Review 4-2

M4 Lynn

1C. In ΔABC angle $A = 41^\circ$, angle $B = 72^\circ$, and side $a = 15\,cm$. Find sides b and c and the area.

$\frac{b}{\sin 72} = \frac{15}{\sin 41}$

$b \sin 41 = 15 \sin 72$

$\boxed{b = 21.745\ cm}$

$\frac{c}{\sin 67} = \frac{15}{\sin 41}$

$c \sin 41 = 15 \sin 67$

$\boxed{c = 21.046\ cm}$

$Area = \frac{1}{2}(15)(21.745)\sin 67$

$\boxed{Area = 150.121\ cm^2}$

2C. In ΔABC side $a = 11\,m$, side $b = 12\,m$, and side $c = 13\,m$. Find angle A and the area.

$\cos A = \frac{12^2 + 13^2 - 11^2}{2(12)(13)}$

$\cos A = 0.615$

$\boxed{A = 52.020^\circ}$

$s = \frac{1}{2}(11 + 12 + 13)$

$s = 18$

$Area = \sqrt{18(18-11)(18-12)(18-13)}$

$\boxed{Area = 61.482\ m^2}$

3C. In ΔABC angle $B = 56^\circ$, side $a = 98$ ft, and side $b = 85$ ft. Find side c and the area.

$85^2 = 98^2 + c^2 - 2(98)(c)\cos 56$

$0 = c^2 - 109.602c + 2379$

$c = \frac{109.602 \pm \sqrt{109.602^2 - 4(1)(2379)}}{2(1)}$

$\boxed{c = 79.784\ ft,\ 29.818\ ft}$

$Area_1 = \frac{1}{2}(98)(79.784)\sin 56$

$\boxed{Area_1 = 3241.040\ ft^2}$

$Area_2 = \frac{1}{2}(98)(29.818)\sin 56$

$\boxed{Area_2 = 1211.297\ ft^2}$

4C. In ΔABC angle $B = 87^\circ$, angle $C = 24^\circ$, and side $a = 113$ mi. Find sides b and c and the area.

$\frac{b}{\sin 87} = \frac{113}{\sin 69}$

$b \sin 69 = 113 \sin 87$

$\boxed{b = 120.874\ mi}$

$\frac{c}{\sin 24} = \frac{113}{\sin 69}$

$c \sin 69 = 113 \sin 24$

$\boxed{c = 49.231\ mi}$

$Area = \frac{1}{2}(113)(120.874)\sin 24$

$\boxed{Area = 2777.748\ mi^2}$

5C. In ΔABC side $c = 634\,cm$, side $b = 600\,cm$, and angle $B = 78^\circ$ [redacted]. Find side a and the area.

$600^2 = a^2 + 634^2 - 2(a)(634)\cos 78$

$0 = a^2 - 263.632a + 41956$

$a = \frac{263.632 \pm \sqrt{263.632^2 - 4(1)(41956)}}{2(1)}$

a = no solution

These measurements cannot make a triangle

6C. In ΔABC side $a = 19$ ft, side $b = 4$ ft, and side $c = 22$ ft. Find angle C and the area.

$\cos C = \dfrac{19^2 + 4^2 - 22^2}{2(19)(4)}$

$\cos C = -0.704$

$\boxed{C = 134.745^\circ}$

$s = \frac{1}{2}(19 + 4 + 22)$

$s = 22.5$

$Area = \sqrt{22.5(22.5-19)(22.5-4)(22.5-22)}$

$\boxed{Area = 26.990 \text{ ft}^2}$

7C. In ΔABC angle $A = 107^\circ$ [redacted], side $b = 17$ m, and side $c = 25$ m. Find side a and the area.

$a^2 = 17^2 + 25^2 - 2(17)(25)\cos 107$

$a^2 = 1162.516$

$\boxed{a = 34.096 \text{ m}}$

$Area = \frac{1}{2}(17)(25)\sin 107$

$\boxed{Area = 203.215 \text{ m}^2}$

8C. In ΔABC angle $A = 47^\circ$, angle $B = 63^\circ$ [redacted], and side $c = 123$ km. Find sides a and b and the area.

$\dfrac{a}{\sin 47} = \dfrac{123}{\sin 70}$

$a \sin 70 = 123 \sin 47$

$\boxed{a = 95.730 \text{ km}}$

$\dfrac{b}{\sin 63} = \dfrac{123}{\sin 70}$

$b \sin 70 = 123 \sin 63$

$\boxed{b = 116.627 \text{ km}}$

$Area = \frac{1}{2}(95.730)(123)\sin 63$

$\boxed{Area = 5245.692 \text{ km}^2}$

9C. In ΔABC angle $C = 58^\circ$, side $c = 50$ in, and side $a = 67$ in. Find angle A and the area.

