Review 1-3

Write the factored and expanded form of an equation that has the following roots:

NC1. Roots at (5,0) with multiplicity 2 and (9,0)

$\boxed{y = (x-5)(x-5)(x-9)}$

$y = (x^2 - 10x + 25)(x-9)$

$\boxed{y = x^3 - 19x^2 + 115x - 225}$

	x	-9
x^2	x^3	$-9x^2$
$-10x$	$-10x^2$	$90x$
$+25$	$25x$	-225

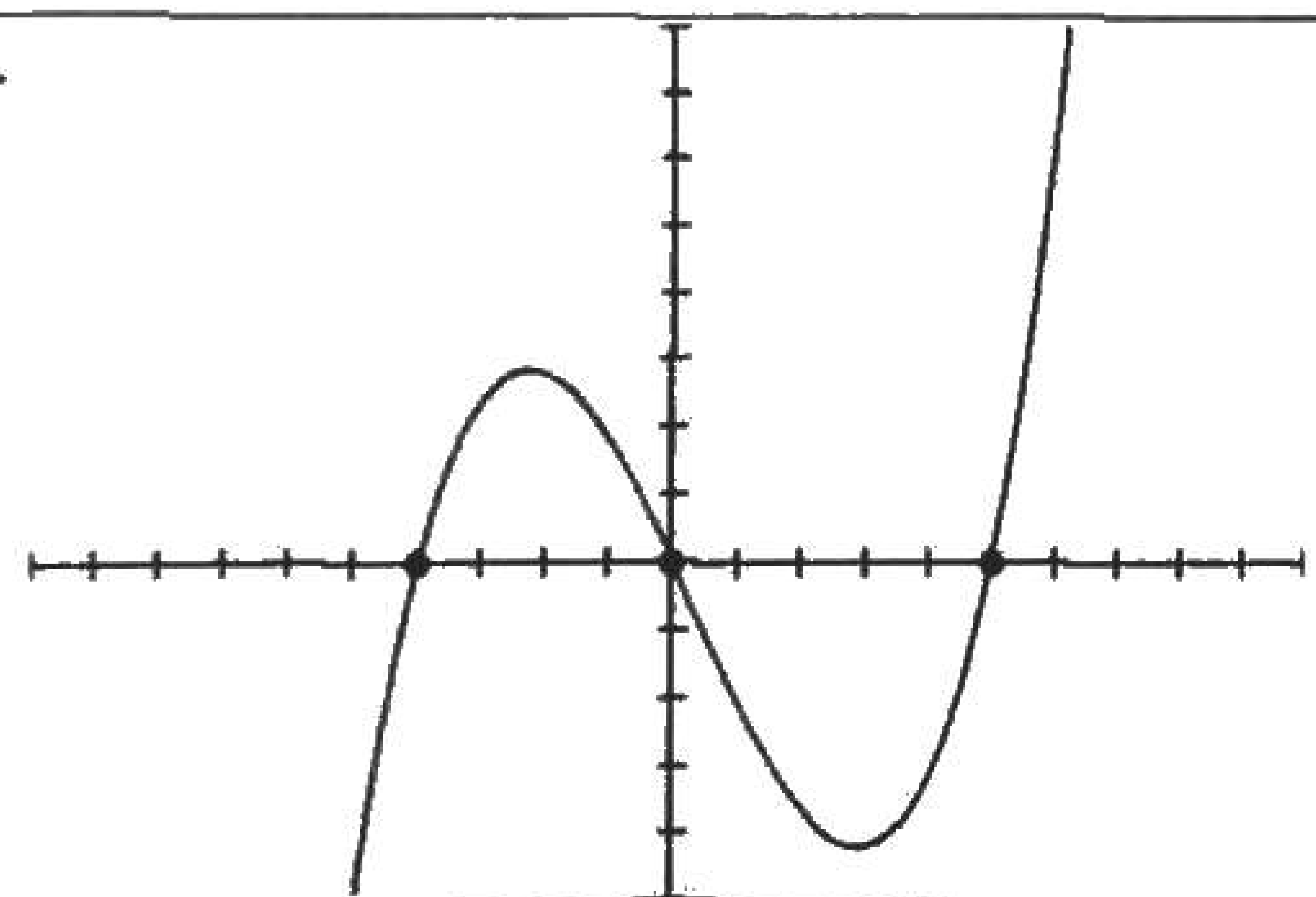
NC2. Roots at $(-1,0)$ and $(-\frac{4}{3},0)$

$\boxed{y = (x+1)(3x+4)}$

$\boxed{y = 3x^2 + 7x + 4}$

Write the equation of the polynomial in factored and expanded form for the following graphs:

NC3.

