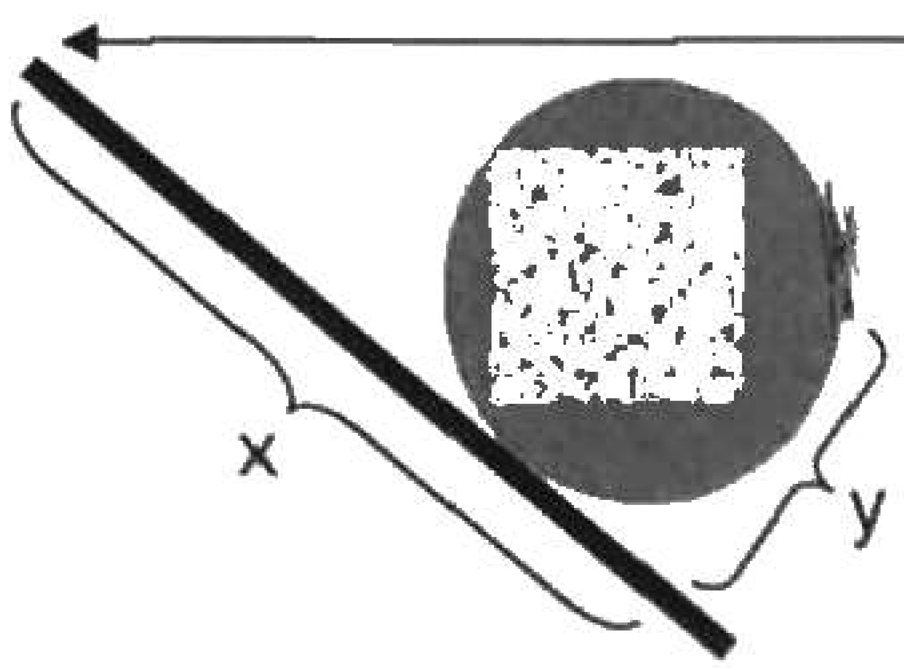
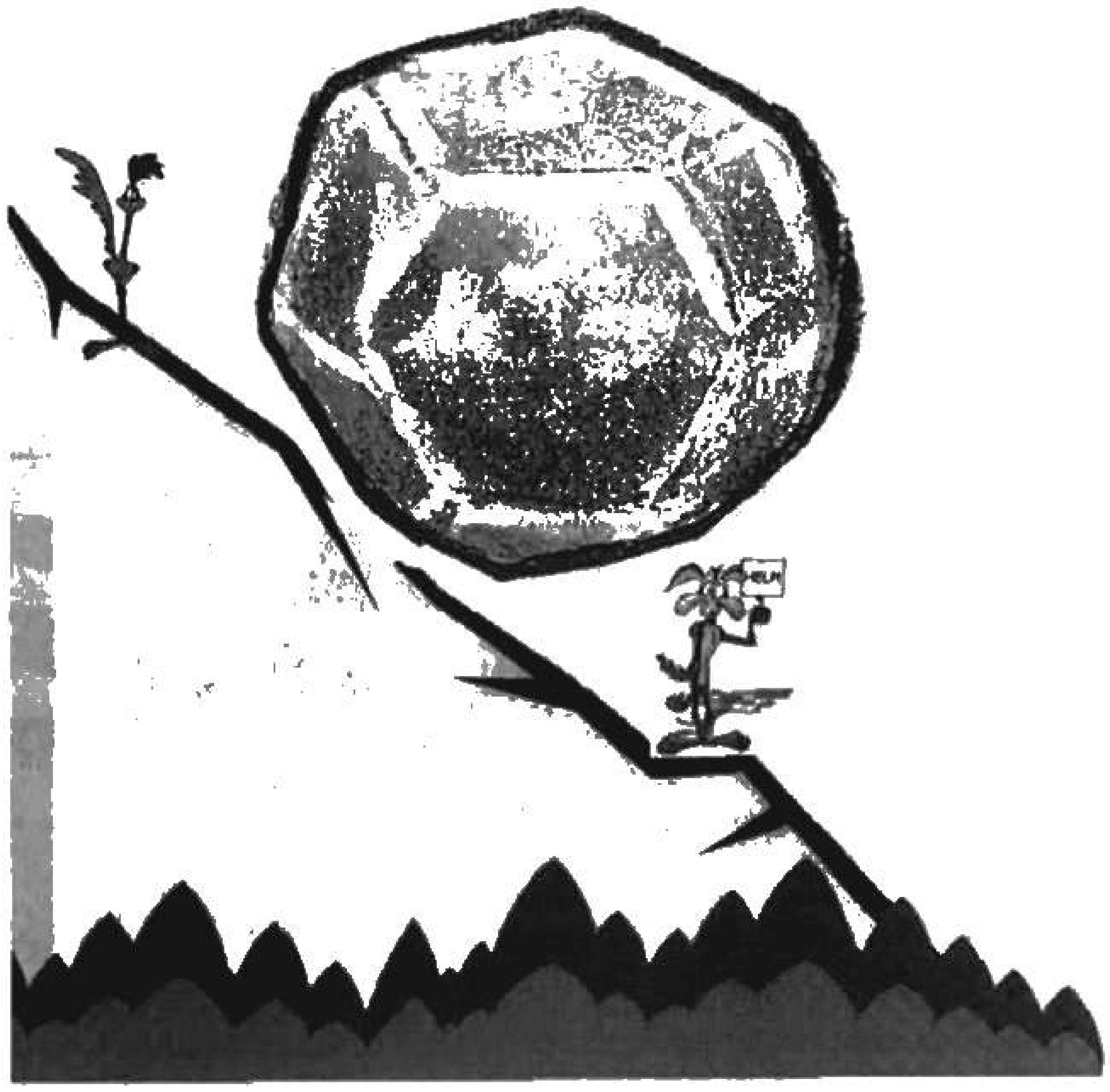
Mc Lennan

