-intercept: $y = -\frac{1}{4}(0-2)^2 + 3$ $y = 2$ $(0, 2)$</p> <p>x-intercepts: $0 = -\frac{1}{4}(x-2)^2 + 3$ $-3 = -\frac{1}{4}(x-2)^2$ $12 = (x-2)^2$ $x-2 = \pm\sqrt{12}$ $x-2 = \pm 2\sqrt{3}$ $x = 2 \pm 2\sqrt{3}$ $(2 \pm 2\sqrt{3}, 0)$</p> |

15. Graph the following function and find the indicated features of the graph:

$$f(x) = \begin{cases} -2x + 3 & \text{if } x < 1 \\ 3x - 2 & \text{if } x \geq 1 \end{cases}$$



(a) Domain:

$$(-\infty, \infty)$$

(b) Range

$$[1, \infty)$$

(c) x-intercepts:

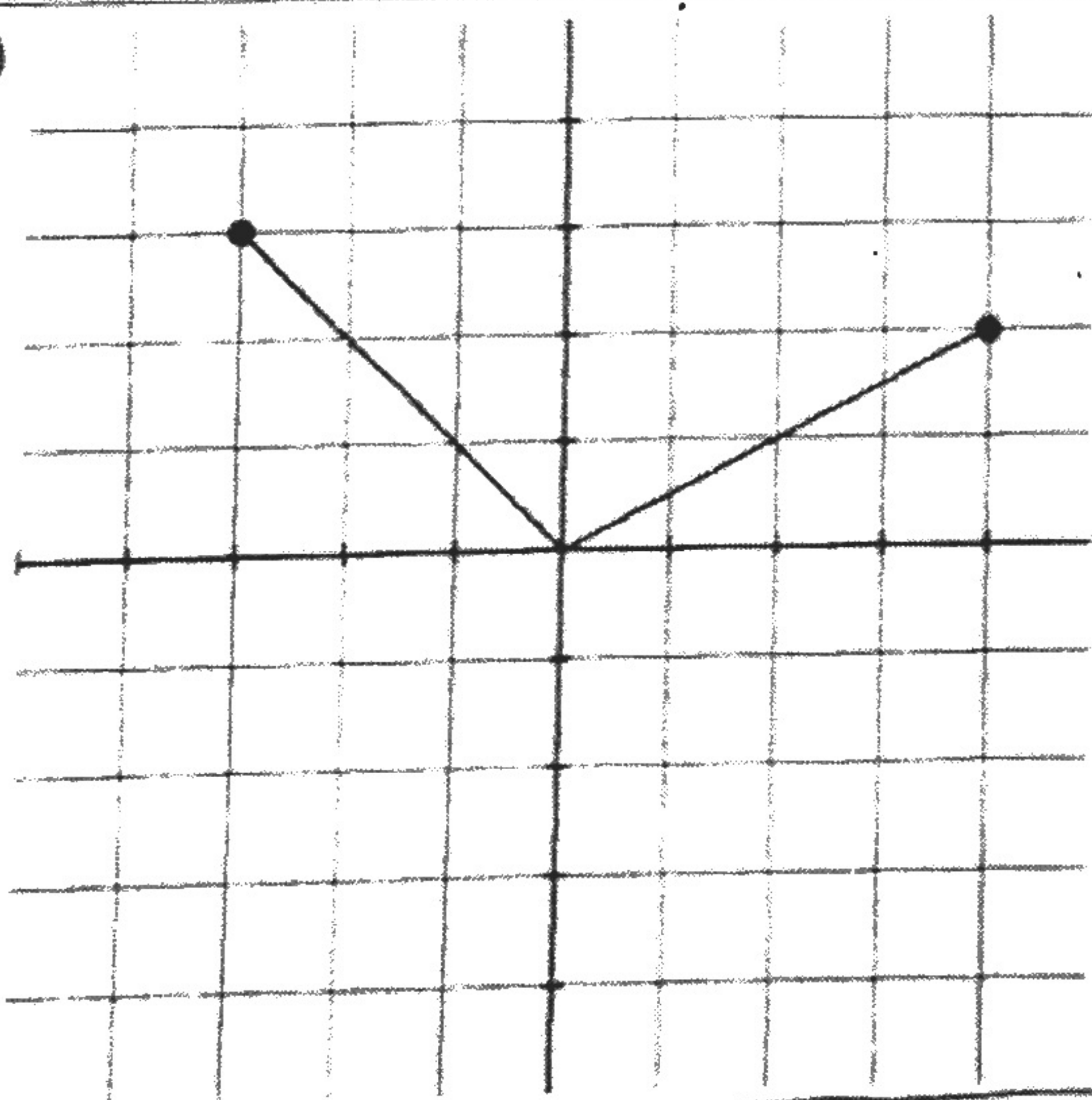
None

(d) y-intercepts:

$$(0, 3)$$

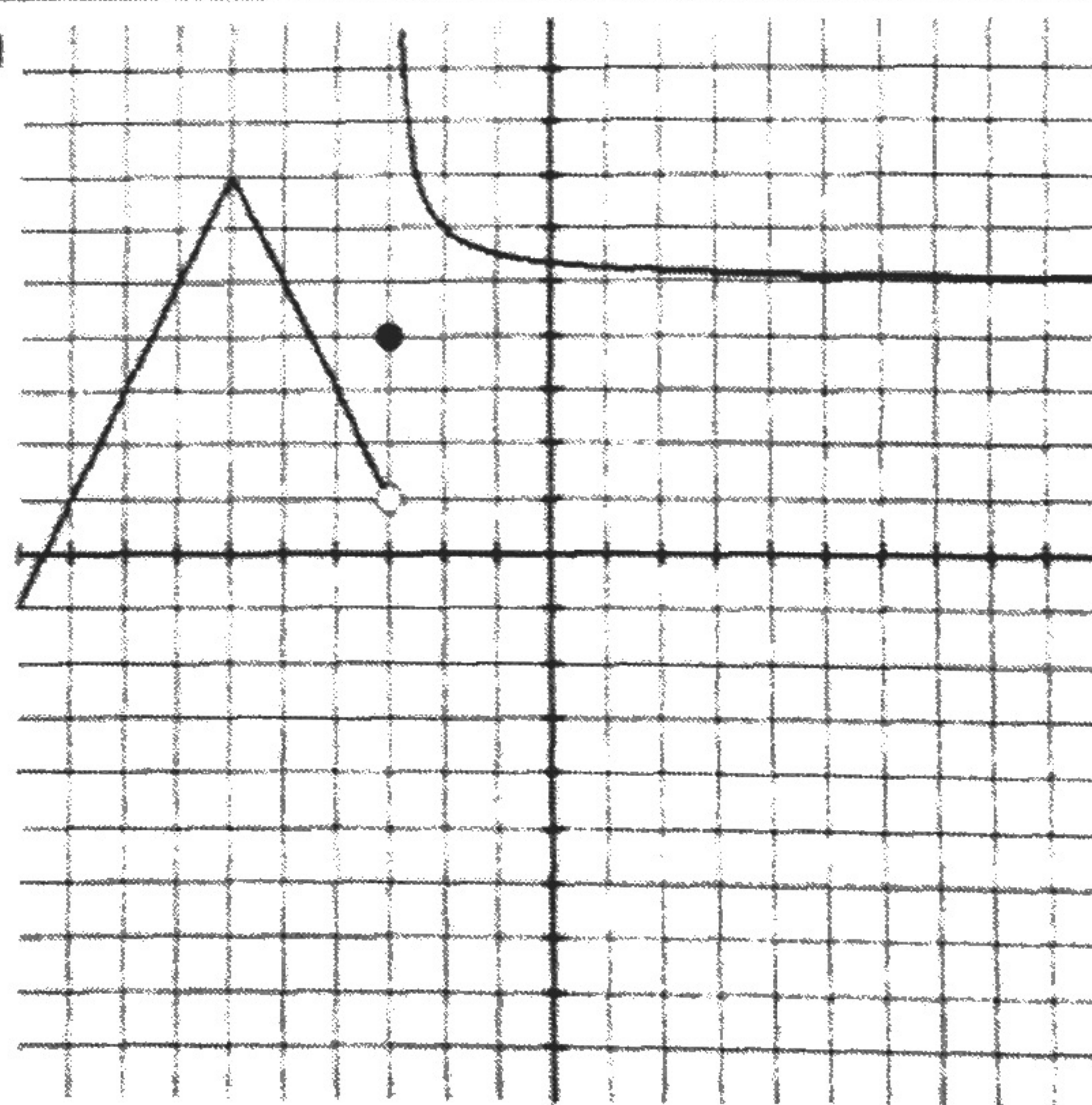
16. The graph of a piecewise function is given. Write a definition for the function.

