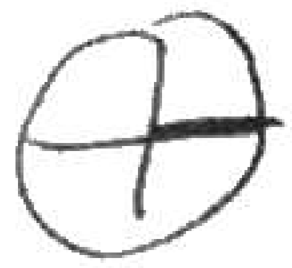
Review 4-1

M4 7am

Solve the following equations on the interval $0 \le x < 2\pi$:

1NC. $\cos x - 1 = 0$



$\cos x = 1$

$x = 0 + 2\pi n$

$\boxed{x = 0}$

2NC. $\csc x + 2 = 0$

$\csc x = -2$

$\sin x = -\frac{1}{2}$

$\sin x = \frac{7\pi}{6} + 2\pi n$ $\qquad$ $\sin x = \frac{11\pi}{6} + 2\pi n$

$\boxed{x = \frac{7\pi}{6}, \frac{11\pi}{6}}$

3NC. $2\sin(4x) - 1 = 0$

$2\sin(4x) = 1$

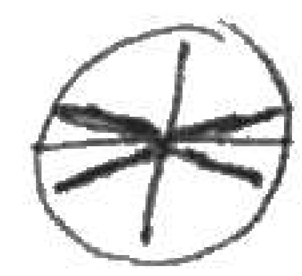
$\sin(4x) = \frac{1}{2}$

$4x = \frac{\pi}{6} + 2\pi n$ $\qquad$ $4x = \frac{5\pi}{6} + 2\pi n$

$x = \frac{\pi}{24} + \frac{\pi}{2} n$ $\qquad$ $x = \frac{5\pi}{24} + \frac{\pi}{2} n$

$\boxed{x = \frac{\pi}{24}, \frac{5\pi}{24}, \frac{13\pi}{24}, \frac{17\pi}{24}, \frac{25\pi}{24}, \frac{29\pi}{24}, \frac{37\pi}{24}, \frac{41\pi}{24}}$

4NC. $\sin x = \dfrac{1}{4\sin x}$



$4\sin^2 x = 1$

$\sin^2 x = \frac{1}{4}$

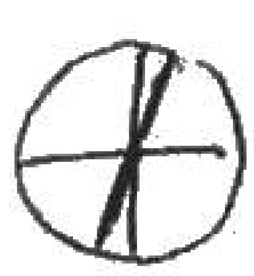
$\sin x = \pm\frac{1}{2}$

$x = \frac{\pi}{6} + 2\pi n$ $\qquad$ $x = \frac{5\pi}{6} + 2\pi n$

$x = \frac{7\pi}{6} + 2\pi n$ $\qquad$ $x = \frac{11\pi}{6} + 2\pi n$

$\boxed{x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}}$

5NC. $\tan(3x) = \sqrt{3}$



$3x = \frac{\pi}{3} + 2\pi n$ $\qquad$ $3x = \frac{4\pi}{3} + 2\pi n$

$x = \frac{\pi}{9} + \frac{2\pi}{3} n$ $\qquad$ $x = \frac{4\pi}{9} + \frac{2\pi}{3} n$

$\boxed{x = \frac{\pi}{9}, \frac{4\pi}{9}, \frac{7\pi}{9}, \frac{10\pi}{9}, \frac{13\pi}{9}, \frac{16\pi}{9}}$