$50^2 = 67^2 + b^2 - 2(67)(b)\cos 58$

$0 = b^2 - 71.009b + 1989$

$b = \dfrac{71.009 \pm \sqrt{71.009^2 - 4(1)(1989)}}{2(1)}$

$b =$ no solution

These measurements cannot make a triangle

10C. In ΔABC angle $A = 74^\circ$, side $a = 59.2$ ft, and side [redacted] b = 60.3 ft. Find angle B and the area.

$59.2^2 = 60.3^2 + c^2 - 2(60.3)(c)\cos 74$

$0 = c^2 - 33.242c + 131.45$

$c = \dfrac{33.242 \pm \sqrt{33.242^2 - 4(1)(131.45)}}{2(1)}$

$c = 28.654,\ 4.587$

$\cos B_1 = \dfrac{59.2^2 + 28.654^2 - 60.3^2}{2(59.2)(28.654)}$

$\cos B_1 = 0.203$

$\boxed{B_1 = 78.272^\circ}$

$\cos B_2 = \dfrac{59.2^2 + 4.587^2 - 60.3^2}{2(59.2)(4.587)}$

$\cos B_2 = -0.203$

$\boxed{B_2 = 101.728^\circ}$

$Area_1 = \frac{1}{2}(59.2)(28.654)\sin 78.272$

$\boxed{Area_1 = 830.464 \text{ ft}^2}$

$Area_2 = \frac{1}{2}(59.2)(4.587)\sin 101.728$

$\boxed{Area_2 = 132.953 \text{ ft}^2}$

11C. A sailboat leaves St. Thomas bound for an island in the British West Indies, 200 miles away. Maintaining a constant speed of 18 miles per hour, but encountering heavy crosswinds and strong currents, the crew finds after 4 hours that the sailboat is off course by 15°. How far is the sailboat from the island at this time? Through what angle should the sailboat turn to correct its course? How much time has been added to the trip because of this (assume the speed remains the same)?