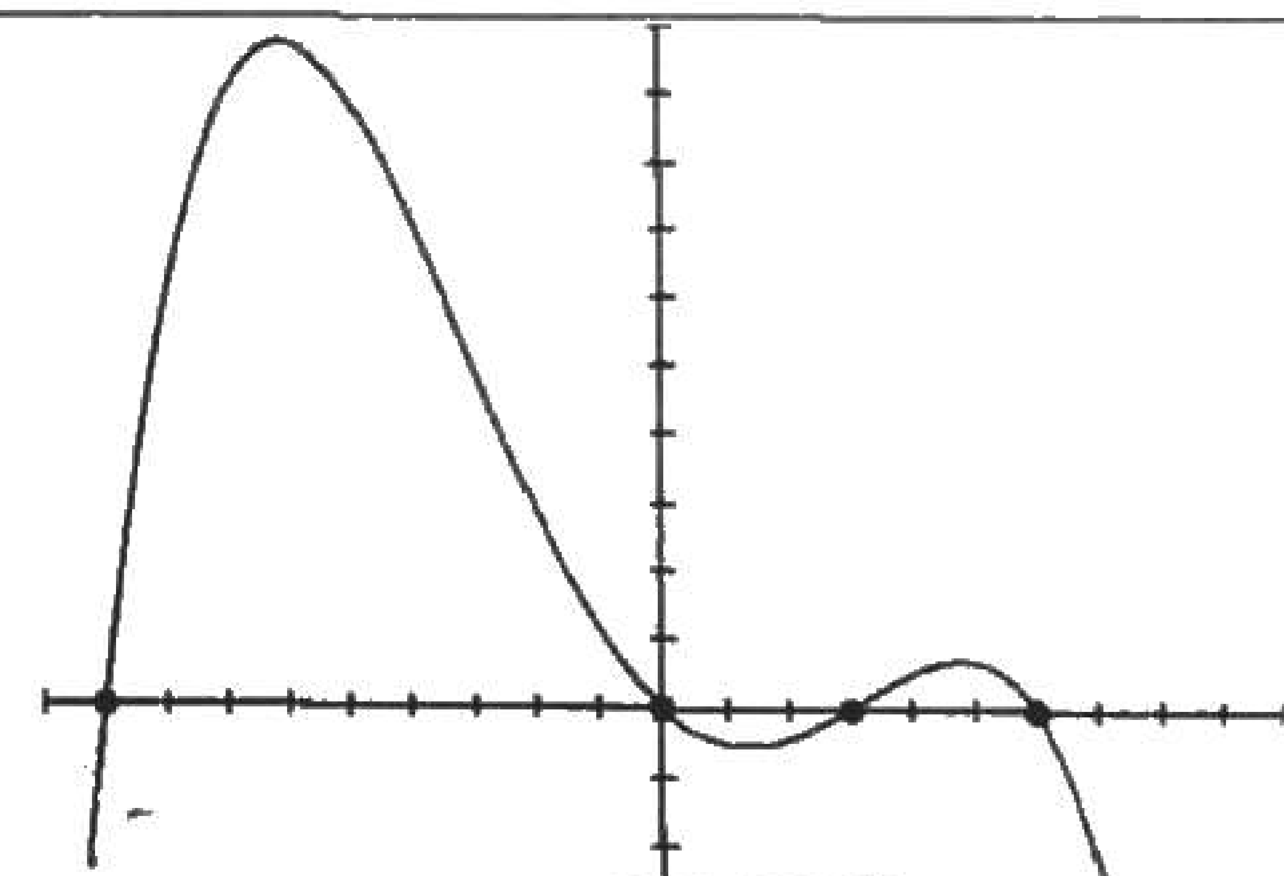


$\boxed{f(x) = (x+4)(x)(x-5)}$

$f(x) = x(x^2 - x - 20)$

$\boxed{f(x) = x^3 - x^2 - 20x}$

NC4.



$\boxed{f(x) = (x+9)(x)(x-3)(x-6)}$

$f(x) = (x^2+9x)(x^2-9x+18)$

$\boxed{f(x) = x^4 - 63x^2 + 162x}$

	x^2	$+9x$
x^2	x^4	$9x^3$
$-9x$	$-9x^3$	$-81x^2$
$+18$	$18x^2$	$162x$

NC5. Sketch a graph that demonstrates everything you know about different types of roots and label where on the graph those types occur. State the degree of your graph.