(a)



$$f(x) = \begin{cases} -x & \text{if } -3 \leq x \leq 0 \\ \frac{1}{2}x & \text{if } 0 < x \leq 4 \end{cases}$$

(b)



$$f(x) = \begin{cases} -2|x+6| + 7 & \text{if } x < -3 \\ 4 & \text{if } x = -3 \\ \frac{1}{x+3} + 5 & \text{if } x > -3 \end{cases}$$

19. Graph the following equations:

| <p>(a) $f(x) = 4 \cdot 2^{\frac{1}{3}(x+3)} - 6$</p> <table border="1" style="width: 100%; border-collapse: collapse; margin-bottom: 10px;"> <thead> <tr> <th>x</th> <th>f(x)</th> </tr> </thead> <tbody> <tr> <td>$-2(3)+3 = -9$</td> <td>$4(1/8)-6 = -5$</td> </tr> <tr> <td>$-1(3)+3 = -6$</td> <td>$4(1/4)-6 = -4$</td> </tr> <tr> <td>$0(3)+3 = 3$</td> <td>$4(1/2)-6 = -2$</td> </tr> <tr> <td>$1(3)+3 = 6$</td> <td>$4(1)-6 = -2$</td> </tr> <tr> <td>$2(3)+3 = 9$</td> <td>$4(2)-6 = 2$</td> </tr> </tbody> </table> | x | f(x) | $-2(3)+3 = -9$ | $4(1/8)-6 = -5$ | $-1(3)+3 = -6$ | $4(1/4)-6 = -4$ | $0(3)+3 = 3$ | $4(1/2)-6 = -2$ | $1(3)+3 = 6$ | $4(1)-6 = -2$ | $2(3)+3 = 9$ | $4(2)-6 = 2$ | <p>(b) $f(x) = -2 \cdot 4^{\frac{1}{4}(x-2)} - 1$</p> <table border="1" style="width: 100%; border-collapse: collapse; margin-bottom: 10px;"> <thead> <tr> <th>x</th> <th>f(x)</th> </tr> </thead> <tbody> <tr> <td>$-2(-2)+2 = 10$</td> <td>$-2(1/16)-1 = -1 1/8$</td> </tr> <tr> <td>$-1(-2)+2 = 6$</td> <td>$-2(1/4)-1 = -1 1/2$</td> </tr> <tr> <td>$0(-2)+2 = 2$</td> <td>$-2(1/2)-1 = -2$</td> </tr> <tr> <td>$1(-2)+2 = -2$</td> <td>$-2(1)-1 = -3$</td> </tr> <tr> <td>$2(-2)+2 = -6$</td> <td>$-2(2)-1 = -5$</td> </tr> </tbody> </table> | x | f(x) | $-2(-2)+2 = 10$ | $-2(1/16)-1 = -1 1/8$ | $-1(-2)+2 = 6$ | $-2(1/4)-1 = -1 1/2$ | $0(-2)+2 = 2$ | $-2(1/2)-1 = -2$ | $1(-2)+2 = -2$ | $-2(1)-1 = -3$ | $2(-2)+2 = -6$ | $-2(2)-1 = -5$ |
|--|-----------------------|------|----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|---------------|-----------------|--------------|--|---|------|------------------|-----------------------|--------------------|----------------------|------------------|------------------|------------------|----------------|----------------|----------------|
| x | f(x) | | | | | | | | | | | | | | | | | | | | | | | | |
| $-2(3)+3 = -9$ | $4(1/8)-6 = -5$ | | | | | | | | | | | | | | | | | | | | | | | | |
| $-1(3)+3 = -6$ | $4(1/4)-6 = -4$ | | | | | | | | | | | | | | | | | | | | | | | | |
| $0(3)+3 = 3$ | $4(1/2)-6 = -2$ | | | | | | | | | | | | | | | | | | | | | | | | |
| $1(3)+3 = 6$ | $4(1)-6 = -2$ | | | | | | | | | | | | | | | | | | | | | | | | |
| $2(3)+3 = 9$ | $4(2)-6 = 2$ | | | | | | | | | | | | | | | | | | | | | | | | |
| x | f(x) | | | | | | | | | | | | | | | | | | | | | | | | |
| $-2(-2)+2 = 10$ | $-2(1/16)-1 = -1 1/8$ | | | | | | | | | | | | | | | | | | | | | | | | |
| $-1(-2)+2 = 6$ | $-2(1/4)-1 = -1 1/2$ | | | | | | | | | | | | | | | | | | | | | | | | |
| $0(-2)+2 = 2$ | $-2(1/2)-1 = -2$ | | | | | | | | | | | | | | | | | | | | | | | | |
| $1(-2)+2 = -2$ | $-2(1)-1 = -3$ | | | | | | | | | | | | | | | | | | | | | | | | |
| $2(-2)+2 = -6$ | $-2(2)-1 = -5$ | | | | | | | | | | | | | | | | | | | | | | | | |
| <p>(c) $f(x) = 2 \log_3 \left[\frac{1}{9}(x-2) \right] + 5$</p> <table border="1" style="width: 100%; border-collapse: collapse; margin-bottom: 10px;"> <thead> <tr> <th>x</th> <th>f(x)</th> </tr> </thead> <tbody> <tr> <td>$1/9(9)+2 = 3$</td> <td>$-2(2)+5 = -1$</td> </tr> <tr> <td>$1/9(18)+2 = 4$</td> <td>$-1(2)+5 = 3$</td> </tr> <tr> <td>$1/9(27)+2 = 5$</td> <td>$0(2)+5 = 5$</td> </tr> <tr> <td>$1/9(36)+2 = 6$</td> <td>$1(2)+5 = 7$</td> </tr> <tr> <td>$1/9(45)+2 = 7$</td> <td>$2(2)+5 = 9$</td> </tr> </tbody> </table> | x | f(x) | $1/9(9)+2 = 3$ | $-2(2)+5 = -1$ | $1/9(18)+2 = 4$ | $-1(2)+5 = 3$ | $1/9(27)+2 = 5$ | $0(2)+5 = 5$ | $1/9(36)+2 = 6$ | $1(2)+5 = 7$ | $1/9(45)+2 = 7$ | $2(2)+5 = 9$ | <p>(d) $f(x) = -3 \log_4 \left[-\frac{1}{4}(x+1) \right] - 2$</p> <table border="1" style="width: 100%; border-collapse: collapse; margin-bottom: 10px;"> <thead> <tr> <th>x</th> <th>f(x)</th> </tr> </thead> <tbody> <tr> <td>$1/4(-4)+1 = -1$</td> <td>$-2(2)+2 = -4$</td> </tr> <tr> <td>$1/4(-2)+1 = -1/2$</td> <td>$-1(2)+2 = 0$</td> </tr> <tr> <td>$1/4(0)+1 = 1/4$</td> <td>$0(2)+2 = 2$</td> </tr> <tr> <td>$1/4(2)+1 = 3/4$</td> <td>$1(2)+2 = 4$</td> </tr> <tr> <td>$1/4(4)+1 = 2$</td> <td>$2(2)+2 = 6$</td> </tr> </tbody> </table> | x | f(x) | $1/4(-4)+1 = -1$ | $-2(2)+2 = -4$ | $1/4(-2)+1 = -1/2$ | $-1(2)+2 = 0$ | $1/4(0)+1 = 1/4$ | $0(2)+2 = 2$ | $1/4(2)+1 = 3/4$ | $1(2)+2 = 4$ | $1/4(4)+1 = 2$ | $2(2)+2 = 6$ |
| x | f(x) | | | | | | | | | | | | | | | | | | | | | | | | |
| $1/9(9)+2 = 3$ | $-2(2)+5 = -1$ | | | | | | | | | | | | | | | | | | | | | | | | |
| $1/9(18)+2 = 4$ | $-1(2)+5 = 3$ | | | | | | | | | | | | | | | | | | | | | | | | |
| $1/9(27)+2 = 5$ | $0(2)+5 = 5$ | | | | | | | | | | | | | | | | | | | | | | | | |
| $1/9(36)+2 = 6$ | $1(2)+5 = 7$ | | | | | | | | | | | | | | | | | | | | | | | | |
| $1/9(45)+2 = 7$ | $2(2)+5 = 9$ | | | | | | | | | | | | | | | | | | | | | | | | |
| x | f(x) | | | | | | | | | | | | | | | | | | | | | | | | |
| $1/4(-4)+1 = -1$ | $-2(2)+2 = -4$ | | | | | | | | | | | | | | | | | | | | | | | | |
| $1/4(-2)+1 = -1/2$ | $-1(2)+2 = 0$ | | | | | | | | | | | | | | | | | | | | | | | | |
| $1/4(0)+1 = 1/4$ | $0(2)+2 = 2$ | | | | | | | | | | | | | | | | | | | | | | | | |
| $1/4(2)+1 = 3/4$ | $1(2)+2 = 4$ | | | | | | | | | | | | | | | | | | | | | | | | |
| $1/4(4)+1 = 2$ | $2(2)+2 = 6$ | | | | | | | | | | | | | | | | | | | | | | | | |

10. Evaluate the following using the Unit Circle:

| | |
|---|---|
| <p>a) $\cos\left(\frac{27\pi}{4}\right) =$ (Non-Calculator)</p> <p>$\cos\left(\frac{27\pi}{4} - \frac{8\pi}{4} \cdot 3\right) =$ $\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$</p> <p>$\cos\left(\frac{3\pi}{4}\right) =$ $-\frac{\sqrt{2}}{2}$</p> | <p>b) $\sin\left(-\frac{11\pi}{3}\right) =$ (Non-Calculator)</p> <p>$\sin\left(-\frac{11\pi}{3} + \frac{6\pi}{3} \cdot 2\right) =$ $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$</p> <p>$\sin\left(\frac{\pi}{3}\right) =$ $\frac{\sqrt{3}}{2}$</p> |
|---|---|

11. Evaluate the following for angles that are not on the Unit Circle:

| | |
|---|---|
| <p>a) $\csc(x)$ if $\cos(x) = -\frac{4}{5}$ and $\tan(x) < 0$</p> <p>$\csc(x) = \frac{5}{3}$</p> <p>(Non-Calculator)</p> | <p>b) $\cot(x)$ if $\csc(x) = -\frac{13}{12}$ and $\cos(x) > 0$</p> <p>$\cot(x) = -\frac{5}{12}$</p> <p>(Non-Calculator)</p> |
|---|---|