$x^2 = 200^2 + 72^2 - 2(200)(72)\cos 15$

$x^2 = 17365.3362$

$\boxed{x = 131.778 \text{ miles}}$

$\cos\theta = \dfrac{72^2 + 131.778^2 - 200^2}{2(72)(131.778)}$

$\cos\theta = -0.920$

$\theta = 156.870°$

$\phi = 180 - 156.870°$

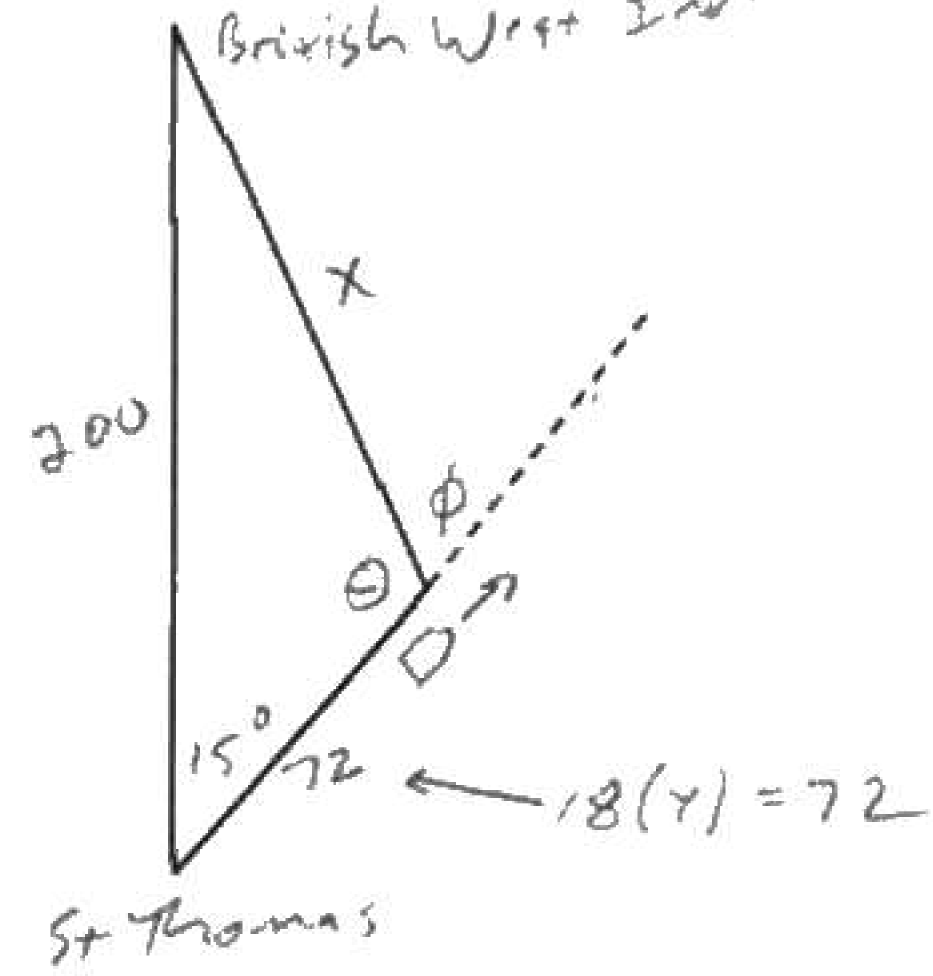
$\boxed{\phi = 23.130°}$

$72 + 131.778 - 200 = 18t$

$3.778 = 18t$

$t = 0.210$ hours

$\boxed{t = 12.592 \text{ minutes}}$

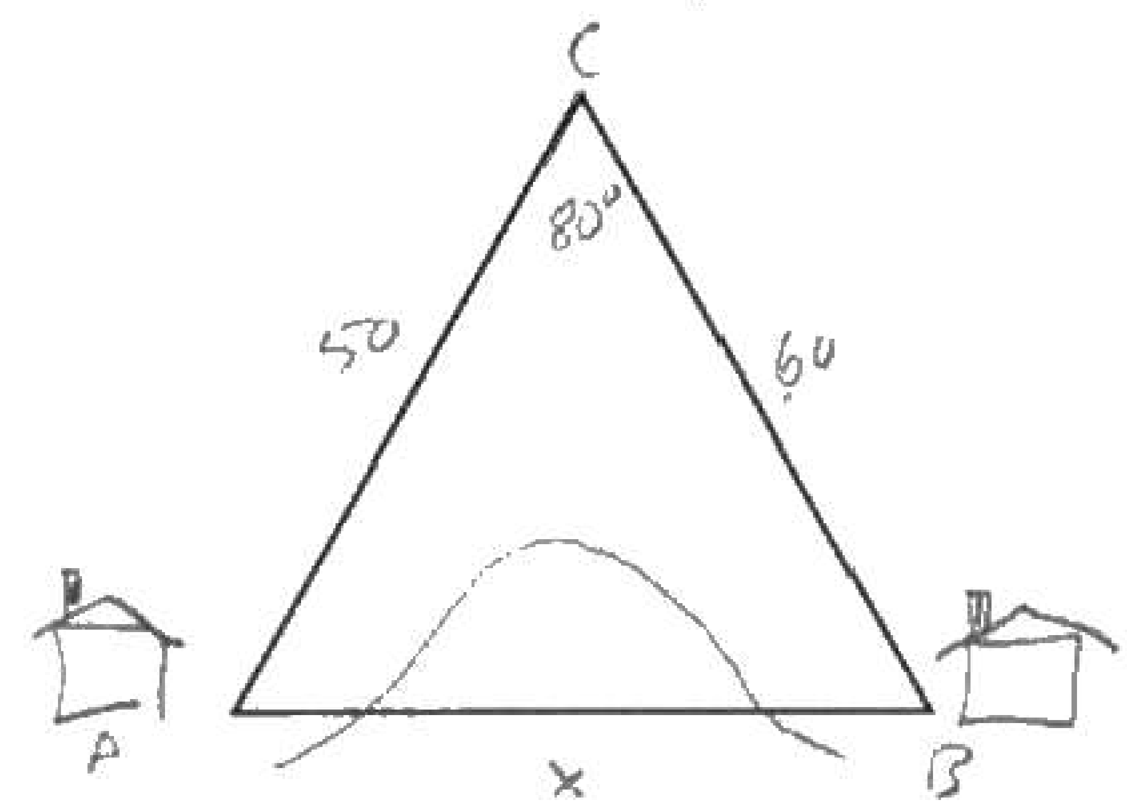


12C. Two homes are located on opposite sides of a small hill. To measure the distance between them, a surveyor walks a distance of 50 ft from house A to point C, uses a transit to measure the angle ACB, which is found to be 80°, and then walks to house B, a distance of 60 ft. How far apart are the houses?