8 roots means

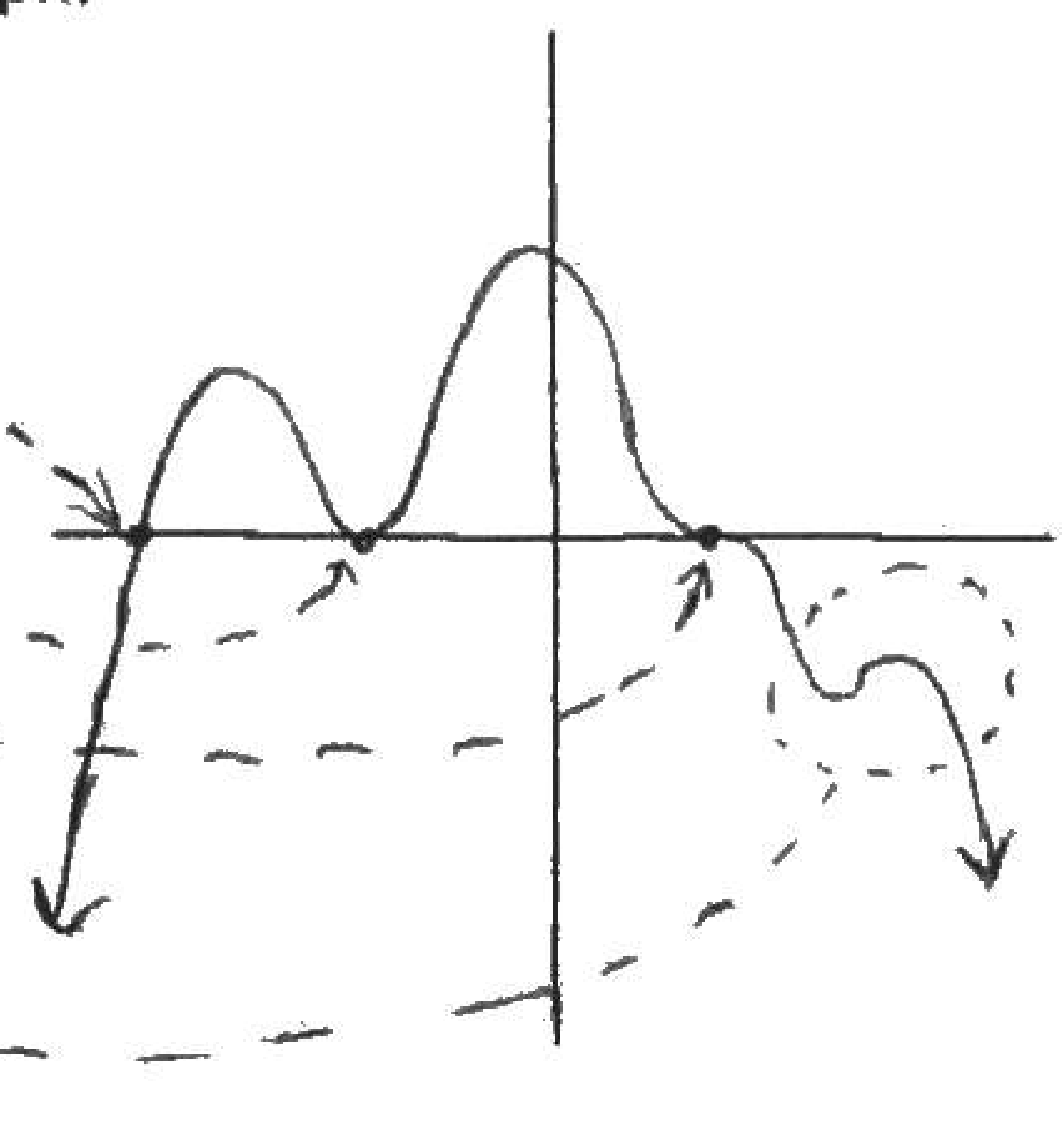
$\boxed{\text{8th degree}}$

single root

double root

triple root

pair of imaginary roots



Use polynomial long division to find each quotient.

NC6. $(3x^4 + 12x^3 + 2x^2 - 4) \div (x + 4)$

$$
\begin{array}{r}
3x^3 \qquad\quad + 2x - 8 \\
x+4 \,\big)\, \overline{3x^4 + 12x^3 + 2x^2 \qquad - 4} \\
-3x^4 - 12x^3 \qquad\qquad\qquad \\
\hline
2x^2 \qquad - 4 \\
-2x^2 - 8x \qquad \\
\hline
-8x - 4 \\
+8x + 32 \\
\hline
28
\end{array}
$$

$$\boxed{3x^3 + 2x - 8 + \frac{28}{x+4}}$$

NC7. $(2x^4 - 3x^3 + x + 1) \div (2x^2 + x + 1)$

$$
\begin{array}{r}
x^2 - 2x + \frac{1}{2} \\
2x^2+x+1 \,\big)\, \overline{2x^4 - 3x^3 \qquad + x + 1} \\
-2x^4 - x^3 - x^2 \qquad\qquad \\
\hline
-4x^3 - x^2 + x + 1 \\
+4x^3 + 2x^2 + 2x \qquad \\
\hline
x^2 + 3x + 1 \\
-x^2 - \frac{1}{2}x - \frac{1}{2} \\
\hline
\frac{5}{2}x + \frac{1}{2}
\end{array}
$$

$$x^2 - 2x + \frac{1}{2} + \frac{\left(\frac{5}{2}x + \frac{1}{2}\right)(2)}{(2x^2 + x + 1)(2)}$$

$$\boxed{x^2 - 2x + \frac{1}{2} + \frac{5x+1}{4x^2+2x+2}}$$

Use synthetic division to find each quotient.

NC8. $(9x^4 - 9x^2 + 2x - 3) \div (x - 2)$

$$
\begin{array}{r|rrrrr}
2 & 9 & 0 & -9 & 2 & -3 \\
 & & 18 & 36 & 54 & 112 \\
\hline
 & 9 & 18 & 27 & 56 & 109
\end{array}
$$

$$\boxed{9x^3 + 18x^2 + 27x + 56 + \frac{109}{x-2}}$$

NC9. $(2x^4 - 3x^2 + 1) \div (x + 1)$

$$
\begin{array}{r|rrrrr}
-1 & 2 & 0 & -3 & 0 & 1 \\
 & & -2 & 2 & 1 & -1 \\
\hline
 & 2 & -2 & -1 & 1 & 0
\end{array}
$$

$$\boxed{2x^3 - 2x^2 - x + 1}$$

List all possible rational roots, write the polynomials in factored form, and list all roots of the following polynomial functions. Sketch a graph #11.