12. For each equation, identify the amplitude, period, phase displacement, and vertical shift. Use them to graph two periods of the function:

a) $y = 12 \cos\left[\frac{2}{7}(x - 2)\right] - 5$ (Calculator)

amplitude: 12

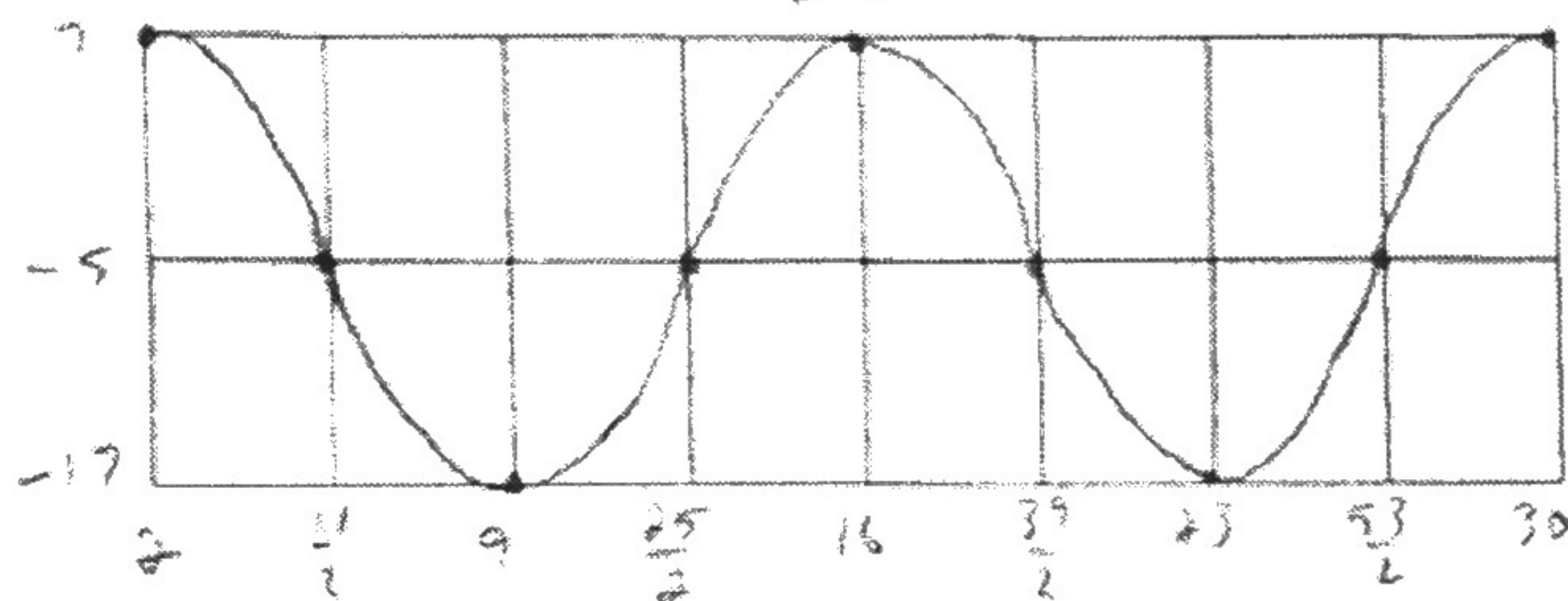
period: $\frac{2\pi}{\frac{2}{7}} = 7\pi$

phase displacement: 2

vertical shift: -5

graph:

x-scale = $\frac{\pi}{2} \left(\frac{7}{\pi}\right) = \frac{7}{2}$



b) $y = -6 \sin\left[\frac{2}{3}\left(x - \frac{\pi}{4}\right)\right] + 2$ (Calculator)

amplitude: 6

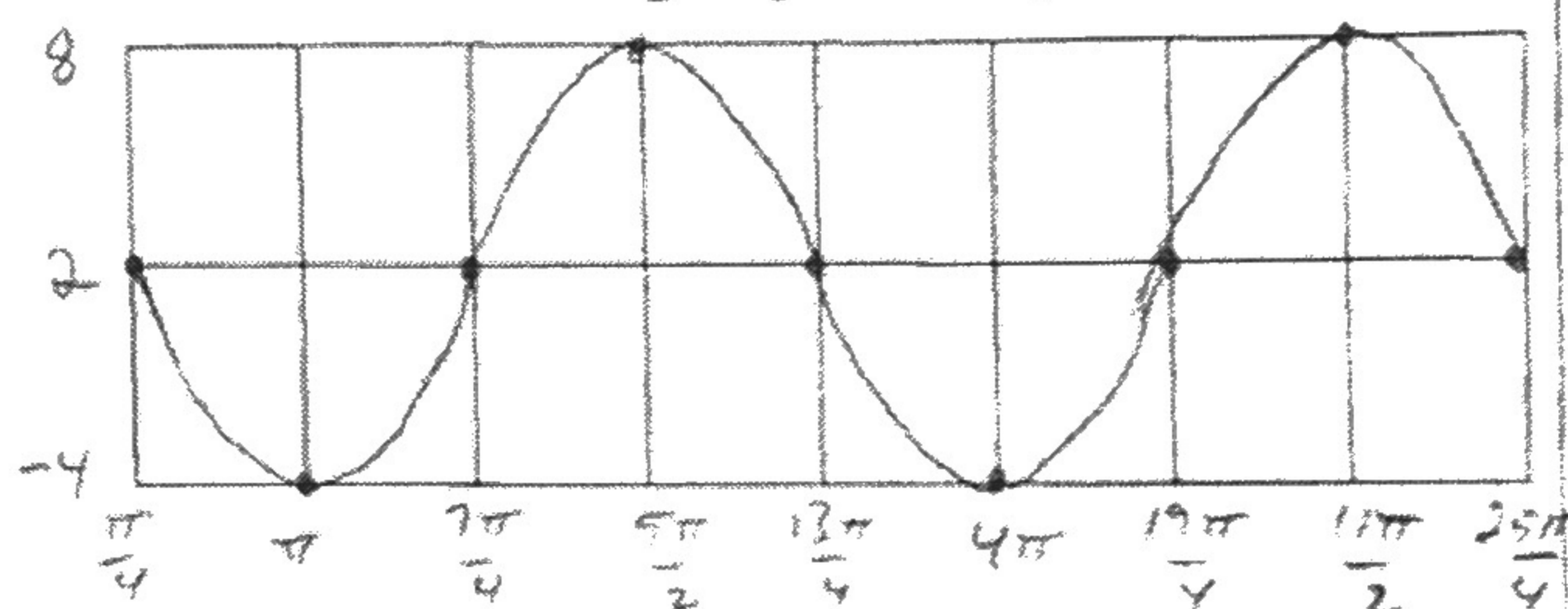
period: $\frac{2\pi}{\frac{2}{3}} = 3\pi$

phase displacement: $\frac{\pi}{4}$

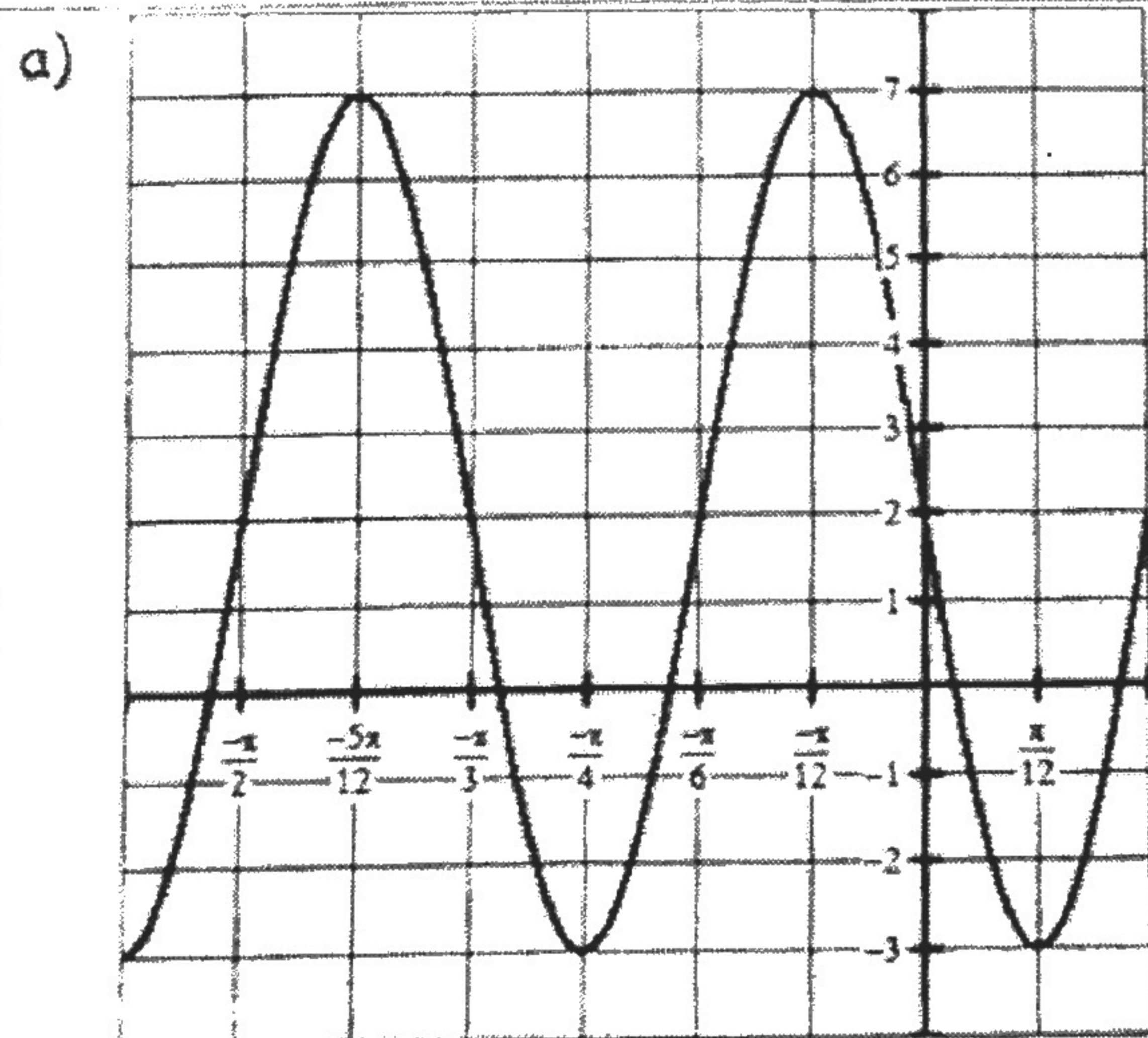
vertical shift: 2

graph:

x-scale = $\frac{\pi}{2} \cdot \frac{3}{2} = \frac{3\pi}{4}$



13. Write both the sine and cosine equations of the following graphs with the first positive phase shift that has a positive amplitude:



$P = 0 - \left(-\frac{\pi}{3}\right)$
 $P = \frac{\pi}{3}$
 sine:
 $C = -\frac{\pi}{6} + \frac{\pi}{3}$
 $C = -\frac{\pi}{6} + \frac{2\pi}{6}$
 $C = \frac{\pi}{6}$
 cosine:
 $C = -\frac{\pi}{2} + \frac{\pi}{3}$

$y = 5 \sin\left[6\left(x - \frac{\pi}{6}\right)\right] + 2$

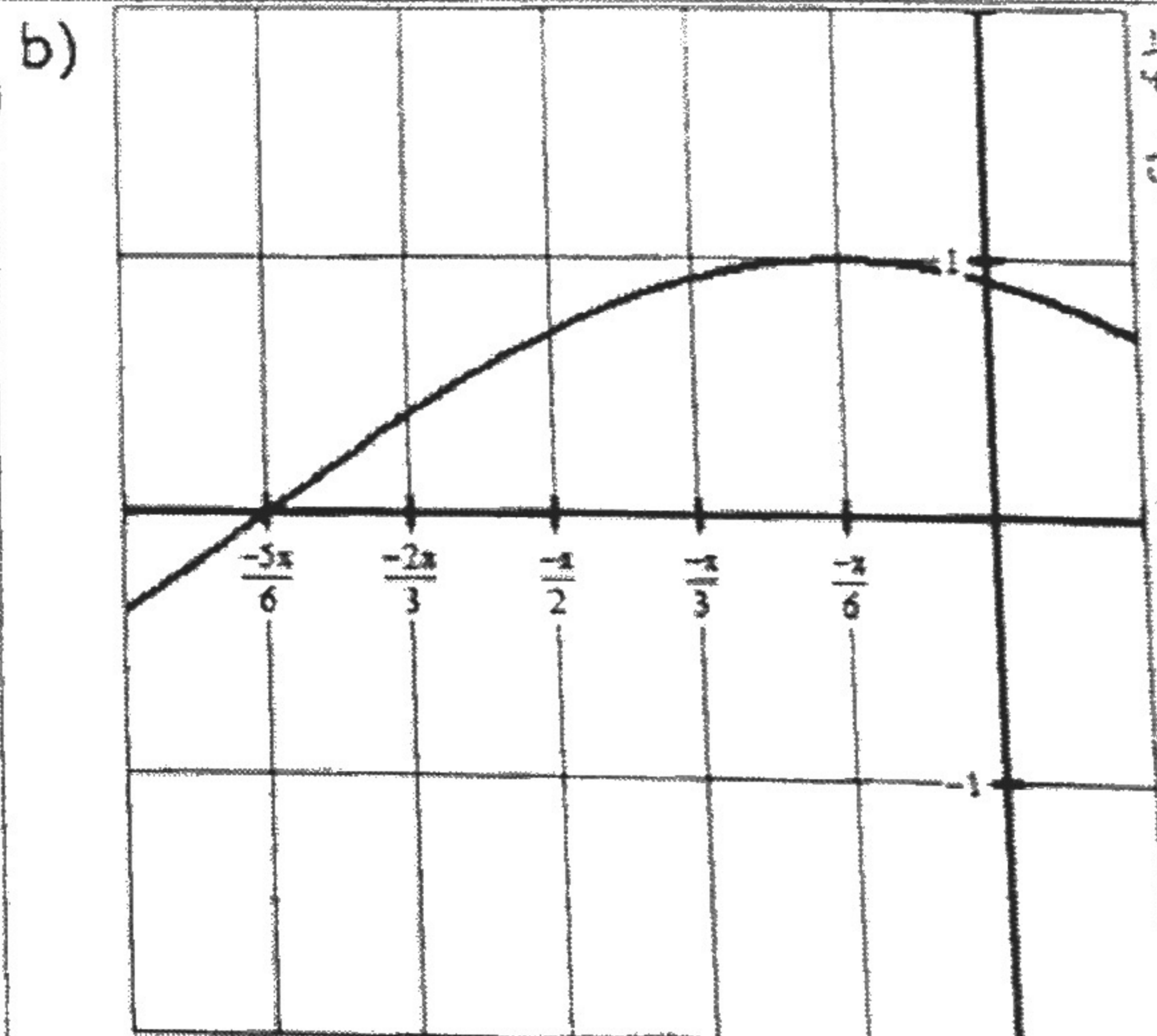
$B = \frac{2\pi}{\frac{3}{4}} = \frac{8\pi}{3}$

$B = 6$

$C = \frac{3\pi}{12} = \frac{\pi}{4}$

$y = 5 \cos\left[6\left(x - \frac{\pi}{4}\right)\right] + 2$

(Non-Calculator)



$\frac{1}{4}P = -\frac{\pi}{6} - \left(-\frac{\pi}{2}\right)$
 $\frac{1}{4}P = \frac{4\pi}{6} - \frac{3\pi}{6} = \frac{\pi}{6}$
 $\frac{1}{4}P = \frac{\pi}{6}$
 $P = \frac{4\pi}{3}$
 sine:
 $C = -\frac{5\pi}{6} + \frac{4\pi}{3}$
 $C = -\frac{5\pi}{6} + \frac{8\pi}{6} = \frac{3\pi}{6} = \frac{\pi}{2}$
 $C = \frac{11\pi}{6}$
 cosine:
 $C = -\frac{\pi}{6} + \frac{4\pi}{3}$
 $C = -\frac{\pi}{6} + \frac{8\pi}{6} = \frac{7\pi}{6}$

$y = \sin\left[\frac{3}{4}\left(x - \frac{11\pi}{6}\right)\right]$

$B = \frac{2\pi}{\frac{3}{4}} = \frac{8\pi}{3}$

$B = \frac{3}{4}$

$y = \cos\left[\frac{3}{4}\left(x - \frac{5\pi}{2}\right)\right]$

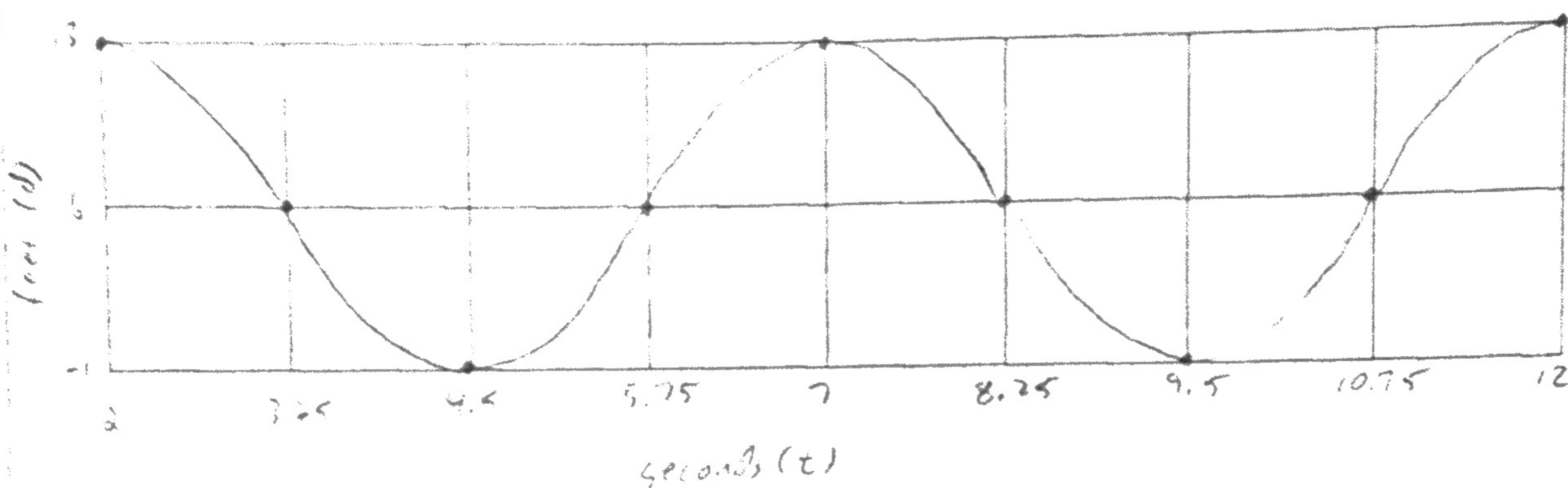
(Non-Calculator)

14 Water Wheel Problem: Suppose a water wheel rotates at 12 revolutions per minute (rpm), the diameter of the wheel is 14 feet, and 1 foot is submerged in the water. You start your stopwatch. Two seconds later, point P on the rim of the wheel is at its greatest height. Assume that P's distance, d (in feet), above the water is a sinusoidal function of the number of seconds, t , the stopwatch reads.