6NC. $8\sin\left(\dfrac{x}{2}\right) - 8 = 0$

$8\sin\left(\frac{x}{2}\right) = 8$

$\sin\left(\frac{x}{2}\right) = 1$

$\frac{x}{2} = \frac{\pi}{2} + 2\pi n$

$x = \pi + 4\pi n$

$\boxed{x = \pi}$

7NC. $2\cos^2(2x)-1=0$

$2\cos^2(2x) = 1$

$\cos^2(2x) = \frac{1}{2}$

$\cos(2x) = \pm\sqrt{\frac{1}{2}}$

$\cos(2x) = \pm\frac{\sqrt{2}}{2}$

$2x = \frac{\pi}{4} + 2\pi n$ $\quad$ $2x = \frac{3\pi}{4} + 2\pi n$

$x = \frac{\pi}{8} + \pi n$ $\quad$ $x = \frac{3\pi}{8} + \pi n$

$2x = \frac{5\pi}{4} + 2\pi n$ $\quad$ $2x = \frac{7\pi}{4} + 2\pi n$

$x = \frac{5\pi}{8} + \pi n$ $\quad$ $x = \frac{7\pi}{8} + \pi n$

$\boxed{x = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8}}$

8NC. $\sec(3x)=\sqrt{2}$

$\cos(3x) = \frac{1}{\sqrt{2}}$

$\cos(3x) = \frac{\sqrt{2}}{2}$

$3x = \frac{\pi}{4} + 2\pi n$ $\quad$ $3x = \frac{7\pi}{4} + 2\pi n$

$x = \frac{\pi}{12} + \frac{2\pi}{3}n$ $\quad$ $x = \frac{7\pi}{12} + \frac{2\pi}{3}n$

$\boxed{x = \frac{\pi}{12}, \frac{7\pi}{12}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{17\pi}{12}, \frac{23\pi}{12}}$

9NC. $\sin\left(\theta-\frac{\pi}{4}\right)=-\frac{\sqrt{2}}{2}$

$\theta - \frac{\pi}{4} = \frac{5\pi}{4} + 2\pi n$ $\quad$ $\theta - \frac{\pi}{4} = \frac{7\pi}{4} + 2\pi n$

$\theta = \frac{3\pi}{2} + 2\pi n$ $\quad$ $\theta = 2\pi + 2\pi n$

$\boxed{\theta = 0, \frac{3\pi}{2}}$

10NC. $3\tan^2\left(2x-\frac{\pi}{2}\right)-1=0$

$3\tan^2\left(2x-\frac{\pi}{2}\right) = 1$

$\tan^2\left(2x-\frac{\pi}{2}\right) = \frac{1}{3}$

$\tan\left(2x-\frac{\pi}{2}\right) = \pm\sqrt{\frac{1}{3}}$

$\tan\left(2x-\frac{\pi}{2}\right) = \pm\frac{\sqrt{3}}{3}$

$2x - \frac{\pi}{2} = \frac{\pi}{6} + 2\pi n$ $\quad$ $2x - \frac{\pi}{2} = \frac{5\pi}{6} + 2\pi n$

$2x = \frac{2\pi}{3} + 2\pi n$ $\quad$ $2x = \frac{4\pi}{3} + 2\pi n$

$x = \frac{\pi}{3} + \pi n$ $\quad$ $x = \frac{2\pi}{3} + \pi n$

$2x - \frac{\pi}{2} = \frac{7\pi}{6} + 2\pi n$ $\quad$ $2x - \frac{\pi}{2} = \frac{11\pi}{6} + 2\pi n$

$2x = \frac{5\pi}{3} + 2\pi n$ $\quad$ $2x = \frac{7\pi}{3} + 2\pi n$

$x = \frac{5\pi}{6} + \pi n$ $\quad$ $x = \frac{7\pi}{6} + 2\pi n$

$\boxed{x = \frac{\pi}{6}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{4\pi}{3}, \frac{5\pi}{3}, \frac{11\pi}{6}}$

11NC. $2\sin^2 x - 5\sin x = -3$

$2\sin^2 x - 5\sin x + 3 = 0$

$(2\sin x - 3)(\sin x - 1) = 0$

$2\sin x - 3 = 0$ $\quad$ $\sin x - 1 = 0$

$2\sin x = 3$ $\quad$ $\sin x = 1$

$\sin x = \frac{3}{2}$ $\quad$ $x = \frac{\pi}{2} + 2\pi n$

no solution

$\boxed{x = \frac{\pi}{2}}$

12NC. $2\cos^3 x + \cos^2 x = 0$

$\cos^2 x\,(2\cos x + 1) = 0$

$\cos^2 x = 0$ $\quad$ $2\cos x + 1 = 0$

$\cos x = 0$ $\quad$ $2\cos x = -1$

$x = \frac{\pi}{2} + 2\pi n$ $\quad$ $\cos x = -\frac{1}{2}$

$x = \frac{3\pi}{2} + 2\pi n$ $\quad$ $x = \frac{2\pi}{3} + 2\pi n$

$x = \frac{4\pi}{3} + 2\pi n$

$\boxed{x = \frac{\pi}{2}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{3\pi}{2}}$

Prove the following:

13NC. Show that $\cot(-x) = -\cot x$ for $x = \dfrac{13\pi}{6}$ using the graph to the right:

$\cot\left(-\frac{13\pi}{6}\right) =$

$\cot\left(-\frac{13\pi}{6} + \frac{12\pi}{6}\right) =$

$\cot\left(-\frac{\pi}{6}\right) =$

$-\frac{3}{\sqrt{3}} =$

$\boxed{-\sqrt{3}}$

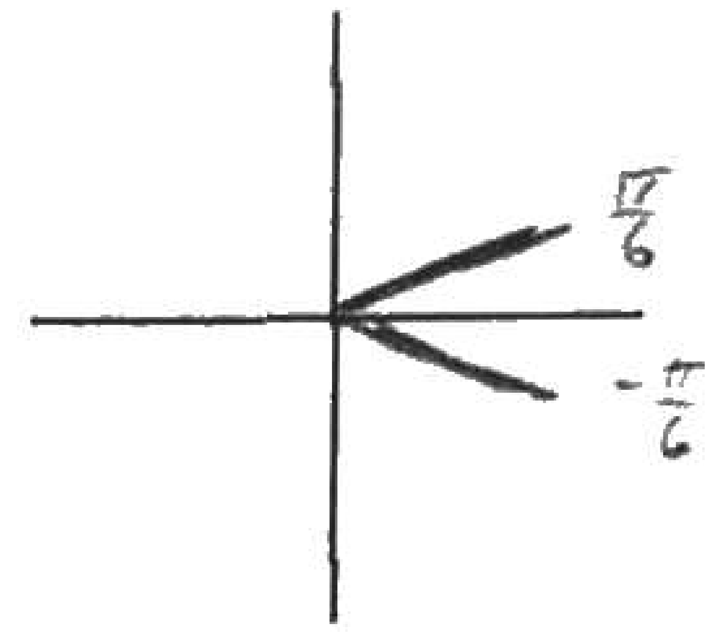
$-\cot\left(\frac{13\pi}{6}\right) =$

$-\cot\left(\frac{13\pi}{6} - \frac{12\pi}{6}\right) =$

$-\cot\left(\frac{\pi}{6}\right) =$

$-\frac{3}{\sqrt{3}} -$

$\boxed{-\sqrt{3}}$



14NC. Show that: $\sec(-x) = \sec x$ for $x = \dfrac{14\pi}{3}$ using the graph to the right:

$\sec\left(-\frac{14\pi}{3}\right) =$

$\sec\left[-\frac{14\pi}{3} + \frac{6\pi}{3}(2)\right] =$

$\sec\left(-\frac{2\pi}{3}\right) =$

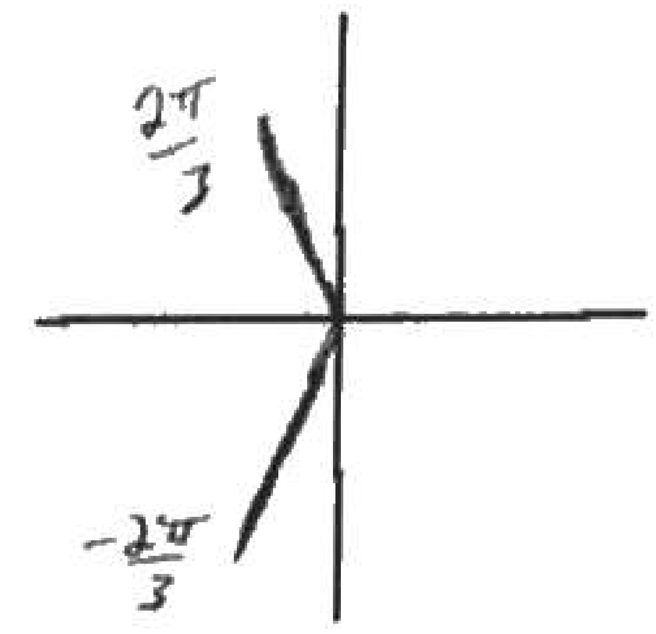
$\boxed{-2}$

$\sec\left(\frac{14\pi}{3}\right) =$

$\sec\left[\frac{14\pi}{3} - \frac{6\pi}{3}(2)\right] =$

$\sec\left(\frac{2\pi}{3}\right) =$

$\boxed{-2}$



15NC. Prove:

$$\frac{1+\tan x}{\sin x} - \sec x = \csc x$$

$\frac{1}{\sin x} + \frac{\tan x}{\sin x} - \frac{1}{\cos x} = \frac{1}{\sin x}$

$\frac{1}{\sin x} + \frac{\cancel{\sin x}}{\cos x}\left(\frac{1}{\cancel{\sin x}}\right) - \frac{1}{\cos x} = \frac{1}{\sin x}$