C10. $f(x) = 6x^4 - 11x^3 - 5x^2 + 8x - 4$

All possible rational roots:

	±1	±2	±4
±1	±1	±2	±4
±2	$\pm\frac{1}{2}$		
±3	$\pm\frac{1}{3}$	$\pm\frac{2}{3}$	$\pm\frac{4}{3}$
±6	$\pm\frac{1}{6}$		

$$\begin{array}{r|rrrrr} -1 & 6 & -11 & -5 & 8 & -4 \\ & & -6 & 17 & -12 & 4 \\ \hline 2 & 6 & -17 & 12 & -4 & 0 \\ & & 12 & -10 & 4 & \\ \hline & 6 & -5 & 2 & 0 & \end{array}$$

$\frac{5 \pm \sqrt{25-4(6)(2)}}{2(6)}$

$\frac{5 \pm \sqrt{25-48}}{12}$

$\frac{5 \pm \sqrt{-23}}{12}$

$\frac{5 \pm i\sqrt{23}}{12}$

$\boxed{f(x) = (x+1)(x-2)(6x^2-5x+2)}$

$\boxed{\text{Roots: } (-1,0),\ (2,0),\ \left(\frac{5 \pm i\sqrt{23}}{12},\ 0\right)}$

NC11. $f(x) = 5x^3 + 3x^2 - 32x + 12$

All possible rational roots:

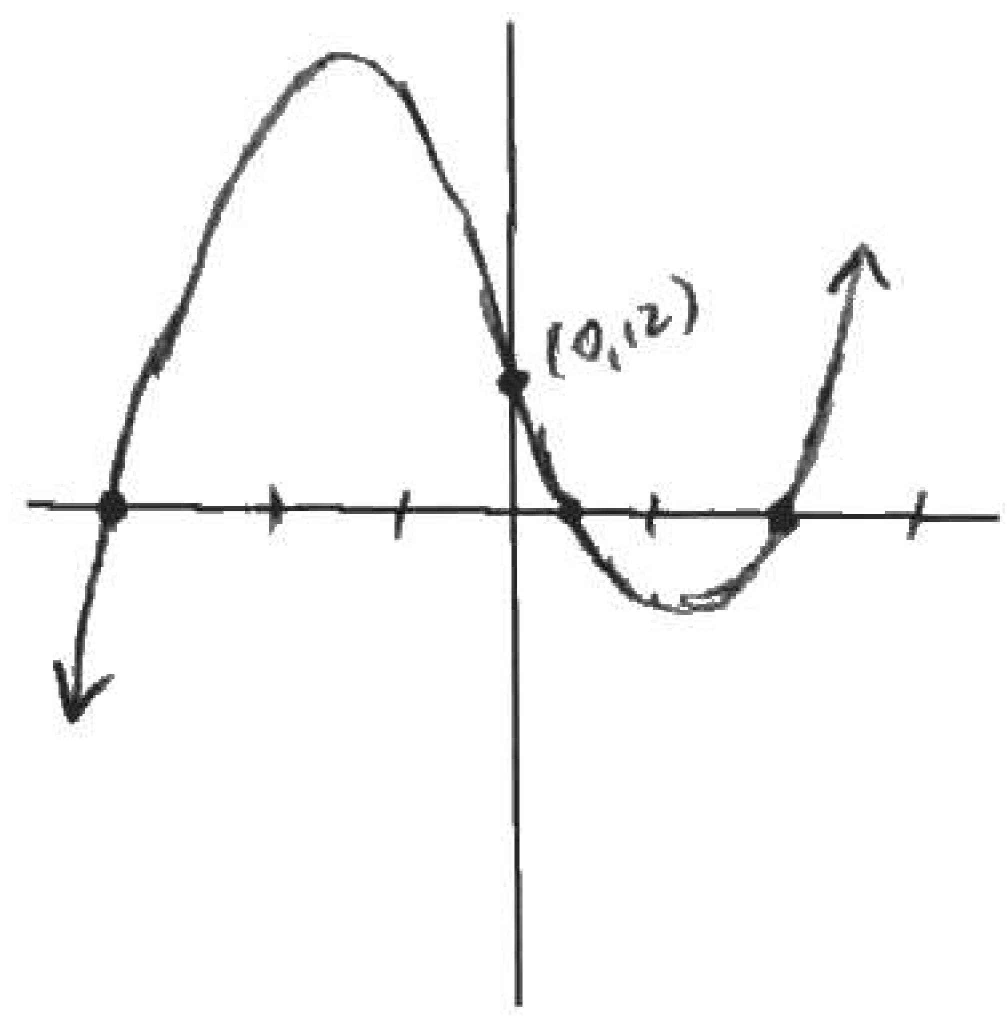
	±1	±2	±3	±4	±6	±12
±1	±1	±2	±3	±4	±6	±12
±5	$\pm\frac{1}{5}$	$\pm\frac{2}{5}$	$\pm\frac{3}{5}$	$\pm\frac{4}{5}$	$\pm\frac{6}{5}$	$\pm\frac{12}{5}$

$$\begin{array}{r|rrrr} 2 & 5 & 3 & -32 & 12 \\ & & 10 & 26 & -12 \\ \hline & 5 & 13 & -6 & 0 \end{array}$$

$f(x) = (x-2)(5x^2+13x-6)$

$\boxed{f(x) = (x-2)(5x-2)(x+3)}$

$\boxed{\text{Roots: } (2,0),\ \left(\frac{2}{5},0\right),\ (-3,0)}$



C12. $f(x) = 3x^3 + 2x^2 - 14x - 4$

All possible rational roots:

	±1	±2	±4
±1	±1	±2	±4
±3	$\pm\frac{1}{3}$	$\pm\frac{2}{3}$	$\pm\frac{4}{3}$

$$\begin{array}{r|rrrr} 2 & 3 & 2 & -14 & -4 \\ & & 6 & 16 & 4 \\ \hline & 3 & 8 & 2 & 0 \end{array}$$