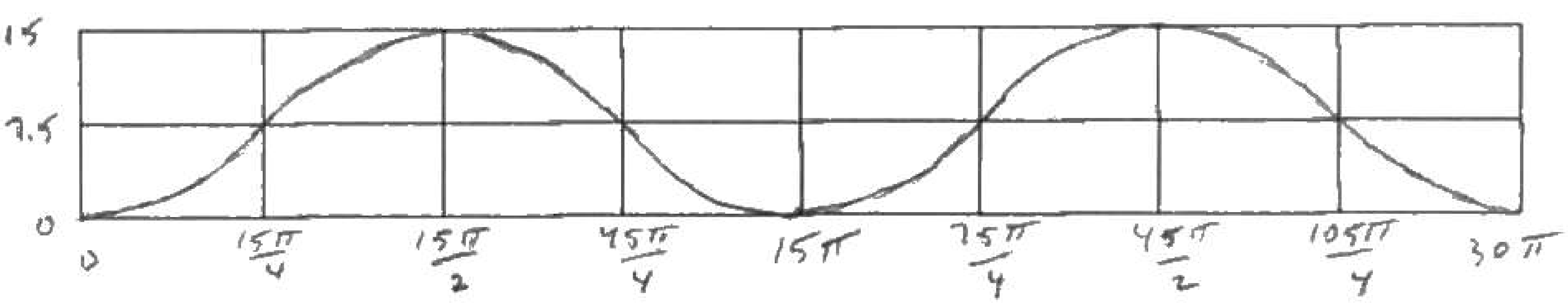
Review 3-3

Wile E Coyote Problem: The coyote falls victim to the trap he set for the roadrunner and is squashed by (and is now stuck to) a large boulder rolling down the mountainside over him. The diameter of the boulder is 15 feet. As the boulder continues to roll (with the flattened coyote still stuck to it), the coyote's distance from the mountainside, "y", in feet varies sinusoidally with his distance travelled down the mountain, "x", in feet.





1C) Sketch a graph of this sinusoidal function.



2C) Write a cosine function expressing distance from the ground as a function of distance travelled down the mountainside.

$p: 15\pi$ $\quad B = \frac{2\pi}{15\pi}$

$B = \frac{2}{15}$

$$\boxed{y = 7.5\cos\left[\frac{2}{15}\left(x - \frac{15\pi}{2}\right)\right] + 7.5}$$

3C) Predict the coyote's distance from the ground when the boulder has rolled 1000 feet.

$y = 7.5\cos\left[\frac{2}{15}\left(1000 - \frac{15\pi}{2}\right)\right] + 7.5$ $\qquad y = \boxed{6.125 \text{ feet}}$

4C) What are the first four times the coyote is a height of 2 feet from the ground? **You MUST show the algebraic steps.**

$2 = 7.5\cos\left[\frac{2}{15}\left(x - \frac{15\pi}{2}\right)\right] + 7.5$

$-5.5 = 7.5\cos\left[\frac{2}{15}\left(x - \frac{15\pi}{2}\right)\right]$

$-0.733 = \cos\left[\frac{2}{15}\left(x - \frac{15\pi}{2}\right)\right]$

$\frac{2}{15}\left(x - \frac{15\pi}{2}\right) = 2.394 + 2\pi n$ $\qquad \frac{2}{15}\left(x - \frac{15\pi}{2}\right) = -2.394 + 2\pi n$