$\frac{1}{\sin x} + \frac{1}{\cos x} - \frac{1}{\cos x} = \frac{1}{\sin x}$

$\frac{1}{\sin x} = \frac{1}{\sin x}$

16NC. Prove:

$$\frac{\csc x}{\sin x} - \frac{\cot x}{\tan x} = 1$$

$\frac{1}{\sin x}\left(\frac{1}{\sin x}\right) - \frac{\cos x}{\sin x}\left(\frac{\cos x}{\sin x}\right) = 1$

$\frac{1}{\sin^2 x} - \frac{\cos^2 x}{\sin^2 x} = 1$

$\frac{1-\cos^2 x}{\sin^2 x} = 1$

$\frac{\sin^2 x}{\sin^2 x} = 1$

$1 = 1$

17NC. Prove:

$$\frac{\sec^2 x}{\cot x} - \tan^3 x = \tan x$$

$\frac{1}{\cos^2 x}\left(\frac{\sin x}{\cos x}\right) - \frac{\sin^3 x}{\cos^3 x} = \frac{\sin x}{\cos x}$

$\frac{\sin x - \sin^3 x}{\cos^3 x} = \frac{\sin x}{\cos x}$

$\frac{\sin x\,(1-\sin^2 x)}{\cos^3 x} = \frac{\sin x}{\cos x}$

$\frac{\sin x\,(\cos^2 x)}{\cos^3 x} = \frac{\sin x}{\cos x}$

$\frac{\sin x}{\cos x} = \frac{\sin x}{\cos x}$

18NC. Prove:

$$\frac{\tan^2 x + 1}{\tan^2 x} = \csc^2 x$$

$\frac{\sec^2 x}{\tan^2 x} = \csc^2 x$

$\frac{1}{\cancel{\cos^2 x}}\left(\frac{\cancel{\cos^2 x}}{\sin^2 x}\right) = \frac{1}{\sin^2 x}$

$\frac{1}{\sin^2 x} = \frac{1}{\sin^2 x}$

19NC. Prove:

$$(\sin x + \cos x)^2 + (\sin x - \cos x)^2 = 2$$

$\sin^2 x + 2\sin x\cos x + \cos^2 x + \sin^2 x - 2\sin x\cos x + \cos^2 x = 2$

$(\sin^2 x + \cos^2 x) + (\sin^2 x + \cos^2 x) = 2$

$1 + 1 = 2$

$2 = 2$

20NC. Prove:

$$\frac{1-\sin^2 x}{1+\cot^2 x} = \sin^2 x \cos^2 x$$

$\dfrac{\cos^2 x}{\csc^2 x} = \sin^2 x \cos^2 x$

$\dfrac{\cos^2 x}{1/\sin^2 x} = \sin^2 x \cos^2 x$

$\cos^2 x \sin^2 x = \sin^2 x \cos^2 x$

21NC. Prove:

$$\frac{\tan x}{1+\sec x} + \frac{1+\sec x}{\tan x} = 2\csc x$$

$\dfrac{\tan x(\sec x - 1)}{(\sec x + 1)(\sec x - 1)} + \dfrac{1+\sec x}{\tan x} = 2\csc x$

$\dfrac{\tan x(\sec x - 1)}{\sec^2 x - 1} + \dfrac{1+\sec x}{\tan x} = 2\csc x$

$\dfrac{\tan x(\sec x - 1)}{\tan^2 x} + \dfrac{1+\sec x}{\tan x} = 2\csc x$

$\dfrac{\sec x - 1}{\tan x} + \dfrac{1+\sec x}{\tan x} = 2\csc x$

$\dfrac{2\sec x}{\tan x} = 2\csc x$

$2\left(\dfrac{1}{\cos x}\right)\left(\dfrac{\cos x}{\sin x}\right) = 2\left(\dfrac{1}{\sin x}\right)$