$\frac{-8 \pm \sqrt{64-4(3)(2)}}{2(3)}$

$\frac{-8 \pm \sqrt{64-24}}{6}$

$\frac{-8 \pm \sqrt{40}}{6}$

$\frac{-8 \pm 2\sqrt{10}}{6}$

$\frac{-4 \pm \sqrt{10}}{3}$

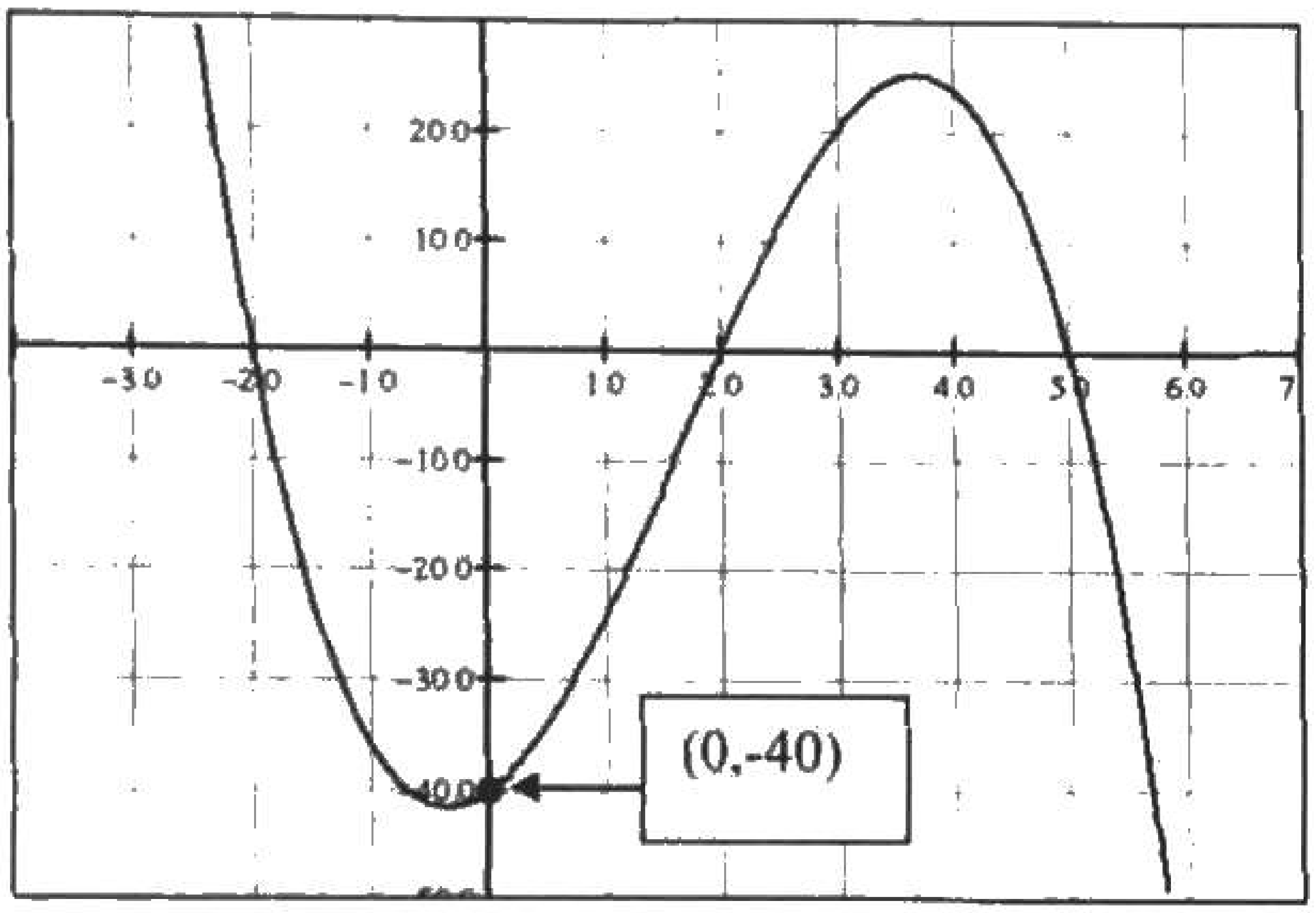
$\boxed{f(x) = (x-2)(3x^2+8x+2)}$

$\boxed{\text{Roots: } (2,0),\ \left(\frac{-4 \pm \sqrt{10}}{3},\ 0\right)}$

13. State in your own words the Fundamental Theorem of Algebra.

When taking into account multiple roots (double, triple, etc.) as well as imaginary roots (which come in pairs), the number of roots is exactly equal to the degree of the polynomial function.

C14. Find write the equation for the following polynomial graph in expanded form:



$f(x) = a(x+2)(x-2)(x-5)$

$-40 = a(2)(-2)(-5)$

$-40 = 20a$

$a = -2$

$f(x) = -2(x+2)(x^2-7x+10)$

$f(x) = -2(x^3-5x^2-4x+20)$

$\boxed{f(x) = -2x^3+10x^2+8x-40}$

	x	$+2$
x^2	x^3	$2x^2$
$-7x$	$-7x^2$	$-14x$
10	$10x$	20

Write the particular equation in expanded form of the polynomial function that has the indicated roots and passes through the given point.

C15. Zeros at $(3, 0)$, $(2, 0)$ and $(-4, 0)$ and passes through the point $(-2, 2)$.