12 rev = 1 min 12 rev = 60 sec (rev = 5 sec)

a) Sketch a graph of d versus t .

(Calculator)



b) Write the particular equation expressing d in terms of t as a cosine function. Use the first positive phase shift that has a positive amplitude.

(Calculator)

$$d = 7 \cos \left[\frac{2\pi}{5} (t - 2) \right] + 6$$

c) Write the particular equation expressing d in terms of t , as a sine function. Use the first positive phase shift that has a positive amplitude.

(Calculator)

$C = 5.75 - 5$
 $C = 0.75$

$$d = 7 \sin \left[\frac{2\pi}{5} (t - 0.75) \right] + 6$$

d) When $t = 7.7$ is P underwater or above? How far?

$$d = 7 \cos \left[\frac{2\pi}{5} (7.7 - 2) \right] + 6 \quad (\text{Calculator})$$

$d = 12.462$

12.462 feet above the water

e) Find the first positive time at which P emerges from the water.

$$0 = 7 \cos \left[\frac{2\pi}{5} (t - 2) \right] + 6 \quad (\text{Calculator})$$

$$-6 = 7 \cos \left[\frac{2\pi}{5} (t - 2) \right]$$

$$-\frac{6}{7} = \cos \left[\frac{2\pi}{5} (t - 2) \right]$$

$$2.600 = \frac{2\pi}{5} (t - 2) \quad \text{OR} \quad -2.600 = \frac{2\pi}{5} (t - 2)$$

$$2.069 = t - 2$$

$$t = 4.069 + 5n$$

$$-2.069 = t - 2$$

$$t = -0.069 + 5n$$

$$t = 4.931 + 5n$$

4.069 seconds

f) How high is P when you started your stopwatch?

$$d = 7 \cos \left[\frac{2\pi}{5} (0 - 2) \right] + 6 \quad (\text{Calculator})$$

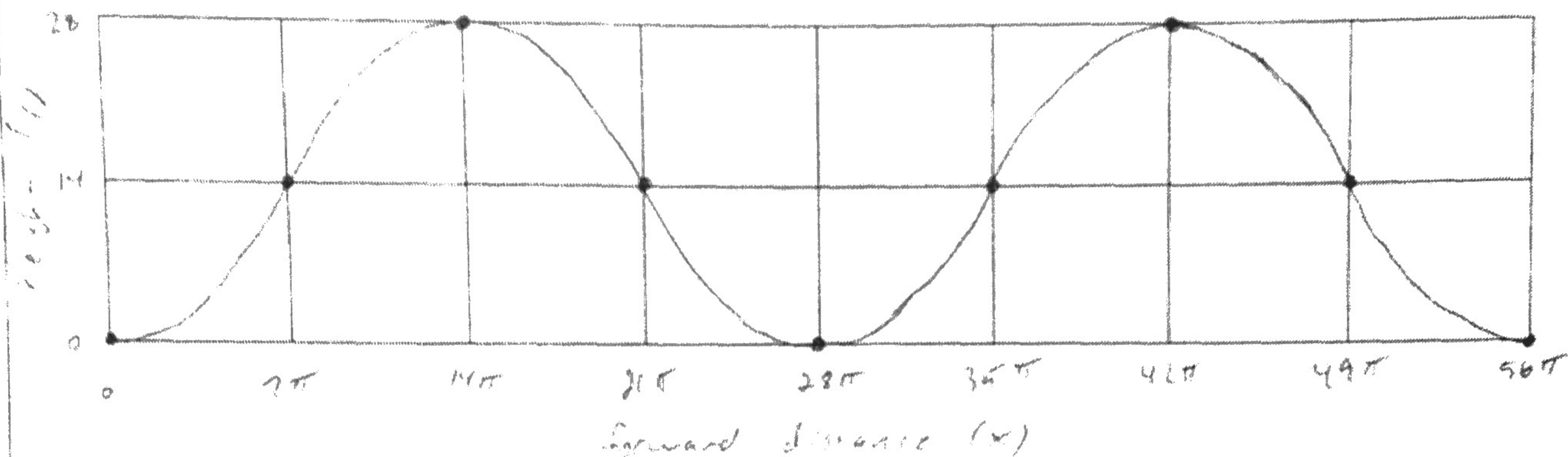
$d = 2.577$

2.577 feet above the water

15. Pebble-in-the-Tire Problem: As you stop your car at a traffic light, a pebble becomes wedged between the tires treads. When you start off, the distance of the pebble from the pavement, y , varies sinusoidally with the forward distance you have traveled, x . The diameter of the wheel is 28 inches.

a) Sketch a graph of y versus x .

(Calculator)



b) Write the particular equation expressing y in terms of x as a cosine function. Use the first positive phase shift.

(Calculator)

$$B = \frac{2\pi}{28\pi}$$

$$B = \frac{1}{14}$$

$$y = 14 \cos \left[\frac{1}{14} (x - 14\pi) \right] + 14$$

c) Write the particular equation expressing y in terms of x as a sine function. Use the first positive phase shift.

(Calculator)

$$y = 14 \sin \left[\frac{1}{14} (x - 7\pi) \right] + 14$$

d) Predict the distance from the pavement when you have gone 142 inches.

(Calculator)

$$y = 14 \sin \left[\frac{1}{14} (142 - 7\pi) \right] + 14$$

$$y = 24.543 \text{ inches}$$

e) What are the first two distances when the pebble is 17 inches from the pavement?

(Calculator)

$$17 = 14 \cos \left[\frac{1}{14} (x - 14\pi) \right] + 14$$

$$3 = 14 \cos \left[\frac{1}{14} (x - 14\pi) \right]$$

$$\frac{3}{14} = \cos \left[\frac{1}{14} (x - 14\pi) \right]$$

$$1.355 = \frac{1}{14} (x - 14\pi) \quad \text{OR} \quad -1.355 = \frac{1}{14} (x - 14\pi)$$

$$18.968 = x - 14\pi$$

$$-18.968 = x - 14\pi$$

$$x = 62.950 + 28\pi n$$

$$x = 25.015 + 28\pi n$$

$$25.015 \text{ inches and } 62.950 \text{ inches}$$

16. Solve for θ in the indicated domain:

a) $\csc(5\theta - 28) = 2$ for $370^\circ < \theta < 560^\circ$

$\sin(5\theta - 28) = \frac{1}{2}$ 

$5\theta - 28 = 30 + 360n$ $5\theta - 28 = 150 + 360n$

$5\theta = 178 + 360n$ $5\theta = 518 + 360n$

$\theta = 35.6 + 72n$ $\theta = 35.6 + 72n$

$\theta = 371.6, 395.6, 443.6, 467.6,$
 $515.6, 539.6$

(Calculator)

b) $5 - 2\tan(3\theta - \pi) = 7$ for $0 < \theta < 2\pi$

$-2\tan(3\theta - \pi) = 2$ $\tan(3\theta - \pi) = -1$ 

$3\theta - \pi = \frac{3\pi}{4} + 6\pi n$ $3\theta - \pi = \frac{7\pi}{4} + 6\pi n$

$3\theta = \frac{7\pi}{4} + 2\pi n$ $3\theta = \frac{11\pi}{4} + 2\pi n$

$\theta = \frac{7\pi}{12} + \frac{2\pi}{3}n$ $\theta = \frac{11\pi}{12} + \frac{2\pi}{3}n$

$\theta = \frac{3\pi}{12}, \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{5\pi}{4}, \frac{19\pi}{12}, \frac{23\pi}{12}$

(Calculator)

17. Know the trigonometric identities (Calculator)

18. Use the sum and difference identities to evaluate the following:

a) $\sin(15^\circ) =$

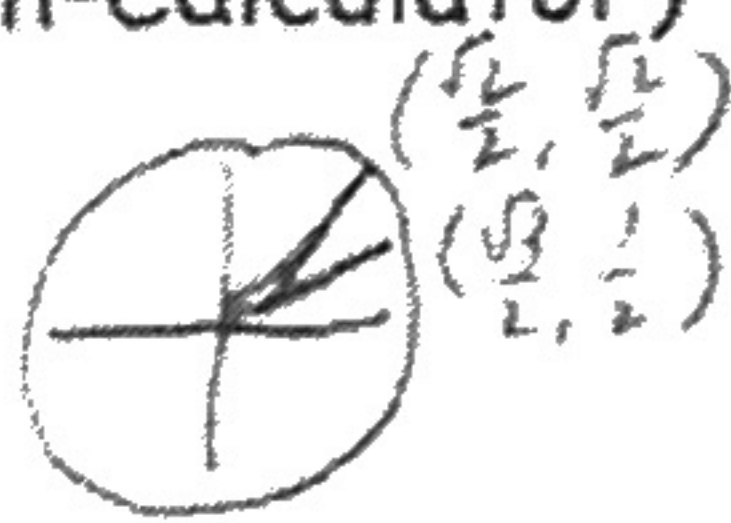
(Non-Calculator)