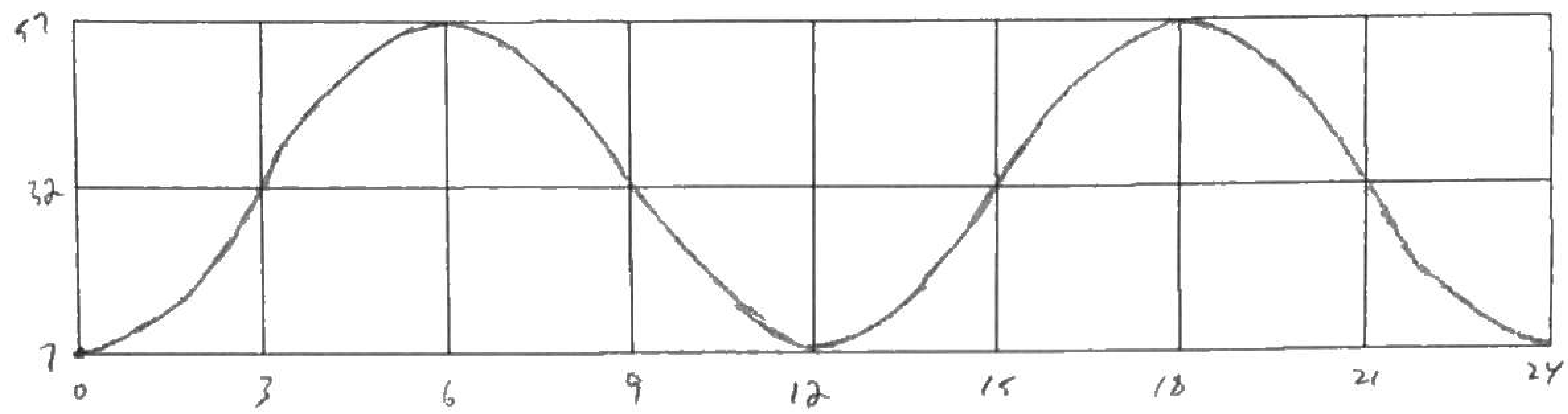
$x - \frac{15\pi}{2} = 17.955 + 15\pi n$ $\qquad x - \frac{15\pi}{2} = -17.955 + 15\pi n$

$x = 41.517 + 15\pi n$ $\qquad x = 5.607 + 15\pi n$

when he has travelled 5.607 ft, 41.517 ft, 52.731 ft, and 88.641 ft down the mountain

Windmill Problem: Jill decides to build a windmill in order to harness the energy of the wind. The blades of the windmill are each 25 feet long, and they clear the ground by 7 feet at their lowest point. The blades make 5 revolutions each minute. As one of the blades passes through the low point, Jill leaps up, grabs the edge of the blade, and starts her stopwatch. Jill's height above the ground varies sinusoidally with time.

5C) Sketch a graph of this sinusoidal function. $P = \frac{60 \text{ sec}}{\text{min}} \cdot \frac{1 \text{ min}}{5 \text{ rev}} = 12$ seconds/revolution



6C) Write an equation which describes Jill's height as a function of time.

$B = \frac{2\pi}{12}$

$B = \frac{\pi}{6}$

$$\boxed{y = 25\cos\left[\frac{\pi}{6}(t-6)\right] + 32}$$

7C) How high is Jill after 181 minutes? Is she moving up or down? How can you tell?

$t = 181(60)$ $\quad y = 25\cos\left[\frac{\pi}{6}(10860-6)\right] + 32 = \boxed{7 \text{ feet}}$

$t = 10860$

Moving up because she is at a low point

8C) What are the first three times that Jill will be at a height of 42 feet?