$f(x) = a(x-3)(x-2)(x+4)$

$2 = a(-5)(-4)(2)$

$2 = 40a$

$a = \frac{1}{20}$

$f(x) = \frac{1}{20}(x-3)(x^2+2x-8)$

$f(x) = \frac{1}{20}(x^3-x^2-14x+24)$

$\boxed{f(x) = \frac{1}{20}x^3 - \frac{1}{20}x^2 - \frac{7}{10}x + \frac{6}{5}}$

	x^2	$+2x$	-8
x	x^3	$2x^2$	$-8x$
-3	$-3x^2$	$-6x$	24

C16. Zeros at $(-1, 0)$, $(2, 0)$ and $(4+i, 0)$ and passes through the point $(4, 9)$.

$f(x) = a(x+1)(x-2)(x-4-i)(x-4+i)$

$f(x) = a(x^2-x-2)(x^2-8x+17)$

$9 = a(10)(1)$

$9 = 10a$

$a = \frac{9}{10}$

$f(x) = \frac{9}{10}(x^4-9x^3+23x^2-x-34)$

$\boxed{f(x) = \frac{9}{10}x^4 - \frac{81}{10}x^3 + \frac{207}{10}x^2 - \frac{9}{10}x - \frac{153}{5}}$

	x	-4	$+i$
x	x^2	$-4x$	ix
-4	$-4x$	16	$-4i$
$-i$	$-ix$	$4i$	$-i^2$

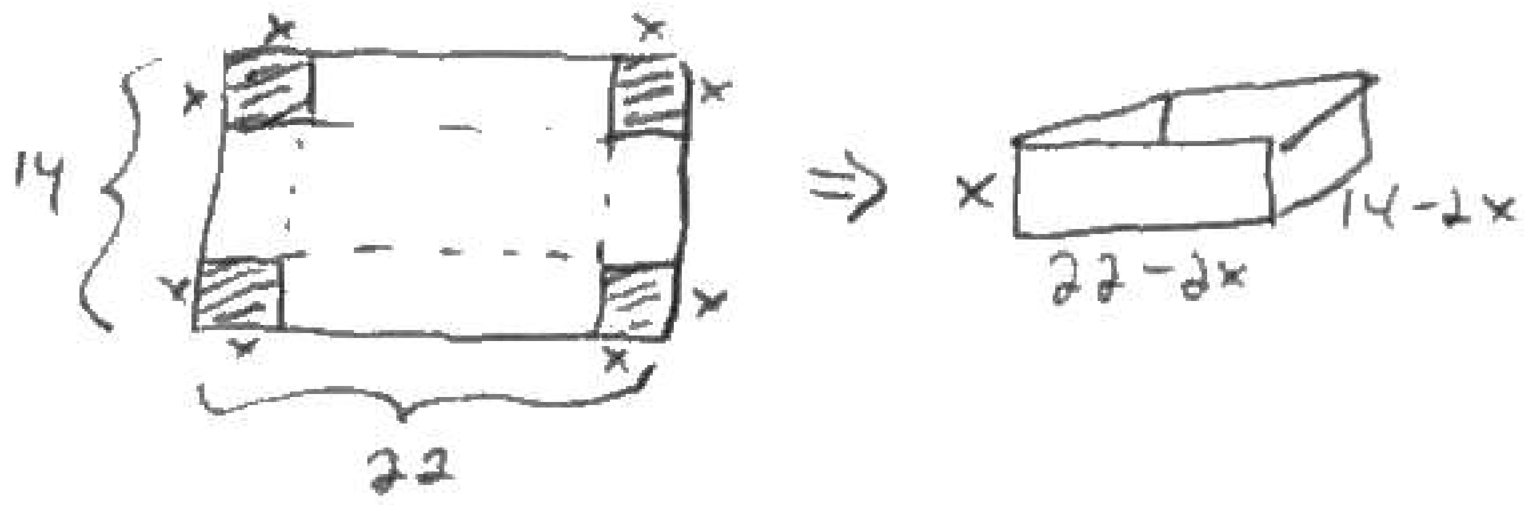
$x^2-8x+16+1$

	x^2	$-8x$	$+17$
x^2	x^4	$-8x^3$	$17x^2$
$-x$	$-x^3$	$8x^2$	$-17x$
-2	$-2x^2$	$16x$	-34

C17. Squares with sides of length **(x)** are cut from the corners of a rectangular piece of sheet metal with dimensions of 14 inches and 22 inches. The metal is then folded to make an open-top box. What is the maximum volume of such a box?

a) Write an equation that represents the volume of the box as a function of x.

$\boxed{V = x(22-2x)(14-2x)}$



b) State a **reasonable** domain for this situation.

$\boxed{[0, 7]}$

c) Find the maximum volume of this box and give a **complete** explanation of how to achieve this maximum.

$(2.785, 385.736)$

By cutting out 2.785" by 2.785" squares from the corners, the box will have a maximum volume of 385.736 in^3.

C18. A rancher has 200 ft of fencing with which to enclose two adjacent rectangular corrals.

a) Write an equation that represents the total area of both corrals as a function of x, the width of the corrals.

