McGram

Review 4-1

NC1. For the following polynomial function, list all possible rational roots, write the polynomial in factored form, and find the actual roots.

$$f(x) = 3x^4 + x^3 - 14x^2 - 32x - 48$$

Possible rational roots:

±1 ±2 ±3 ±4 ±6 ±8 ±12 ±16 ±24 ±48

±1	±1	±2	±3	±4	±6	±8	±12	±16	±24	±48
±3	$\pm\frac{1}{3}$	$\pm\frac{2}{3}$		$\pm\frac{4}{3}$		$\pm\frac{8}{3}$		$\pm\frac{16}{3}$		

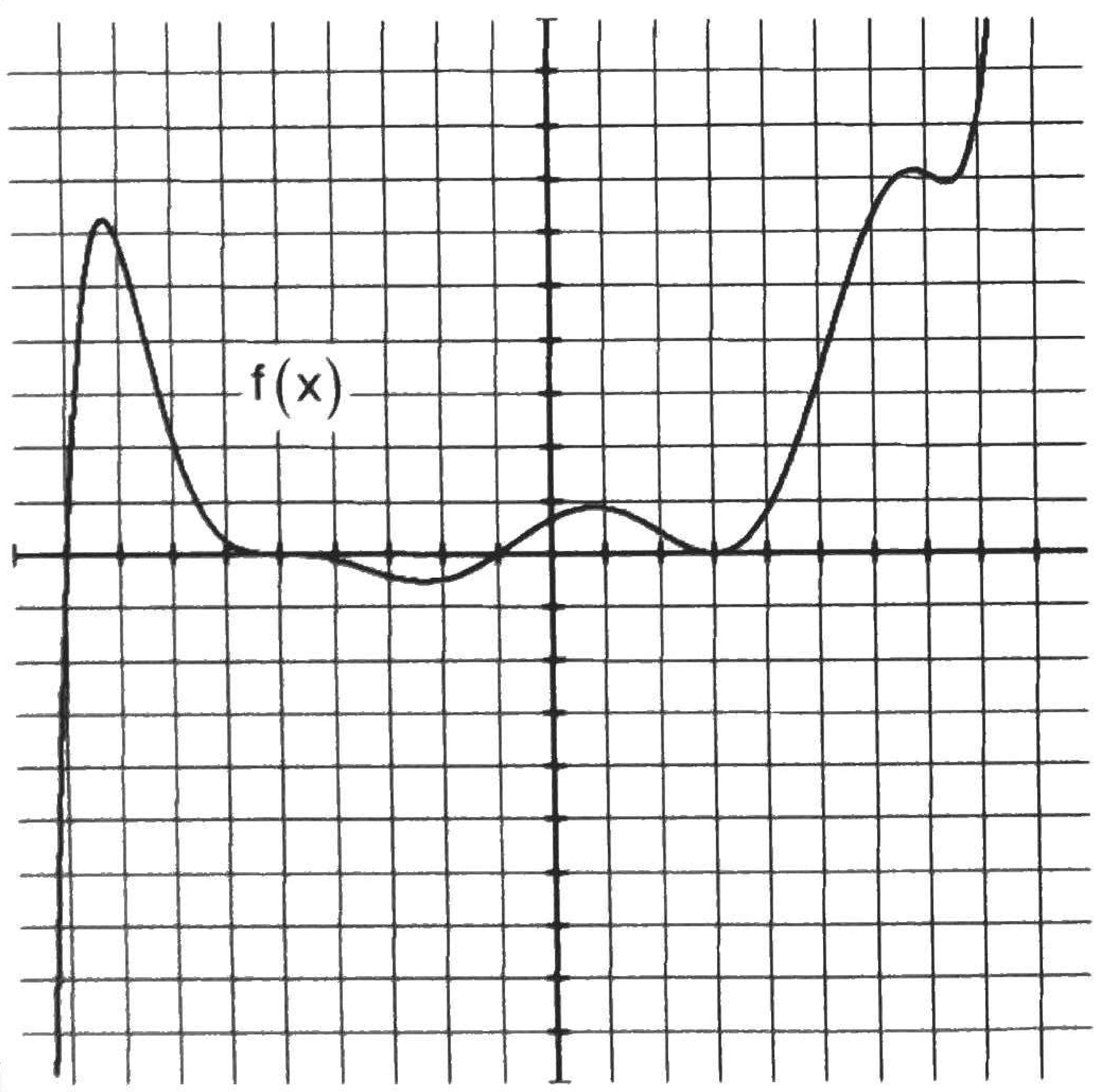
−2	3	1	−14	−32	−48
		−6	10	8	48
3	3	−5	−4	−24	0
		9	12	24	
	3	4	8	0	