$\sin(45 - 30) =$

$\sin 45 \cos 30 - \cos 45 \sin 30 =$

$\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) =$

$\frac{\sqrt{6} - \sqrt{2}}{4}$



b) $\cos(105^\circ) =$

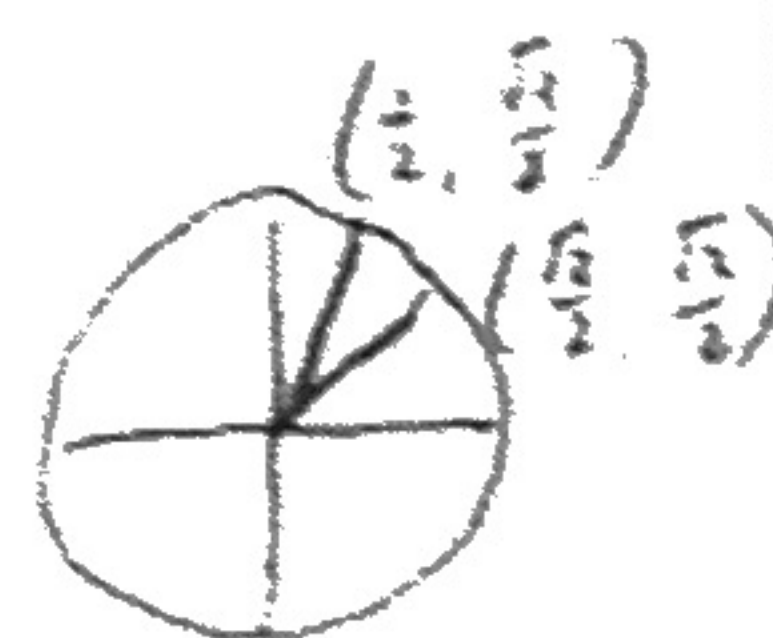
(Non-Calculator)

$\cos(45 + 60) =$

$\cos 45 \cos 60 - \sin 45 \sin 60 =$

$\left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) =$

$\frac{\sqrt{2} - \sqrt{6}}{4}$



19. For the following problems, $\cos A = -\frac{12}{13}$ and $\tan A < 0$, while $\sin B = -\frac{3}{5}$ and $\sec B < 0$.

a) $\cos(A+B) =$

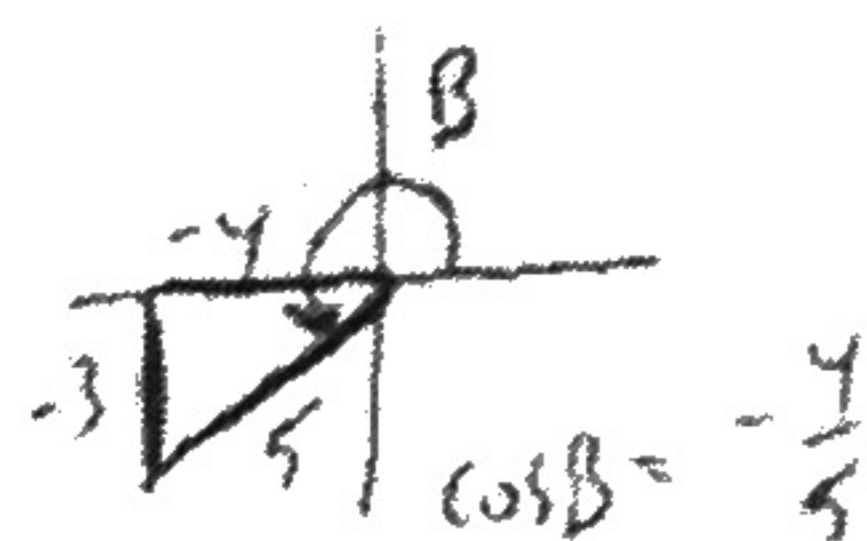
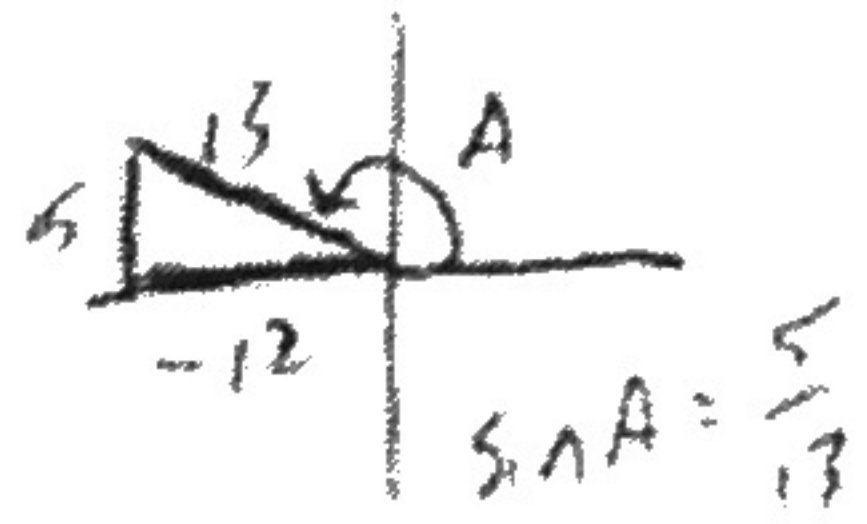
(Non-Calculator)

$\cos A \cos B - \sin A \sin B =$

$\left(-\frac{12}{13}\right)\left(-\frac{4}{5}\right) - \left(\frac{5}{13}\right)\left(-\frac{3}{5}\right) =$

$\frac{48}{65} + \frac{15}{65} =$

$\frac{63}{65}$



b) $\sin(A+B) =$

(Non-Calculator)

$\sin A \cos B + \cos A \sin B =$

$\left(\frac{5}{13}\right)\left(-\frac{4}{5}\right) + \left(-\frac{12}{13}\right)\left(-\frac{3}{5}\right) =$

$-\frac{20}{65} + \frac{36}{65} =$

$\frac{16}{65}$

c) $\cos(2A) =$ (Non-Calculator) d) $\sin(2B) =$ (Non-Calculator)

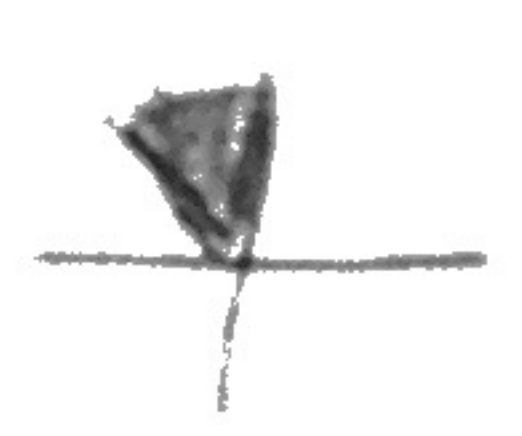
$\cos 2A = 1 - 2\sin^2 A$
 $\cos 2A = 1 - 2\left(\frac{3}{5}\right)^2 = 1 - \frac{18}{25} = \frac{7}{25}$

$\sin 2B = 2\sin B \cos B = 2\left(-\frac{3}{5}\right)\left(-\frac{4}{5}\right) = \frac{24}{25}$

e) $\cos\left(\frac{1}{2}B\right) =$ (Non-Calculator) f) $\sin\left(\frac{1}{2}A\right) =$ (Non-Calculator)

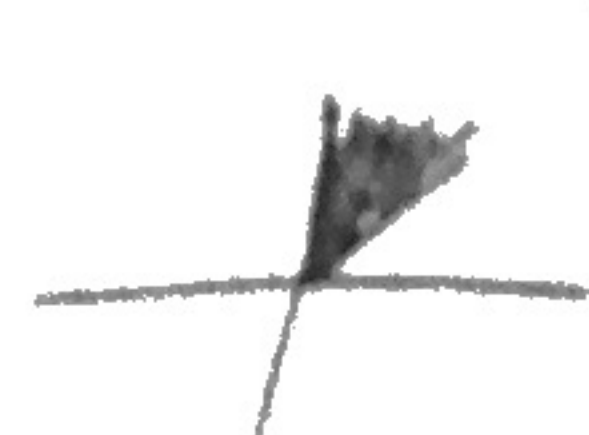
$\cos\left(\frac{1}{2}B\right) = \sqrt{\frac{1 + \cos B}{2}} = \sqrt{\frac{1 + \frac{3}{5}}{2}} = \sqrt{\frac{8}{10}} = \frac{\sqrt{10}}{5}$

$\frac{\pi}{2} < B < \frac{3\pi}{2}$
 $\frac{\pi}{2} < \frac{1}{2}B < \frac{3\pi}{4}$



$\sin\left(\frac{1}{2}A\right) = \sqrt{\frac{1 - \cos A}{2}} = \sqrt{\frac{1 - \left(-\frac{1}{7}\right)}{2}} = \sqrt{\frac{13+12}{26}} = \sqrt{\frac{25}{26}} = \frac{5\sqrt{26}}{26}$

$\frac{\pi}{2} < A < \pi$
 $\frac{\pi}{4} < \frac{1}{2}A < \frac{\pi}{2}$



20. Explain how to identify when to use law of sines or law of cosines. (Non-Calculator)

If two sides and one angle are given, use the first version of Law of Cosines. If one of the given sides is across from the given angle, it will be the ambiguous case. If all three sides are given, use the second version of Law of Cosines to find the missing angle. If two angles and one side are given, use the Law of Sines.