$3x + 2y = 200$

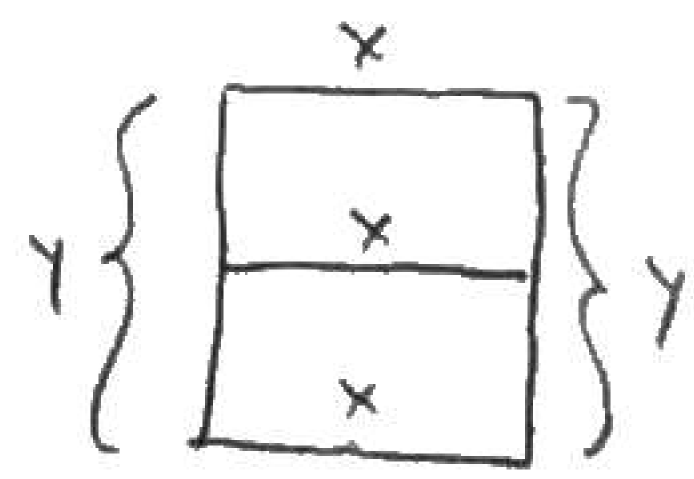
$2y = 200 - 3x$

$y = 100 - \frac{3}{2}x$

$A = xy$

$A = x(100 - \frac{3}{2}x)$

$\boxed{A = 100x - \frac{3}{2}x^2}$



b) State a **reasonable** domain for this situation.

$\boxed{[0, \frac{200}{3}]}$

c) Find the maximum possible area enclosed by both corrals and give a **complete** explanation of how to achieve this maximum.

$(33.333, 1666.667)$

$y = 100 - \frac{3}{2}(33.333)$

$y = 50$

If the entire enclosed area is 33.333 ft wide and 50 ft long, the maximum area will be 1666.667 ft^2

C19. A two-stage rocket is fired straight up. After the first stage stops firing, the rocket slows down until the second stage starts firing. Its altitudes, h(t), in feet above the ground at time, t, after firing are given by the following table:

t	h(t)
10	1865
20	3765
30	5865
40	9365
50	15465
60	25365

a) What type of regression best fits this data (quadratic, cubic, quartic, quintic, etc.)? **<u>You must justify your answer numerically</u>**.

Cubic because $r^2 = 1$

b) Write the equation that describes the height of the rocket, h(t), as a function of time, t.

$h(t) = 0.2t^3 - 11t^2 + 380t - 1035$

d) How long will it take for the rocket to reach 50,000 feet?

75.017 seconds

d) How high is the rocket after 32 seconds?

6414.6 feet

Given f(x) and g(x), find the indicated compositions for problems 20-29:

x	f(x)	g(x)
-3	1	8
0	5	-3
1	6	0
4	5	4

NC20. Find f(g(0)).

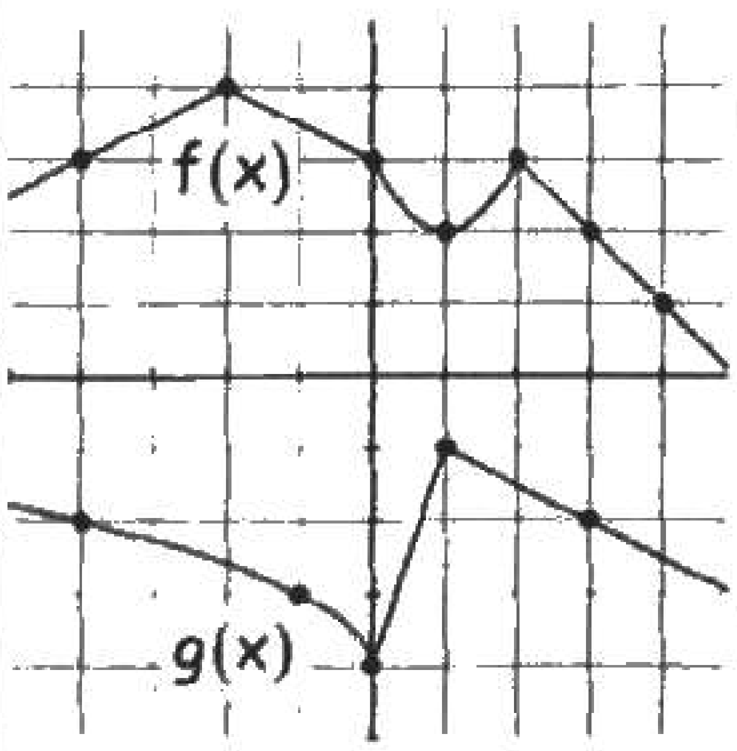
$f(-3) = \boxed{1}$

NC21. Find $(g \circ f)(-3)$.

$g(1) = \boxed{0}$

NC22. Find f(g(4)).

$f(4) = \boxed{5}$



NC23. Find $(g \circ f)(1)$.

$g(2) = \boxed{-1.5}$

NC24. Find $(f \circ g)(0)$.

$f(-4) = \boxed{3}$

NC25. Find g(f(4)).

$g(1) = \boxed{-1}$

$$f(x) = \frac{2}{x-3} \qquad g(x) = \frac{4x-5}{7x+9}$$

NC26. Find $(g \circ f)(-1)$.

$f(-1) = \frac{2}{-1-3} = -\frac{1}{2}$

$g(-\frac{1}{2}) = \frac{(4(-\frac{1}{2})-5)2}{(7(-\frac{1}{2})+9)2} = \frac{-4-10}{-7+18}$

$\boxed{-\frac{14}{11}}$

NC27. Find f(g(2)).

$g(2) = \frac{4(2)-5}{7(2)+9}$

$g(2) = \frac{3}{23}$

$f(\frac{3}{23}) = \frac{2 \cdot 23}{(\frac{3}{23}-3)23} = \frac{46}{3-69} = -\frac{46}{66} = \boxed{-\frac{23}{33}}$

NC28. Find g(f(x)) and its domain.