$42 = 25\cos\left[\frac{\pi}{6}(t-6)\right] + 32$

$10 = 25\cos\left[\frac{\pi}{6}(t-6)\right]$

$0.4 = \cos\left[\frac{\pi}{6}(t-6)\right]$

$\frac{\pi}{6}(t-6) = 1.159 + 2\pi n$

$t - 6 = 2.214 + 12n$

$t = 8.214 + 12n$

$\frac{\pi}{6}(t-6) = -1.159 + 2\pi n$

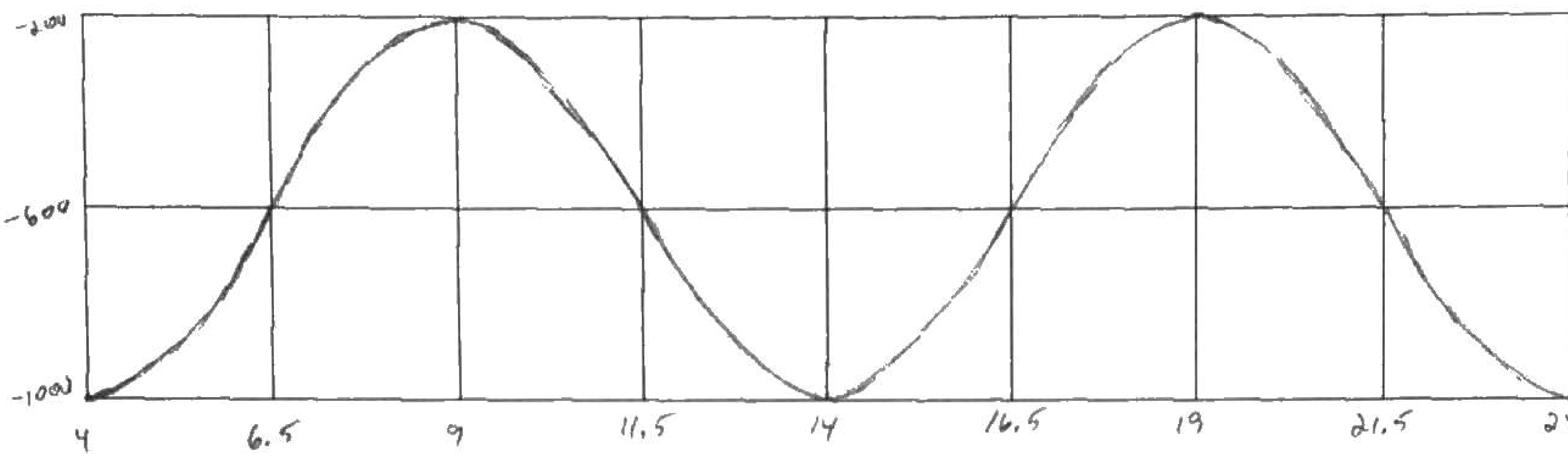
$t - 6 = -2.214 + 12n$

$t = 3.786 + 12n$

$$\boxed{t = 3.786 \text{ sec}, \ 8.214 \text{ sec}, \ 15.786 \text{ sec}}$$

Porpoising Problem: Assume that you are aboard a submarine, submerged in the Pacific Ocean. At time $t = 0$ you make contact with an enemy destroyer. Immediately, you start Porpoising (going deeper and then shallow). At time $t = 4$ minutes, you are at your deepest $y = -1000$ meters. At time $t = 9$ minutes, you reach your shallowest depth, $y = -200$ meters. Assume y varies sinusoidally with t for $t \geq 0$.

9C) Sketch a graph of this sinusoidal function.



10C) Write a cosine function expressing y in terms of t.

$B = \frac{2\pi}{10}$

$B = \frac{\pi}{5}$

$$\boxed{y = 400 \cos\left[\frac{\pi}{5}(t-9)\right] - 600}$$

11C) Your submarine is safe when it is below $y = -300$ meters. At time $t = 0$, was your submarine safe?

$y = 400 \cos\left[\frac{\pi}{5}(0-9)\right] - 600 = -276.393$

$\boxed{\text{No because it was at a depth of } -276.393 \text{ meters}}$

12C) Between what two non-negative times was your submarine first safe? **<u>You MUST show the algebraic steps.</u>**

$-300 = 400 \cos\left[\frac{\pi}{5}(t-9)\right] - 600$

$300 = 400 \cos\left[\frac{\pi}{5}(t-9)\right]$

$0.75 = \cos\left[\frac{\pi}{5}(t-9)\right]$

$\frac{\pi}{5}(t-9) = 0.723 + 2\pi n$

$t-9 = 1.150 + 10n$

$t = 10.150 + 10n$

$\frac{\pi}{5}(t-9) = -0.723 + 2\pi n$

$t-9 = -1.150 + 10n$

$t = 7.850 + 10n$

$$\boxed{t = 0.150 \text{ min}, 7.850 \text{ min}}$$

13C) The equation $y = 40\cos\left[\frac{2\pi}{55}(t-5)\right]+50$ describes your height above the ground, "y", in feet while riding a Ferris Wheel as a function of time, "t", in seconds. If there are low clouds 85 feet above the ground, what percentage of the time will you spend in the clouds?

$85 = 40\cos\left[\frac{2\pi}{55}(t-5)\right]+50$

$35 = 40\cos\left[\frac{2\pi}{55}(t-5)\right]$

$0.875 = \cos\left[\frac{2\pi}{55}(t-5)\right]$

$\frac{2\pi}{55}(t-5) = 0.505 + 2\pi n$ $\qquad$ $\frac{2\pi}{55}(t-5) = -0.505 + 2\pi n$

$t-5 = 4.424 + 55n$ $\qquad$ $t-5 = -4.424 + 55n$

$t = 9.424 + 55n$ $\qquad$ $t = 0.576 + 55n$

$p = \frac{\cancel{2\pi}}{1}\left(\frac{55}{\cancel{2\pi}}\right)$

$p = 55$ seconds

$\frac{9.424 - 0.576}{55} =$

$\frac{8.848}{55} =$

0.161

$\boxed{16.1\%}$

14C) For a marked point on a waterwheel, the equation $y = 7\cos\left[\frac{\pi}{3}(t-1)\right]+5$ describes the height above the water, "y", in feet, as a function of time, "t", in seconds. How long will it take for the point to be submerged for half an hour?