$\dfrac{2}{\sin x} = \dfrac{2}{\sin x}$

22NC. Prove:

$$\csc^4 x - 2\csc^2 x + 1 = \cot^4 x$$

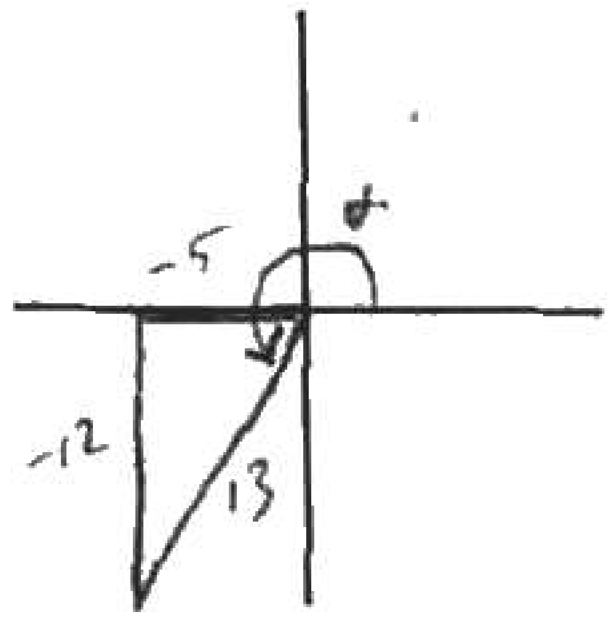
$(\csc^2 x - 1)(\csc^2 x - 1) = \cot^4 x$

$\cot^2 x(\cot^2 x) = \cot^4 x$

$\cot^4 x = \cot^4 x$

23NC. Given the following conditions:

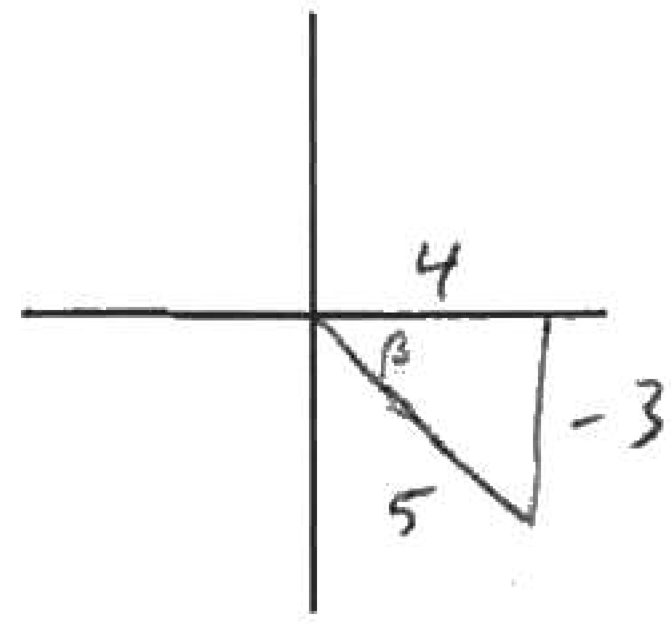
$\tan\alpha = \frac{12}{5}$ where $\pi < \alpha < \frac{3\pi}{2}$



$\sin\alpha = -\frac{12}{13}$

$\cos\alpha = -\frac{5}{13}$

$\cos\beta = \frac{4}{5}$ where $-\frac{\pi}{2} < \beta < 0$



$\sin\beta = -\frac{3}{5}$

find the exact value of:

a) $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$= \left(-\frac{12}{13}\right)\left(\frac{4}{5}\right) + \left(-\frac{5}{13}\right)\left(-\frac{3}{5}\right)$

$= \frac{-48+15}{65}$

$= \boxed{\frac{-33}{65}}$

b) $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$= \left(-\frac{5}{13}\right)\left(\frac{4}{5}\right) - \left(-\frac{12}{13}\right)\left(-\frac{3}{5}\right)$

$= \frac{-20-36}{65}$

$= \boxed{\frac{-56}{65}}$

c) $\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$= \frac{-48-15}{65}$

$= \boxed{\frac{-63}{65}}$

d) $\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$= \frac{-20+36}{65}$

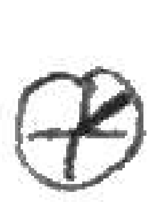
$= \boxed{\frac{16}{65}}$

Find the exact value of each:

24NC. $\sin 165^\circ = \sin(120+45)$

$= \sin 120\cos 45 + \cos 120\sin 45$

$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(-\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$



$= \boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$

25NC. $\cos 285^\circ = \cos(240+45)$

$= \cos 240\cos 45 - \sin 240\sin 45$

$= \left(-\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$= \boxed{\frac{-\sqrt{2}+\sqrt{6}}{4}}$

26NC. $\tan\left(\frac{13\pi}{12}\right) = \tan\left(\frac{4\pi}{12} + \frac{9\pi}{12}\right)$

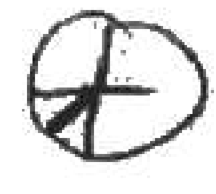
$= \tan\left(\frac{\pi}{3} + \frac{3\pi}{4}\right)$

$= \frac{\tan\frac{\pi}{3} + \tan\frac{3\pi}{4}}{1 - \tan\frac{\pi}{3}\tan\frac{3\pi}{4}}$

$= \frac{\sqrt{3}-1}{1-(\sqrt{3})(-1)}$

$= \boxed{\frac{\sqrt{3}-1}{1+\sqrt{3}}}$

27NC. $\sin\left(\frac{29\pi}{12}\right) = \sin\left(\frac{15\pi}{12} + \frac{14\pi}{12}\right)$



$= \sin\left(\frac{5\pi}{4} + \frac{7\pi}{6}\right)$

$= \sin\frac{5\pi}{4}\cos\frac{7\pi}{6} + \cos\frac{5\pi}{4}\sin\frac{7\pi}{6}$

$= \left(-\frac{\sqrt{2}}{2}\right)\left(-\frac{\sqrt{3}}{2}\right) + \left(-\frac{\sqrt{2}}{2}\right)\left(-\frac{1}{2}\right)$

$= \boxed{\frac{\sqrt{6}+\sqrt{2}}{4}}$

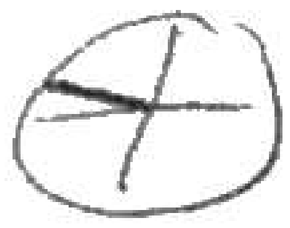
28NC. $\sin\left(\frac{17\pi}{24}\right)\cos\left(\frac{\pi}{8}\right)+\cos\left(\frac{17\pi}{24}\right)\sin\left(\frac{\pi}{8}\right) =$

$\sin\left(\frac{17\pi}{24}+\frac{\pi}{8}\right) =$

$\sin\left(\frac{17\pi}{24}+\frac{3\pi}{24}\right) =$

$\sin\left(\frac{5\pi}{6}\right) =$

$\boxed{\frac{1}{2}}$



29NC. $\frac{\tan 37° - \tan 13°}{1+\tan 37° \tan 13°} =$

$\tan(37-13) =$

$\boxed{\tan 24°}$

30NC. $\frac{\tan\left(\frac{11\pi}{12}\right)+\tan\left(\frac{5\pi}{12}\right)}{1-\tan\left(\frac{11\pi}{12}\right)\tan\left(\frac{5\pi}{12}\right)} =$