$x^2 = 50^2 + 60^2 - 2(50)(60)\cos 80$

$x^2 = 5058.111$

$\boxed{x = 71.120 \text{ ft}}$



13C. Rebecca, the navigator of a ship at sea, spots two lighthouses that she knows to be 2 miles apart along a straight shoreline. She determines that the angles formed between two line-of-sight observations of the lighthouses and the line from the ship directly to shore are 12° and 30°. How far is the ship from each of the lighthouses (two different distances). How far is the ship from shore?

$\dfrac{a}{\sin 60} = \dfrac{2}{\sin 42}$

$a \sin 42 = 2 \sin 60$

$\boxed{a = 2.589 \text{ miles}}$

$\dfrac{b}{\sin 78} = \dfrac{2}{\sin 42}$

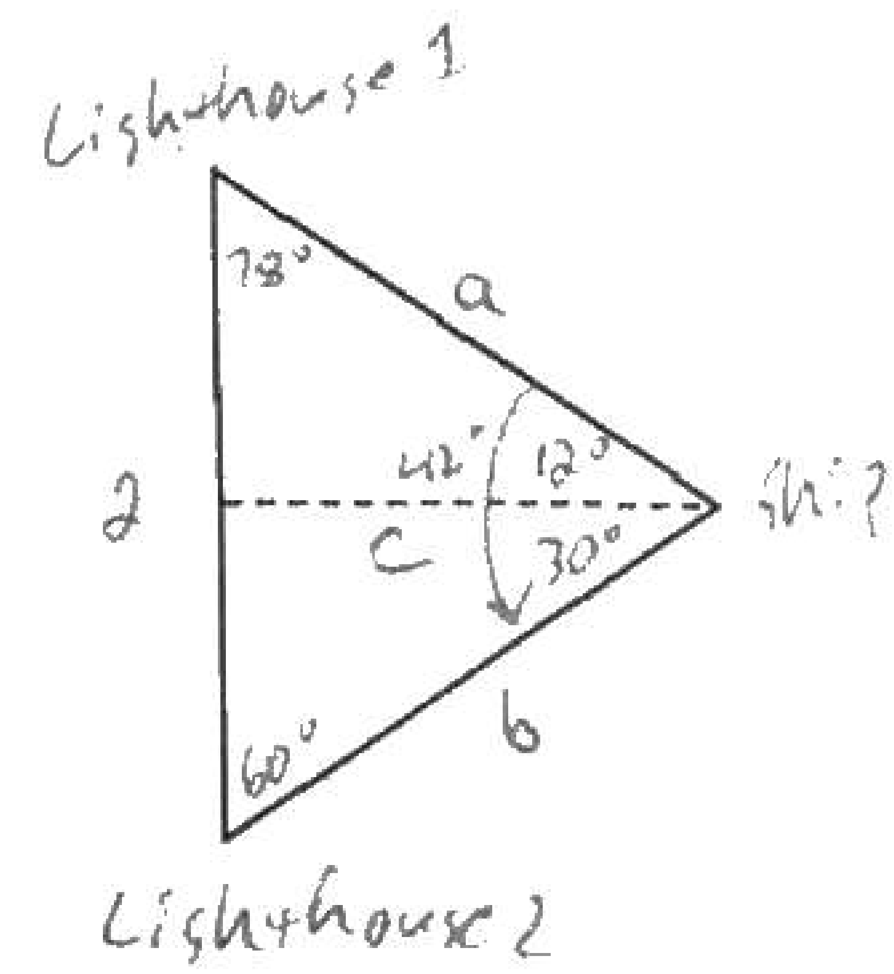
$b \sin 42 = 2 \sin 78$

$\boxed{b = 2.924 \text{ miles}}$

$\dfrac{c}{\sin 60} = \dfrac{2.924}{\sin 90}$

$c \sin 90 = 2.924 \sin 60$

$\boxed{c = 2.532 \text{ miles}}$



14C. An aircraft is spotted by two observers who are 5' 9" tall and 1000 ft apart. As the airplane passes over the line joining them, each observer takes a sighting of the angle of elevation to the plane – the two measurements are 35° and 40°. How far is the plane from each observer when the sightings were taken? How high is the airplane?