$0 = 7\cos\left[\frac{\pi}{3}(t-1)\right]+5$

$-5 = 7\cos\left[\frac{\pi}{3}(t-1)\right]$

$-5/7 = \cos\left[\frac{\pi}{3}(t-1)\right]$

$\frac{\pi}{3}(t-1) = 2.366 + 2\pi n$ $\qquad$ $\frac{\pi}{3}(t-1) = -2.366 + 2\pi n$

$t-1 = 2.260 + 6n$ $\qquad$ $t-1 = -2.260 + 6n$

$t = 3.260 + 6n$ $\qquad$ $t = -1.260 + 6n$

$t = 4.740 + 6n$

$p = \frac{2\cancel{\pi}}{1}\left(\frac{3}{\cancel{\pi}}\right)$

$p = 6$ seconds

$4.740 - 3.260 = 1.480$

$\frac{30\ \cancel{min}}{1} \cdot \frac{60\ sec}{\cancel{min}} = 1800$ sec

$\frac{1800}{1.480} = 1216.007$ revolutions

$1216.007\ \cancel{rev}\left(\frac{6\ sec}{\cancel{rev}}\right)$

$\boxed{7296.040 \text{ seconds}}$

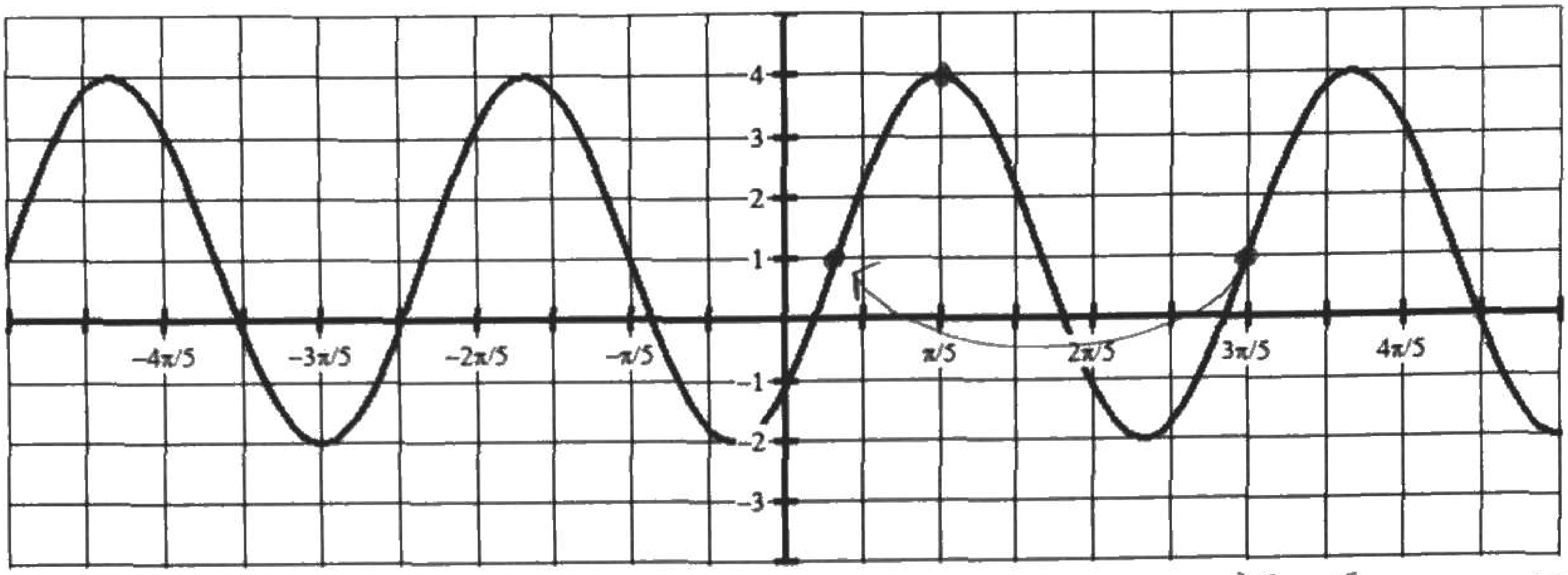
15C. Explain what the "$\cos^{-1}$" and the "$\sin^{-1}$" buttons do on the calculator, including how multiple answers are derived and why the calculator might return an error message.

"$\cos^{-1}$" of a number gives the angle that has that number as a cosine value. Since cosine is the x-coordinate, there is another answer at the negative value of the answer.

"$\sin^{-1}$" of a number gives the angle that has that number as a sine value. Since sine is the y-coordinate, there is another answer at π minus the value of the answer.

Since sine and cosine values are between -1 and 1, anything outside that domain will give an error for $\sin^{-1}$ or $\cos^{-1}$.

Write an equation for a sinusoid function describing it using the **first positive phase shift**.



$\frac{3}{4}P = \frac{3\pi}{5} - \frac{\pi}{5}$ $\qquad B = \frac{2\pi}{1}\left(\frac{15}{8\pi}\right)$

$\frac{3}{4}P = \frac{2\pi}{5}$ $\qquad B = \frac{15}{4}$

$P = \frac{8\pi}{15}$

16NC) As a cosine function

$$\boxed{y = 3\cos\left[\frac{15}{4}\left(x - \frac{\pi}{5}\right)\right] + 1}$$

17NC) As a sine function

$$\boxed{y = 3\sin\left[\frac{15}{4}\left(x - \frac{\pi}{15}\right)\right] + 1}$$