$g(f(x)) = \frac{(4(\frac{2}{x-3})-5)(x-3)}{(7(\frac{2}{x-3})+9)(x-3)}$

$g(f(x)) = \frac{8-5x+15}{14+9x-27}$

$\boxed{g(f(x)) = \frac{23-5x}{9x-13}}$

$9x - 13 \neq 0$, $9x \neq 13$, $x \neq \frac{13}{9}$

$x - 3 \neq 0$, $x \neq 3$

$\boxed{(-\infty, \frac{13}{9}) \cup (\frac{13}{9}, 3) \cup (3, \infty)}$

NC29. Find $(f \circ g)(x)$ and its domain

$f(g(x)) = \frac{2(7x+9)}{(\frac{4x-5}{7x+9}-3)(7x+9)}$

$f(g(x)) = \frac{14x+18}{4x-5-21x-27}$

$\boxed{f(g(x)) = \frac{14x+18}{-17x-32}}$

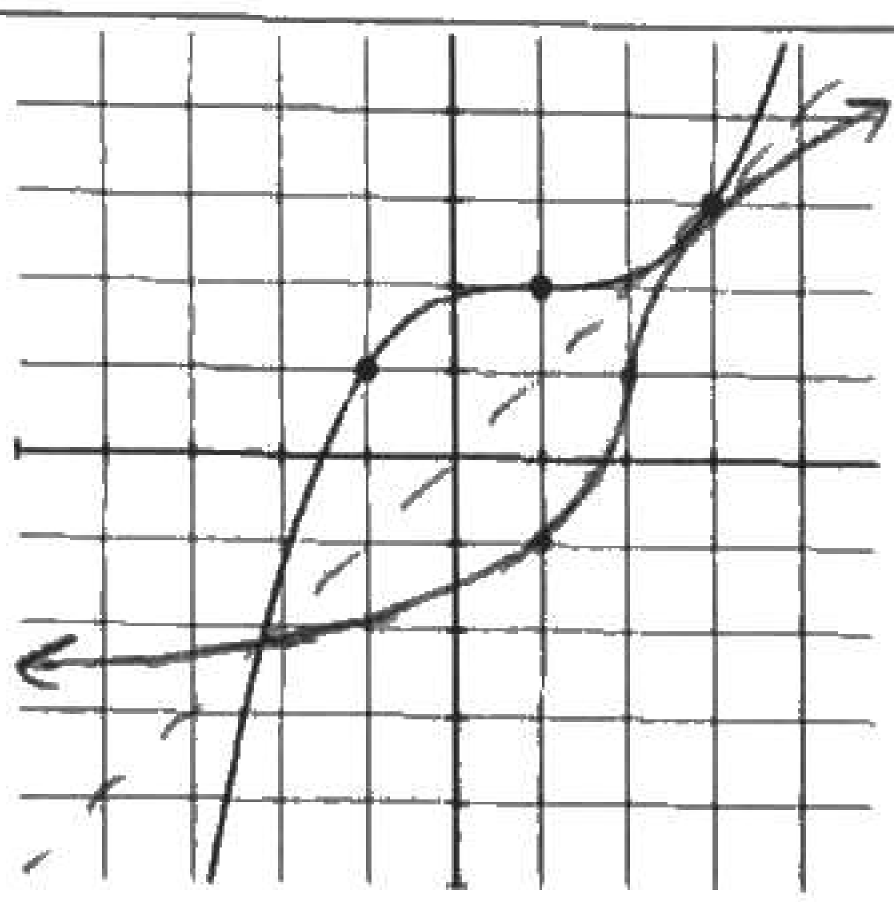
$-17x - 32 \neq 0$, $-17x \neq 32$, $x \neq -\frac{32}{17}$

$7x + 9 \neq 0$, $7x \neq -9$, $x \neq -\frac{9}{7}$

$\boxed{(-\infty, -\frac{32}{17}) \cup (-\frac{32}{17}, -\frac{9}{7}) \cup (-\frac{9}{7}, \infty)}$

For problems 30 and 31, state whether the function is one-to-one and find the inverse:

NC30.



One-to-one? $\boxed{\text{Yes}}$ ← each x-value has a unique y-value (horizontal line test)

Sketch the inverse on the graph

NC31. {(-3, -1), (-2, -5), (1, -1), (2, 7)}

One-to-one? $\boxed{\text{No}}$ ← Two x-values give a y-value of -1.

Inverse: $\boxed{\{(-1,-3),(-5,-2),(-1,1),(7,2)\}}$

For problems 32 and 33, find the inverse of the given one-to-one functions, as well as the domain and range of both:

NC32. $f(x) = \dfrac{2x-5}{x+4}$

$(y+4)x = \dfrac{2y-5}{\cancel{y+4}}\cancel{(y+4)}$

$xy + 4x = 2y - 5$

$xy - 2y = -4x - 5$

$y(x-2) = -(4x+5)$

$\boxed{f^{-1}(x) = -\dfrac{4x+5}{x-2}}$

f(x) domain: $(-\infty, -4) \cup (-4, \infty)$
f(x) range: $(-\infty, 2) \cup (2, \infty)$

$f^{-1}(x)$ domain: $(-\infty, 2) \cup (2, \infty)$
$f^{-1}(x)$ range: $(-\infty, -4) \cup (-4, \infty)$

NC33. $f(x) = x^2 + 2$

$x = y^2 + 2$

$x - 2 = y^2$

$\boxed{y = \pm\sqrt{x-2}}$

f(x) domain: $(-\infty, \infty)$
f(x) range: $[2, \infty)$

Inverse domain: $[2, \infty)$
Inverse range: $(-\infty, \infty)$