21. Find the find all missing sides, angles, and area of the following triangles:

a) Side $x = 52$ cm, side $y = 38$ cm, and angle $X = 25^\circ$ (Calculator)

$52^2 = 38^2 + z^2 - 2(38)(z)\cos 25$ $\cos Y = \frac{52^2 + 83.898^2 - 38^2}{2(52)(83.898)}$

$0 = z^2 - 68.879z - 1260$

$z = \frac{68.879 \pm \sqrt{68.879^2 - 4(1)(-1260)}}{2(1)}$ $\cos Y = 0.991$ $\text{Area} = \frac{1}{2}(52)(38)\sin 137.011$

$z = \frac{68.879 \pm 78.916}{2}$ $Y = 17.985^\circ$ $\text{Area} = 673.677 \text{ cm}^2$

$z = 83.898, -15.018$ $Z = 180 - 25 - 17.985$ $Z = 137.011^\circ$

b) Side a = 36 cm, side b = 39 cm, and side c = 14 cm.

(Calculator)

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$C = 180 - 67.323 - 91.649$$

$$C = 21.028^\circ$$

$$\cos A = 0.386$$

$$\cos B = -0.089$$

$$s = \frac{1}{2}(36 + 39 + 14) = 44.5$$

$$A = 67.323^\circ$$

$$B = 91.649^\circ$$

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)} = 251.896 \text{ cm}^2$$

c) Side x = 20 cm, angle Y = 52°, and angle X = 37°.

(Calculator)

$$Z = 180 - 52 - 37$$

$$\frac{y}{\sin Y} = \frac{x}{\sin X}$$

$$\frac{z}{\sin Z} = \frac{x}{\sin X}$$

$$\text{Area} = \frac{1}{2}(20)(36.188) \sin 91$$

$$Z = 91^\circ$$

$$\sin 37 y = 20 \sin 52$$

$$\sin 37 z = 20 \sin 91$$

$$\text{Area} = 361.838 \text{ cm}^2$$

$$y = 26.188 \text{ cm}$$

$$z = 33.228 \text{ cm}$$

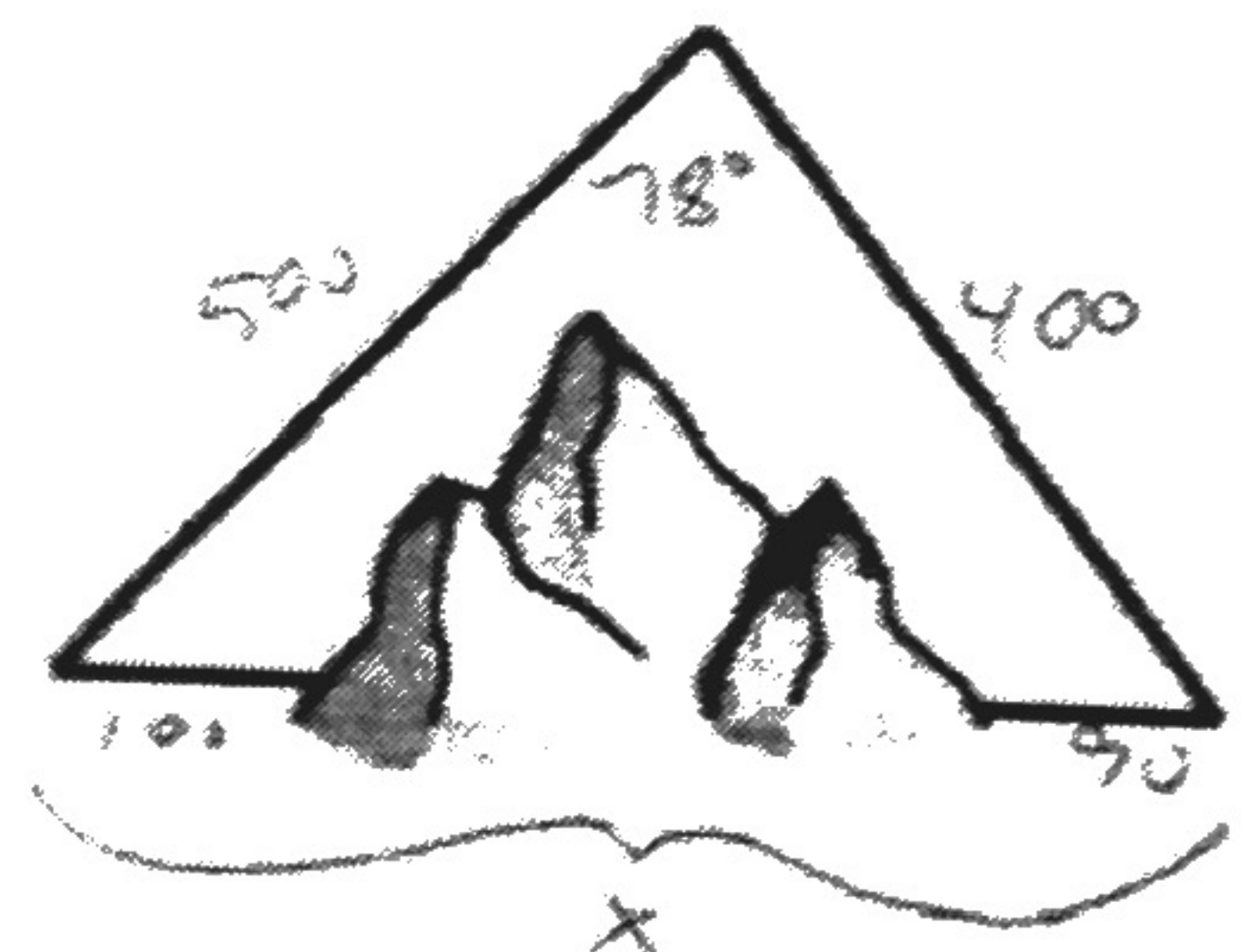
22. You are determining the length of a straight tunnel that is to be dug through a mountain. You measure 100 meters backwards from where the tunnel is to start, then turn and walk 500 meters to a point north of the mountain. You then turn 78° and walk 400 meters to a point which is 90 meters from the where the tunnel is to exit. How long will the tunnel be?

$$x^2 = 500^2 + 400^2 - 2(500)(400) \cos 78$$

$$x = 571.695$$

$$571.695 - 100 - 90$$

$$381.695 \text{ meters}$$



(Calculator)

23. You go sailing on a lake during vacation. You sail 2540 meters from the dock, turn 53° to port (left), then sail for a little while more. To go back to the dock, you turn 154°, also to port. How far did you travel after the first turn? How far is it back to the dock?

After first turn:

Back to the dock:

$$\frac{x}{\sin 27} = \frac{2540}{\sin 26}$$

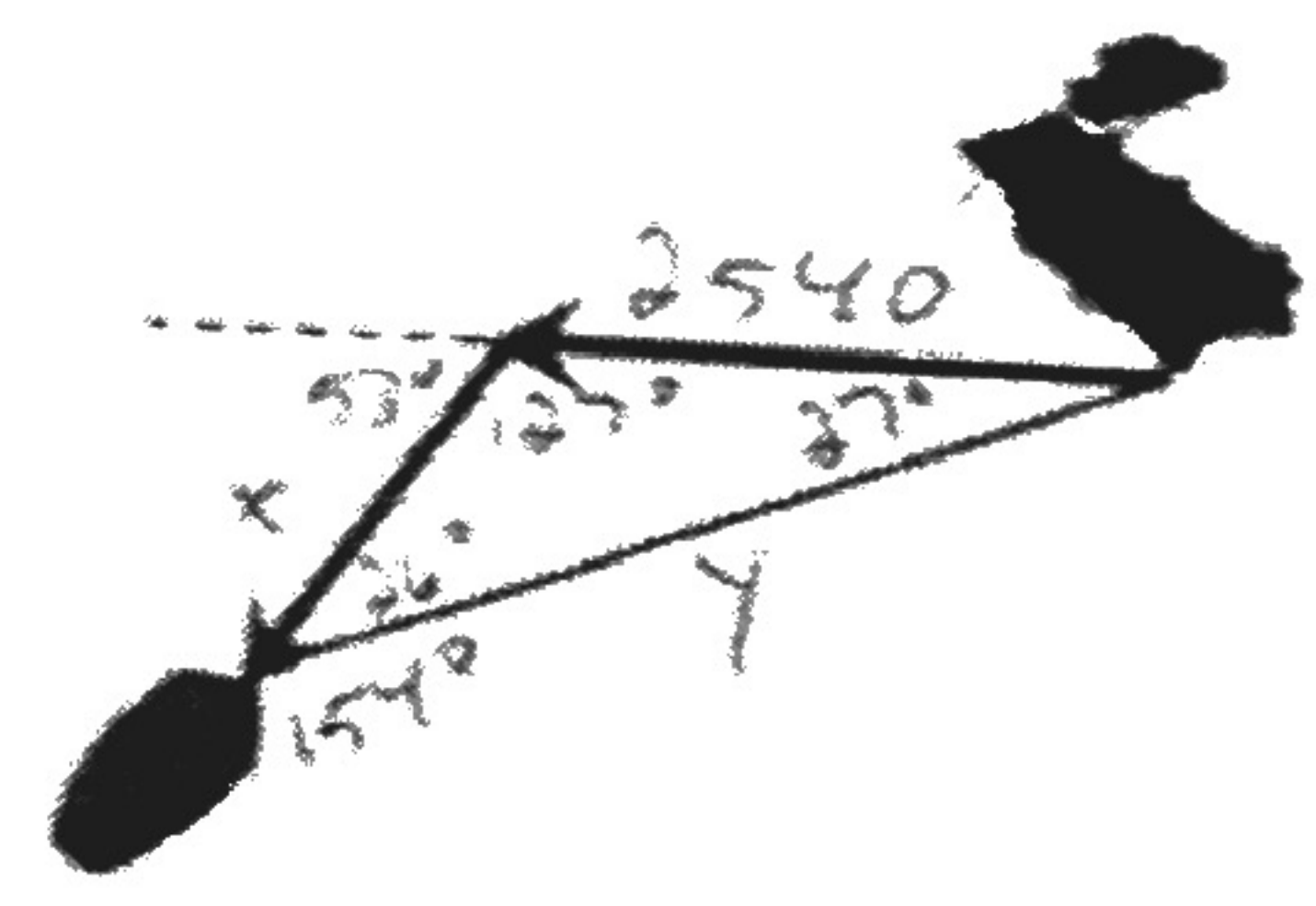
$$\frac{y}{\sin 127} = \frac{2540}{\sin 26}$$

$$\sin 26 x = 2540 \sin 27$$

$$\sin 26 y = 2540 \sin 127$$

$$x = 2630.501 \text{ meters}$$

$$y = 4627.435 \text{ meters}$$



(Calculator)