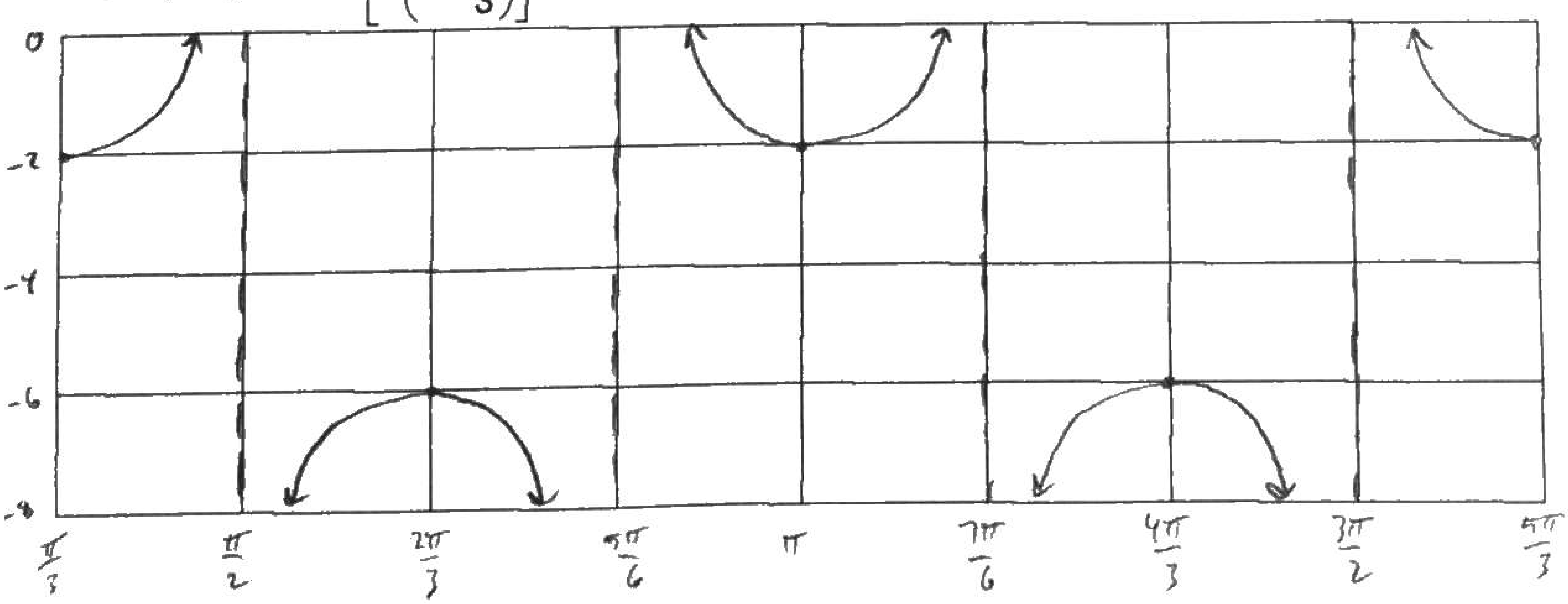
$\frac{3\pi}{5} - \frac{8\pi}{15} =$

$\frac{9\pi}{15} - \frac{8\pi}{15} =$

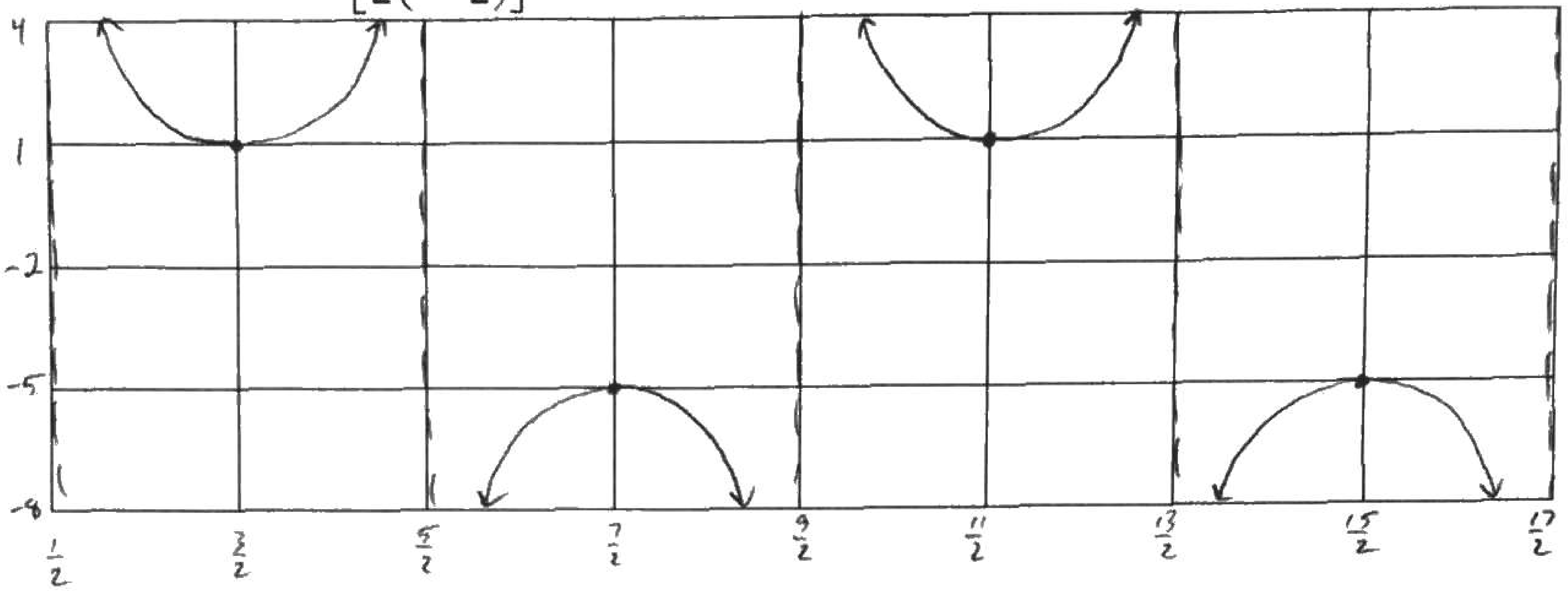
$\frac{\pi}{15}$

Graph the following functions:

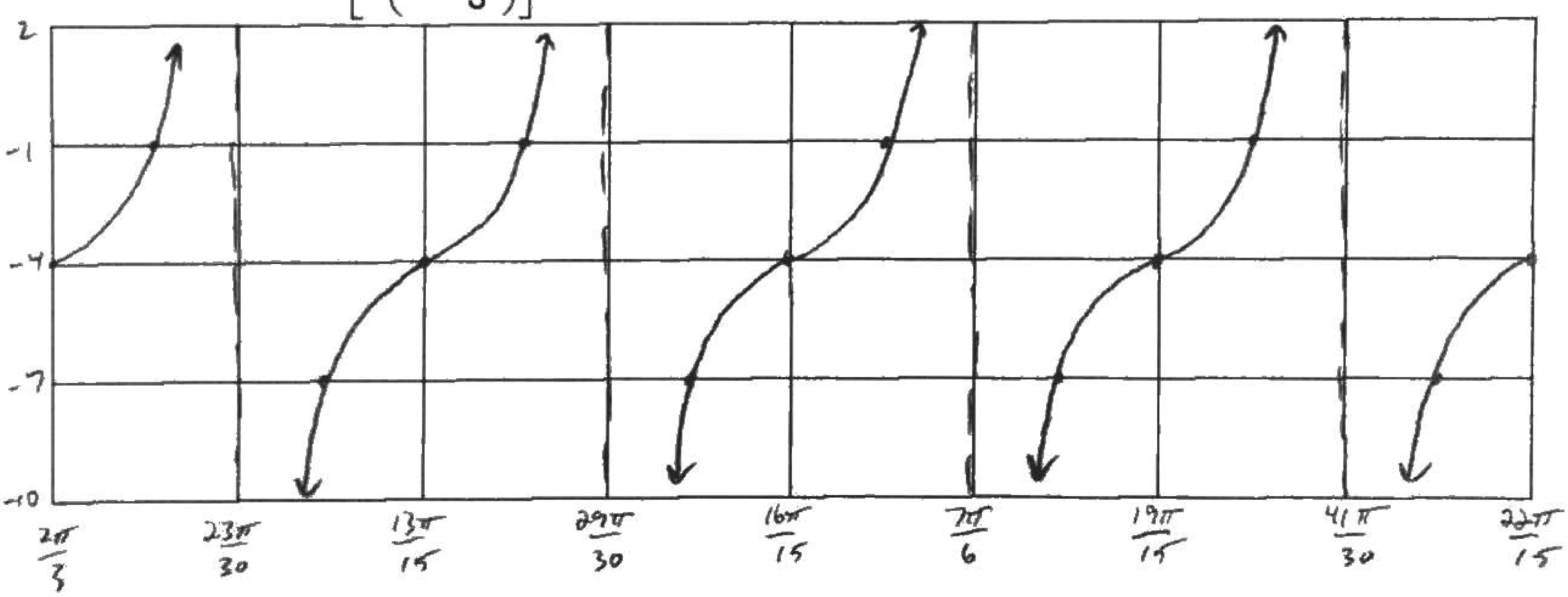
18NC) $y = 2\sec\left[3\left(x - \frac{\pi}{3}\right)\right] - 4$ $\qquad$ x-scale: $\frac{\pi}{2}\left(\frac{1}{3}\right) = \frac{\pi}{6}$



19NC) $y = 3\csc\left[\frac{\pi}{2}\left(x - \frac{1}{2}\right)\right] - 2$ x-scale: $\frac{\pi}{2}\left(\frac{2}{\pi}\right) = 1$



20NC) $y = 3\tan\left[5\left(x - \frac{2\pi}{3}\right)\right] - 4$ x-scale: $\frac{\pi}{2}\left(\frac{1}{5}\right) = \frac{\pi}{10}$



21NC) The tangent function can have a vertical dilation, such as $y = 3\tan\theta$, but the tangent function does not have an amplitude. Explain why not.

the function stretches vertically by a factor of 3, but it does not have an amplitude because the tangent function follows the asymptotes up to ∞ and down to $-\infty$ every time, regardless of the vertical stretch.