$\frac{a}{\sin 40} = \frac{1000}{\sin 105}$

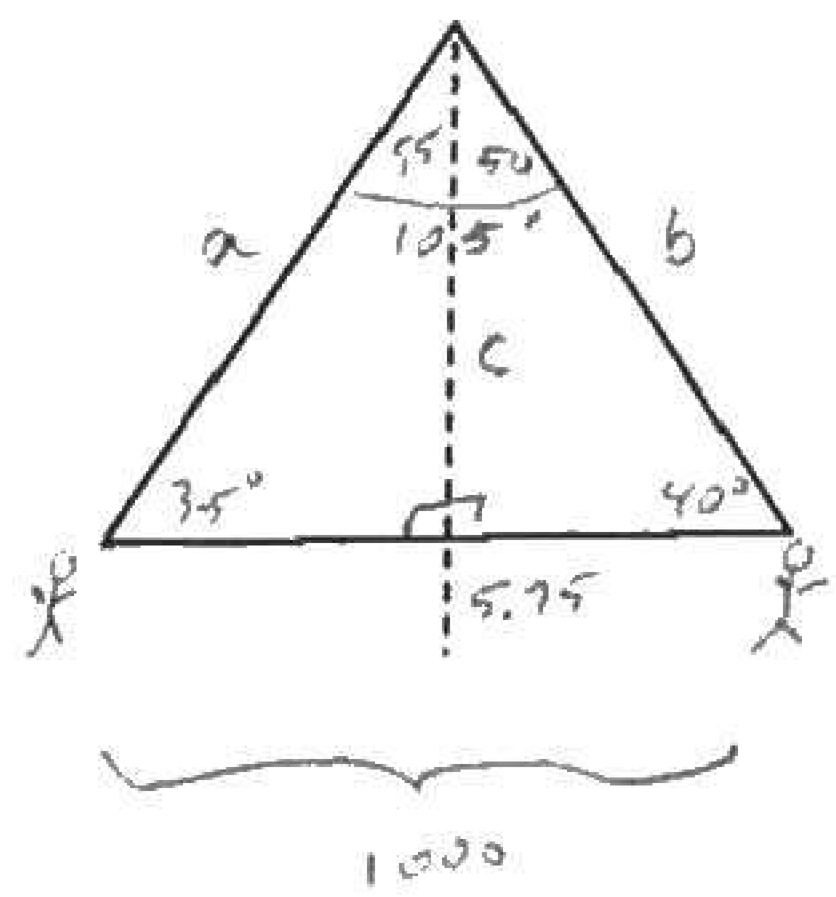
$a \sin 105 = 1000 \sin 40$

$\boxed{a = 665.463 \text{ ft}}$

$\frac{b}{\sin 35} = \frac{1000}{\sin 105}$

$b \sin 105 = 1000 \sin 35$

$\boxed{b = 593.810 \text{ ft}}$



15C. In attempting to fly from city A to city B, an aircraft followed a course that was 10° in error. After flying a distance of 50 mi, the pilot corrected the course by turning at point C and flying 70 miles farther (thus arrive at city B). If the constant speed of the aircraft was 250 miles per hour, how much time was lost due to the error?

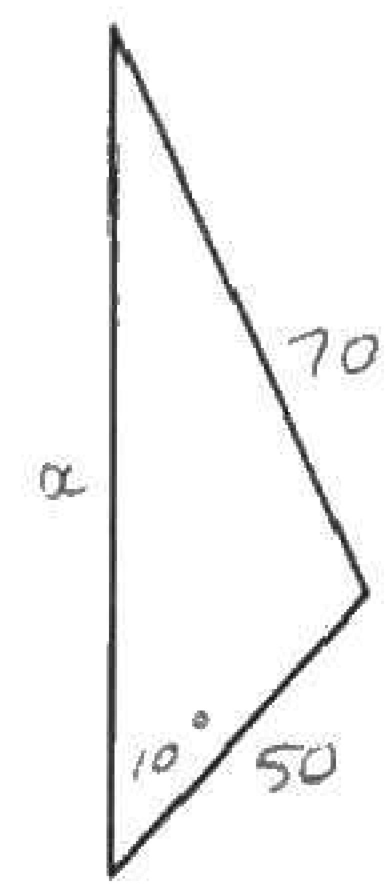
$70^2 = a^2 + 50^2 - 2(a)(50)\cos 10$

$0 = a^2 - 98.481a - 2400$

$a = \frac{98.481 \pm \sqrt{98.481^2 - 4(1)(-2400)}}{2(1)}$

$a = 118.700, \; -20.219$

$a = 118.700$ miles



$\frac{118.770 \text{ miles}}{1} \cdot \frac{1 \text{ hr}}{250 \text{ miles}} = 0.475 \text{ hours} = 28.488 \text{ minutes}$

$\frac{(50+70) \text{ miles}}{1} \cdot \frac{1 \text{ hr}}{250 \text{ miles}} = 0.48 \text{ hours} = 28.8 \text{ minutes}$

$28.8 - 28.488 = 0.312 \text{ minutes} = \boxed{18.722 \text{ seconds}}$