$$\boxed{f(x) = (x+2)(x-3)(3x^2+4x+8)}$$

$$x = \frac{-4 \pm \sqrt{16-4(3)(8)}}{2(3)} = \frac{-4 \pm \sqrt{-80}}{6} = \frac{-4 \pm 4i\sqrt{5}}{6} = \frac{-2 \pm 2i\sqrt{5}}{3}$$

roots:

$$\boxed{(-2,0),\ (3,0),\ \left(\frac{-2 \pm 2i\sqrt{5}}{3}, 0\right)}$$



NC2. $f(x)$ has a single root at what x-value(s)?

$\boxed{x = -9, -1}$

NC3. $f(x)$ has a double root at what x-value(s)?

$\boxed{x = 3}$

NC4. $f(x)$ has a triple root at what x-value(s)?

$\boxed{x = -5}$

NC5. $f(x)$ has imaginary root(s) caused by the part of the graph near what x-value(s)?

Near x = 6 to x = 8

NC6. What is the lowest possible degree for $f(x)$?

9th

NC7. Solve the following inequality:

$$10x^3 + 3x^2 - 4x < 0$$

$x(10x^2 + 3x - 4) < 0$

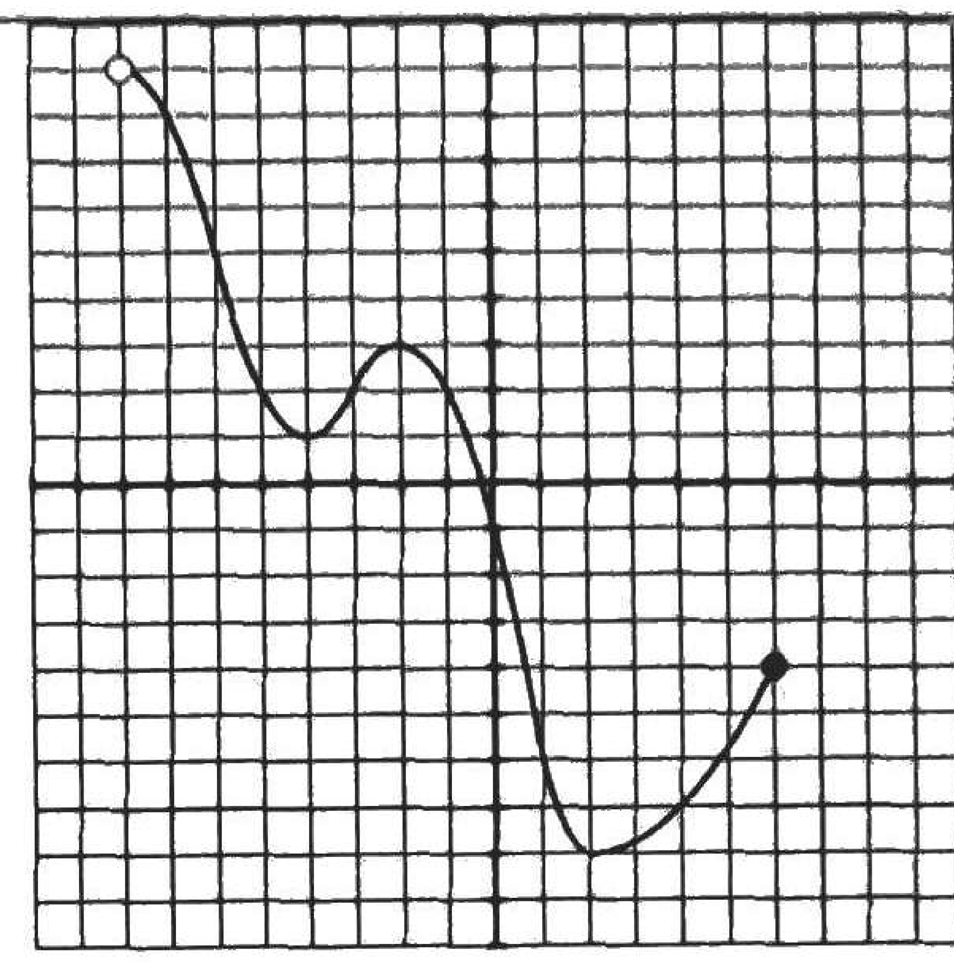
$x(5x+4)(2x-1) < 0$

$x = 0, -\frac{4}{5}, \frac{1}{2}$

$f(x)$	$-$		$+$		$-$		$+$
x		$-\frac{4}{5}$		0		$\frac{1}{2}$	

$\boxed{\left(-\infty, -\frac{4}{5}\right) \cup \left(0, \frac{1}{2}\right)}$

NC8.