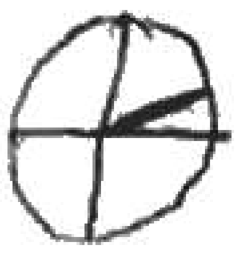
Evaluate each of the following:

22NC) $\cos\frac{37\pi}{6} =$

$\cos\left[\frac{37\pi}{6} - \frac{12\pi}{6}(3)\right] =$

$\cos\frac{\pi}{6} =$

$\boxed{\frac{\sqrt{3}}{2}}$

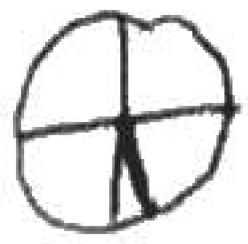


23NC) $\sin\frac{17\pi}{3} =$

$\sin\left[\frac{17\pi}{3} - \frac{6\pi}{3}(2)\right] =$

$\sin\frac{5\pi}{3} =$

$\boxed{-\frac{\sqrt{3}}{2}}$

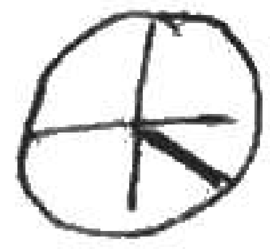


24NC) $\tan\frac{39\pi}{4} =$

$\tan\left[\frac{39\pi}{4} - \frac{8\pi}{4}(4)\right] =$

$\tan\frac{7\pi}{4} =$

$\boxed{-1}$



Find the first 6 positive values of x in radian measure.

25NC) $\cos x = \frac{\sqrt{2}}{2}$

$x = \frac{\pi}{4} + 2\pi n$

$x = \frac{7\pi}{4} + 2\pi n$

$\boxed{\frac{\pi}{4}, \frac{9\pi}{4}, \frac{17\pi}{4}, \frac{7\pi}{4}, \frac{15\pi}{4}, \frac{23\pi}{4}}$

26NC) $\sin x = \frac{1}{2}$

$x = \frac{\pi}{6} + 2\pi n$

$x = \frac{5\pi}{6} + 2\pi n$

$\boxed{\frac{\pi}{6}, \frac{13\pi}{6}, \frac{25\pi}{6}, \frac{5\pi}{6}, \frac{17\pi}{6}, \frac{29\pi}{6}}$

27NC) $\tan x = -\sqrt{3}$

$x = \frac{2\pi}{3} + 2\pi n$

$x = \frac{5\pi}{3} + 2\pi n$

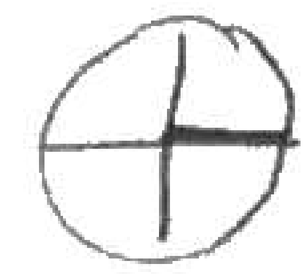
$\boxed{\frac{2\pi}{3}, \frac{8\pi}{3}, \frac{14\pi}{3}, \frac{5\pi}{3}, \frac{11\pi}{3}, \frac{17\pi}{3}}$

Solve for x:

28NC)

$$6\cos\left(2x-\frac{\pi}{5}\right)-4=2$$

$$6\cos\left(2x-\frac{\pi}{5}\right)=6$$



$$\cos\left(2x-\frac{\pi}{5}\right)=1$$

$$2x-\frac{\pi}{5}=\cos^{-1}(1)$$

$$2x-\frac{\pi}{5}=0+2\pi n$$

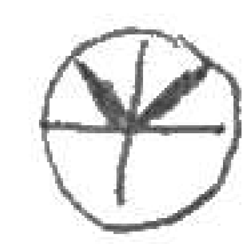
$$2x=\frac{\pi}{5}+2\pi n$$

$$\boxed{x=\frac{\pi}{10}+\pi n}$$

29NC)

$$2\sin\left(\frac{x}{4}+\frac{\pi}{3}\right)=\sqrt{2}$$

$$\sin\left(\frac{x}{4}+\frac{\pi}{3}\right)=\frac{\sqrt{2}}{2}$$



$$\frac{x}{4}+\frac{\pi}{3}=\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$$

$$\frac{x}{4}+\frac{\pi}{3}=\frac{\pi}{4}+2\pi n$$

$$\frac{x}{4}=-\frac{\pi}{12}+2\pi n$$

$$x=-\frac{4\pi}{12}+8\pi n$$

$$\boxed{x=-\frac{\pi}{3}+8\pi n}$$

$$\frac{x}{4}+\frac{\pi}{3}=\frac{3\pi}{4}+2\pi n$$

$$\frac{x}{4}=\frac{5\pi}{12}+2\pi n$$

$$x=\frac{20\pi}{12}+8\pi n$$

$$\boxed{x=\frac{5\pi}{3}+8\pi n}$$