26. The Northwestern and the Time Bandit are crab fishing in the Bearing Sea. The Northwestern is 100 miles west and 20 miles north of Dutch Harbor, while the Time Bandit is 10 miles east and 40 miles north. The Time Bandit is moving at a speed of 10 mph south and 9 mph west, while the Northwestern is moving at a speed of 8 mph south and 11 mph east. Write a pair of parametric equations for each fishing boat and determine if they hit.

$$\begin{aligned} X_N &= 11t - 100 \\ Y_N &= 20 - 8t \end{aligned}$$

$$10t - 100 = 10 - 9t$$

$$20t = 110$$

$$t = 5.5$$

$$X_N = -39.5$$

$$Y_N = -24$$

$$X_{TB} = -39.5$$

$$Y_{TB} = -15$$

They do not hit because $t = 5.5$ hours is the only time when the x-coordinates are the same, but the y-coordinates are different.

(Calculator)

27. A submarine fires a torpedo from a point 84 meters west and 57 meters north of a sonar buoy at an aircraft carrier which is 56 meters east and 95 meters north of the same buoy. The torpedo moves with an eastward speed of 82 meters per second and a northward speed of 34 meters per second, while the aircraft carrier moves with an eastward speed of 12 meters per second and a northward speed of 15 meters per second. Write a pair of parametric equations for the torpedo and the aircraft carrier and determine if they hit.

$$\begin{aligned} X_T &= 82t - 84 \\ Y_T &= 34t + 57 \end{aligned}$$

$$82t - 84 = 12t + 56$$

$$70t = 140$$

$$t = 2$$

$$\begin{aligned} X_{AC} &= 12t + 56 \\ Y_{AC} &= 15t + 95 \end{aligned}$$

$$X_T = 80$$

$$Y_T = 125$$

$$X_{AC} = 80$$

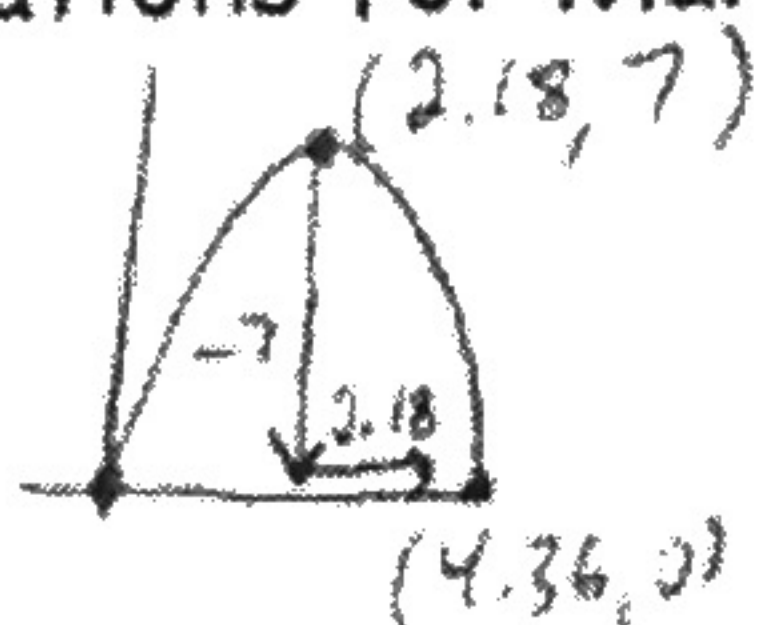
$$Y_{AC} = 125$$

They hit because at $t = 2$ seconds they are both 80 meters east and 125 meters north of the sonar buoy.

(Calculator)

28. Mario is running at a constant rate of 5.2 feet per second. When he jumps, he can stay in the air for 4.36 seconds during which time he reaches a maximum height of 7 feet. When he jumps, a fireball is launched at him from 7 feet behind him at a height of 11 feet. The fireball is moving at a horizontal speed of 8 feet per second and a vertical speed of -2 feet per second. Write a pair of parametric equations for Mario and the Fireball and determine if they hit.

$$\begin{aligned} X_M &= 5.2t \\ Y_M &= -1.473(t - 2.18)^2 + 7 \end{aligned}$$



$$a = \frac{-7}{2.18^2} = -1.473$$

(Calculator)

$$\begin{aligned} X_F &= 8t - 7 \\ Y_F &= 11 - 2t \end{aligned}$$

$$5.2t = 8t - 7$$

$$-2.8t = -7$$

$$t = 2.5$$

$$X_M = 13$$

$$Y_M = 6.849$$

$$X_F = 13$$

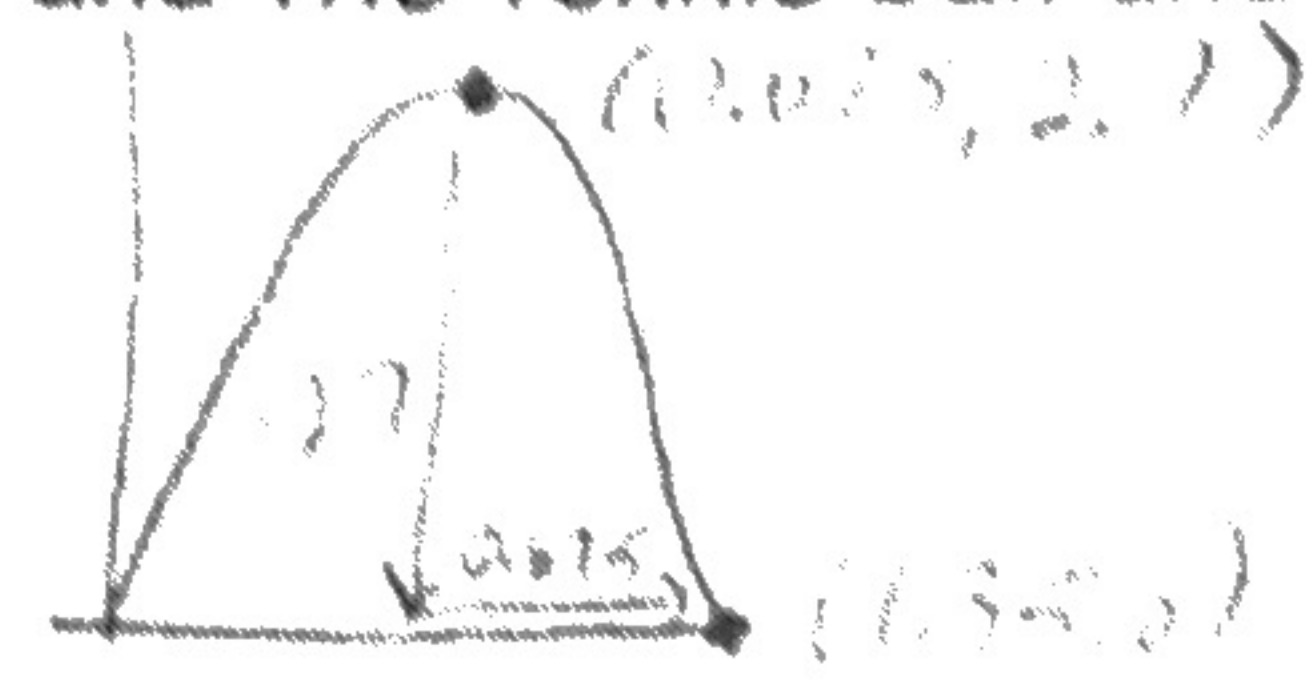
$$Y_F = 6$$

They do not hit because $t = 2.5$ seconds is the only time when the x-coordinates are the same, but the fireball is 0.849 feet below Mario's feet.

29. John is bouncing on a pogo stick that is moving him forward at a rate of 2.4 ft/sec. Each time he bounces, he stays in the air for 1.35 seconds and reaches a maximum height of 2.7 feet. Bob is hiding in a bush to ambush John. When John passes by, Bob throws a tennis ball at him when he is 10 feet behind John. The tennis ball is travelling at a horizontal velocity of 20.2 ft/sec and a vertical velocity of 15.3 ft/sec. He releases it 6 feet above the ground. Write a pair of parametric equations for John and the tennis ball and determine if they hit.

$$x_j = 2.4t$$

$$y_j = -5.926(t - 0.675)^2 + 2.7$$



$a = \frac{2.7}{0.675^2} = 5.926$ (Calculator)

$$x_{TB} = 20.2t - 10$$

$$y_{TB} = 6 + 15.3t - 5.926t^2$$

$$2.4t = 20.2t - 10$$

$$-17.8t = -10$$

$$t = 0.562$$

$$x_j = 1.348$$

$$y_j = 2.024$$

$$x_{TB} = 1.348$$

$$y_{TB} = 12.725$$

They do not hit because 0.562 seconds is the only time when the x-coordinates are the same, but the tennis ball is 10.101 feet above his feet

... of parametric