Intervals of increasing and decreasing:

Increasing on $(-4,-2)$ and $(2,6]$

Decreasing on $(-8,-4)$ and $(-2,2)$

Local maxima and minima:

Local maximum of 3 at $x=-2$ and -4 at $x=6$

Local minimum of 1 at $x=-4$ and -8 at $x=2$

NC9. Use polynomial long division to find the quotient.

$$(15x^5 - 26x^4 + 30x^3 - 25x^2 + 13x - 10) \div (5x^2 - 2x + 4)$$

$3x^3 - 4x^2 + 2x - 1$

$5x^2 - 2x + 4 \;\big)\; \cancel{15x^5} - 26x^4 + 30x^3 - 25x^2 + 13x - 10$

$- \cancel{15x^5} + 6x^4 - 12x^3$

$-\cancel{20x^4} + 18x^3 - 25x^2 + 13x - 10$

$+\cancel{20x^4} - 8x^3 + 16x^2$

$\cancel{10x^3} - 9x^2 + 13x - 10$

$-\cancel{10x^3} + 4x^2 - 8x$

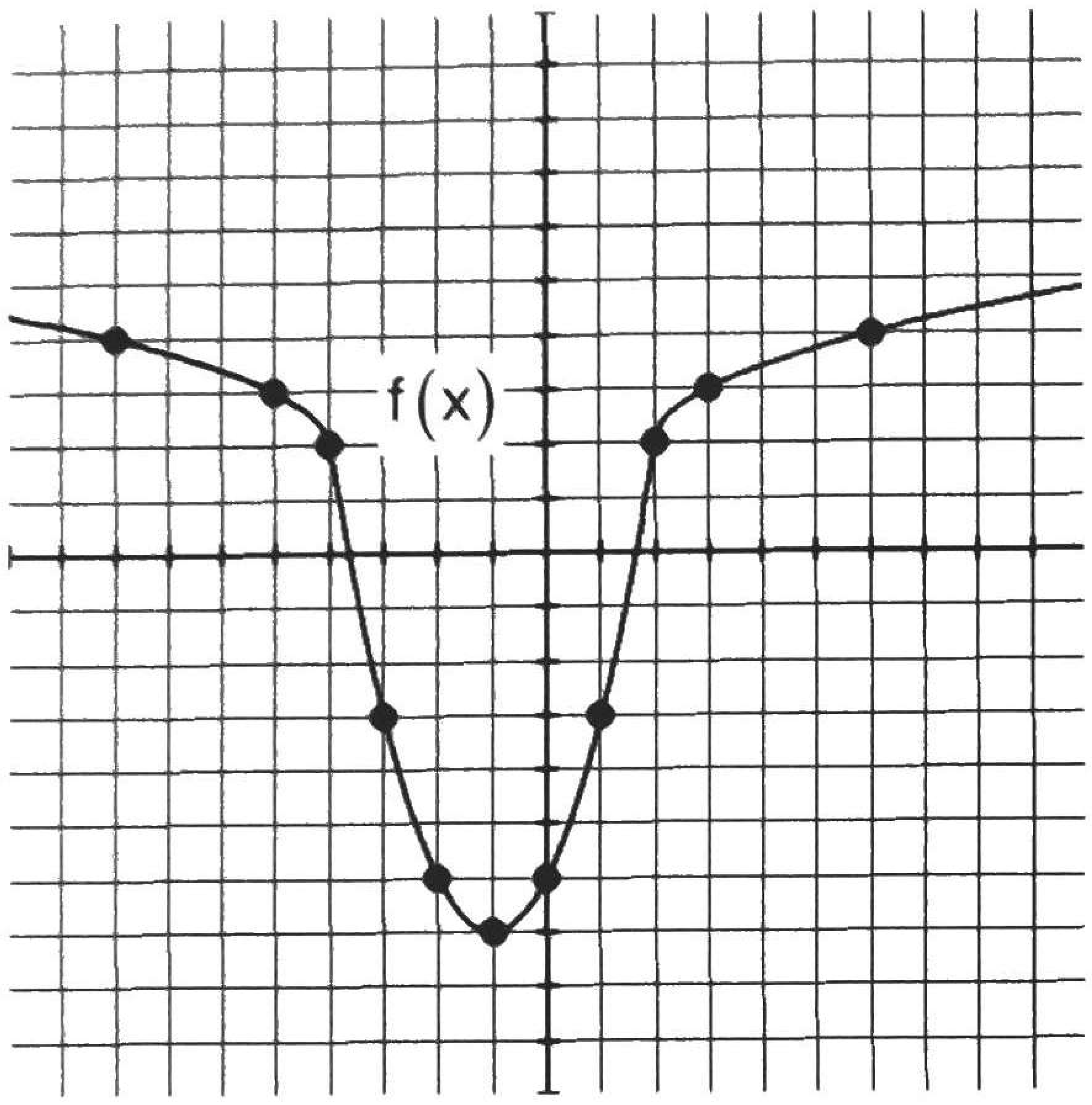
$-5x^2 + 5x - 10$

$+5x^2 - 2x + 4$

$3x - 6$

$\boxed{3x^3 - 4x^2 + 2x - 1 + \frac{3x-6}{5x^2-2x+4}}$

NC10. Use the following graph for f(x) and table for g(x) to find the indicated compositions:



x	-3	-1	0	3	4	6	7
g(x)	2	6	1	-4	6	3	10

a) Find $g(f(-5))$ =

$g(3) =$

$\boxed{-4}$

b) Find $g(g(-1))$ =

$g(6) =$

$\boxed{3}$

c) Find $f(g(4))$ =

$f(6) =$

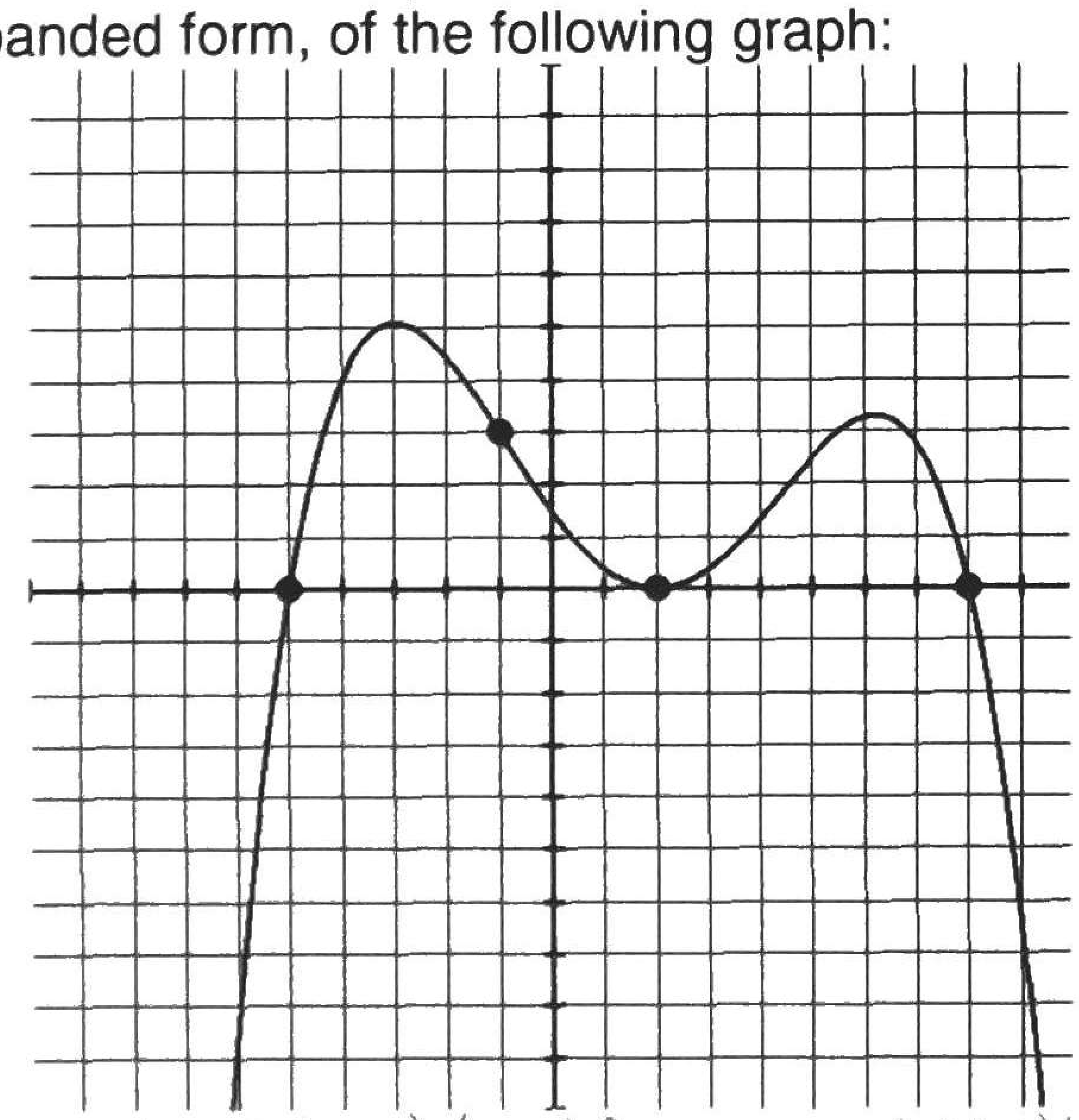
$\boxed{4}$

d) Find $f(f(3))$ =

$f(3) =$

$\boxed{3}$

NC11. Write the equation, in both factored and expanded form, of the following graph:



$f(x) = a(x+5)(x-8)(x-2)^2$

$\boxed{f(x) = -\frac{1}{108}(x+5)(x-8)(x-2)^2}$

$3 = a(4)(-9)(-3)^2$

$1 = a(4)(-3)(-3)^2$

$1 = -108a$

$a = -\frac{1}{108}$

$f(x) = -\frac{1}{108}(x^2-3x-40)(x^2-4x+4)$

$f(x) = -\frac{1}{108}(x^4-7x^3-24x^2+148x-160)$

$\boxed{f(x) = -\frac{1}{108}x^4 + \frac{7}{108}x^3 + \frac{2}{9}x^2 - \frac{37}{27}x + \frac{40}{27}}$

	x^2	$-4x$	$+4$
x^2	x^4	$-4x^3$	$4x^2$
$-3x$	$-3x^3$	$12x^2$	$-12x$
-40	$-40x^2$	$160x$	-160

NC12. Write the equation, in both factored and expanded form, of the polynomial function that has roots at $(-2, 0)$ and $(3+2i, 0)$ and passes through the point $(1, 2)$.