$\tan\left(\frac{11\pi}{12}+\frac{5\pi}{12}\right) =$

$\tan\left(\frac{4\pi}{3}\right) =$

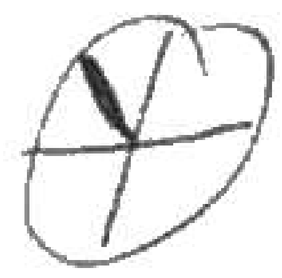
$\boxed{\sqrt{3}}$

31NC. $\cos 146° \cos 11° + \sin 146° \sin 11° =$

$\cos(146-11) =$

$\cos 135° =$

$\boxed{-\frac{\sqrt{2}}{2}}$

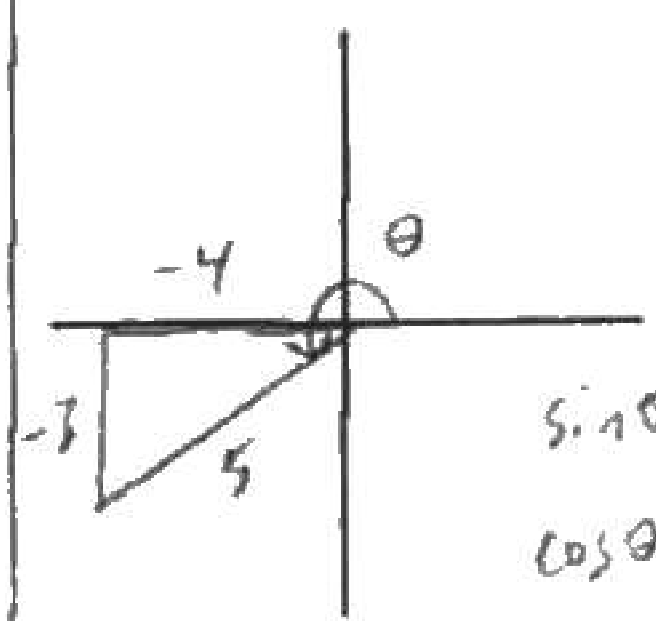


Evaluate each of the following:

32NC. Given $\tan\theta = \frac{3}{4}$ and $\sin\theta < 0$,

$\pi < \theta < \frac{3\pi}{2}$

$\frac{\pi}{2} < \frac{1}{2}\theta < \frac{3\pi}{4}$



$\sin\theta = -\frac{3}{5}$

$\cos\theta = -\frac{4}{5}$

a) find $\cos(2\theta)$ $= 2\cos^2\theta - 1$

$= 2\left(-\frac{4}{5}\right)^2 - 1$

$= \frac{32}{25} - \frac{25}{25}$

$= \boxed{\frac{7}{25}}$

b) find $\cos\left(\frac{1}{2}\theta\right)$ $= -\sqrt{\frac{1+\cos x}{2}}$

$= -\sqrt{\frac{\left(1-\frac{4}{5}\right)5}{2\cdot 5}}$

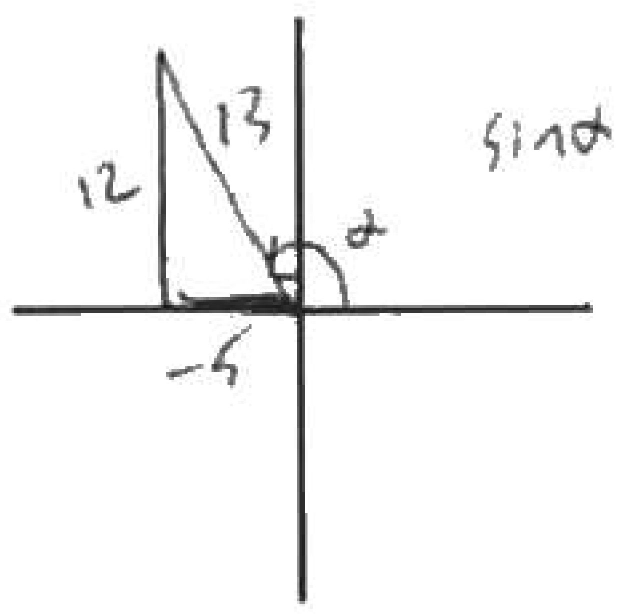
$= -\sqrt{\frac{5-4}{10}}$

$= -\sqrt{\frac{1}{10}}$

$= -\frac{1}{\sqrt{10}}$

$= \boxed{-\frac{\sqrt{10}}{10}}$

33NC. Given $\cos\alpha = -\frac{5}{13}$ and $\tan\alpha < 0$,



$\sin\alpha = \frac{12}{13}$

$\frac{\pi}{2} < \alpha < \pi$

$\frac{\pi}{4} < \frac{1}{2}\alpha < \frac{\pi}{2}$

a) find $\sin(2\alpha)$ $= 2\sin\alpha\cos\alpha$

$= 2\left(\frac{12}{13}\right)\left(-\frac{5}{13}\right)$

$= \boxed{-\frac{120}{169}}$

b) find $\sin\left(\frac{1}{2}\alpha\right)$ $= \sqrt{\frac{1-\cos\alpha}{2}}$

$= \sqrt{\frac{\left(1+\frac{5}{13}\right)13}{2\cdot 13}}$

$= \sqrt{\frac{13+5}{26}}$

$= \sqrt{\frac{18}{26}}$

$= \frac{3\sqrt{2}}{\sqrt{26}}$

$= \boxed{\frac{3\sqrt{52}}{26}}$