$f(x) = a(x+2)(x-3-2i)(x-3+2i)$

$f(x) = a(x+2)(x^2-6x+13)$

$2 = a(3)(8)$

$\boxed{f(x) = \frac{1}{12}(x+2)(x^2-6x+13)}$

$2 = 24a$

$a = \frac{1}{12}$

$f(x) = \frac{1}{12}(x^3-4x^2+x+26)$

$\boxed{f(x) = \frac{1}{12}x^3 - \frac{1}{3}x^2 + \frac{1}{12}x + \frac{13}{6}}$

	x	-3	$+2i$
x	x^2	$-3x$	$\cancel{2ix}$
-3	$-3x$	9	$\cancel{-6i}$
$-2i$	$\cancel{-2ix}$	$\cancel{6i}$	$-4i^2$

$x^2-6x+9+4$

$x^2-6x+13$

	x^2	$-6x$	$+13$
x	x^3	$-6x^2$	$13x$
$+2$	$2x^2$	$-12x$	26

x^3-4x^2+x+26

NC13. Find $g(f(x))$ and its domain:

$$f(x) = \frac{1}{x+3}$$

$x+3 \neq 0$

$x \neq -3$

$$g(x) = \frac{1}{x-2}$$

$$g(f(x)) = \frac{1\,(x+3)}{\left(\frac{1}{x+3} - 2\right)(x+3)}$$

$$g(f(x)) = \frac{x+3}{1-2x-6}$$

$$\boxed{g(f(x)) = \frac{x+3}{-2x-5}}$$

$-2x-5 \neq 0$

$-2x \neq 5$

$x \neq -\frac{5}{2}$

$\boxed{\text{Domain: } (-\infty, -3) \cup (-3, -5/2) \cup (-5/2, \infty)}$

NC14. Find $f(g(x))$ and its domain:

$$f(x) = \frac{1}{x^2 - 4}$$

$$g(x) = \sqrt{3x+1}$$

$3x+1 \geq 0$

$3x \geq -1$

$x \geq -1/3$

$$f(g(x)) = \frac{1}{\sqrt{3x+1}^{\,2} - 4}$$

$$f(g(x)) = \frac{1}{3x+1-4}$$

$$\boxed{f(g(x)) = \frac{1}{3x-3}}$$

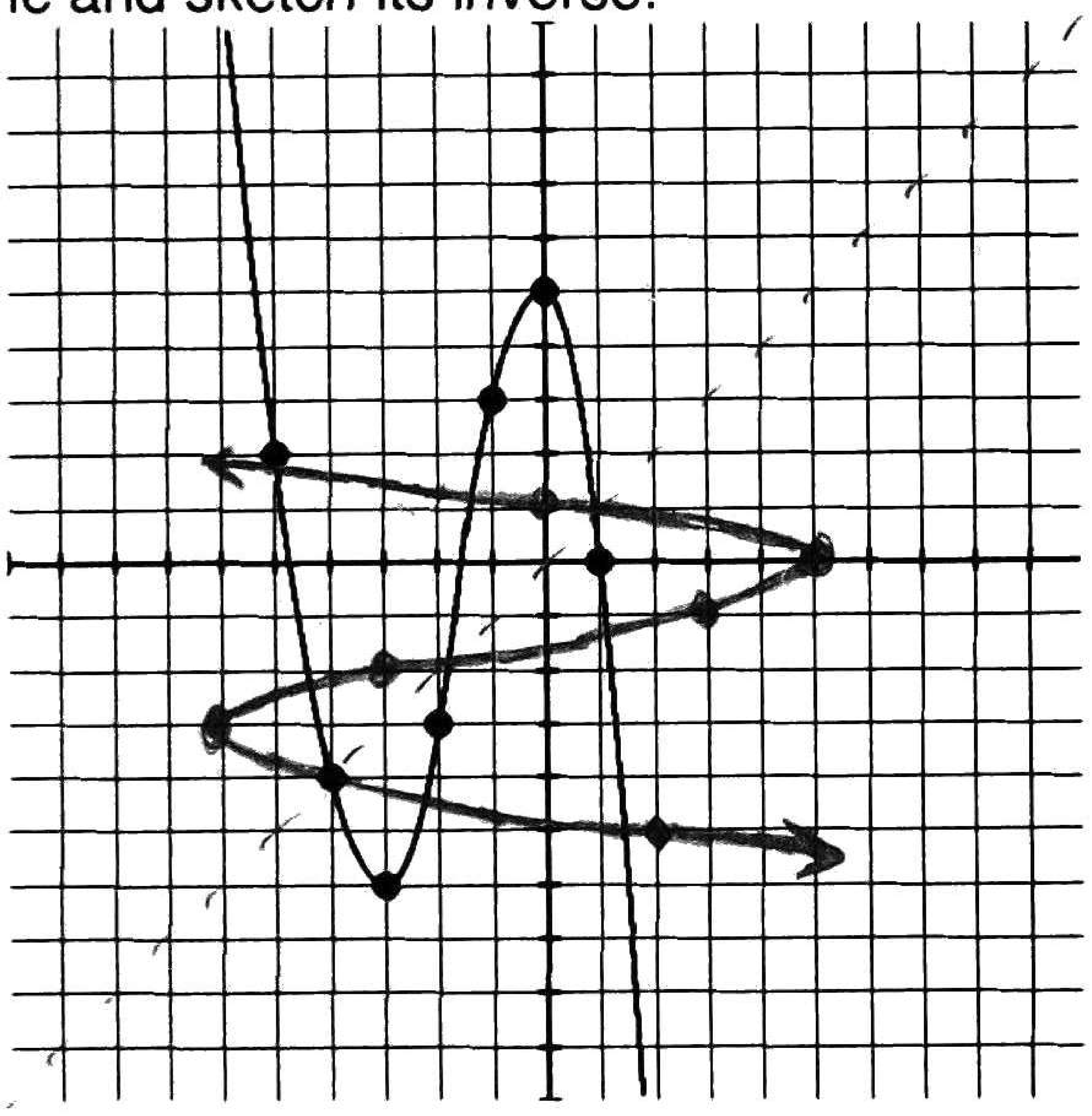
$3x-3 \neq 0$

$3x \neq 3$

$x \neq 1$

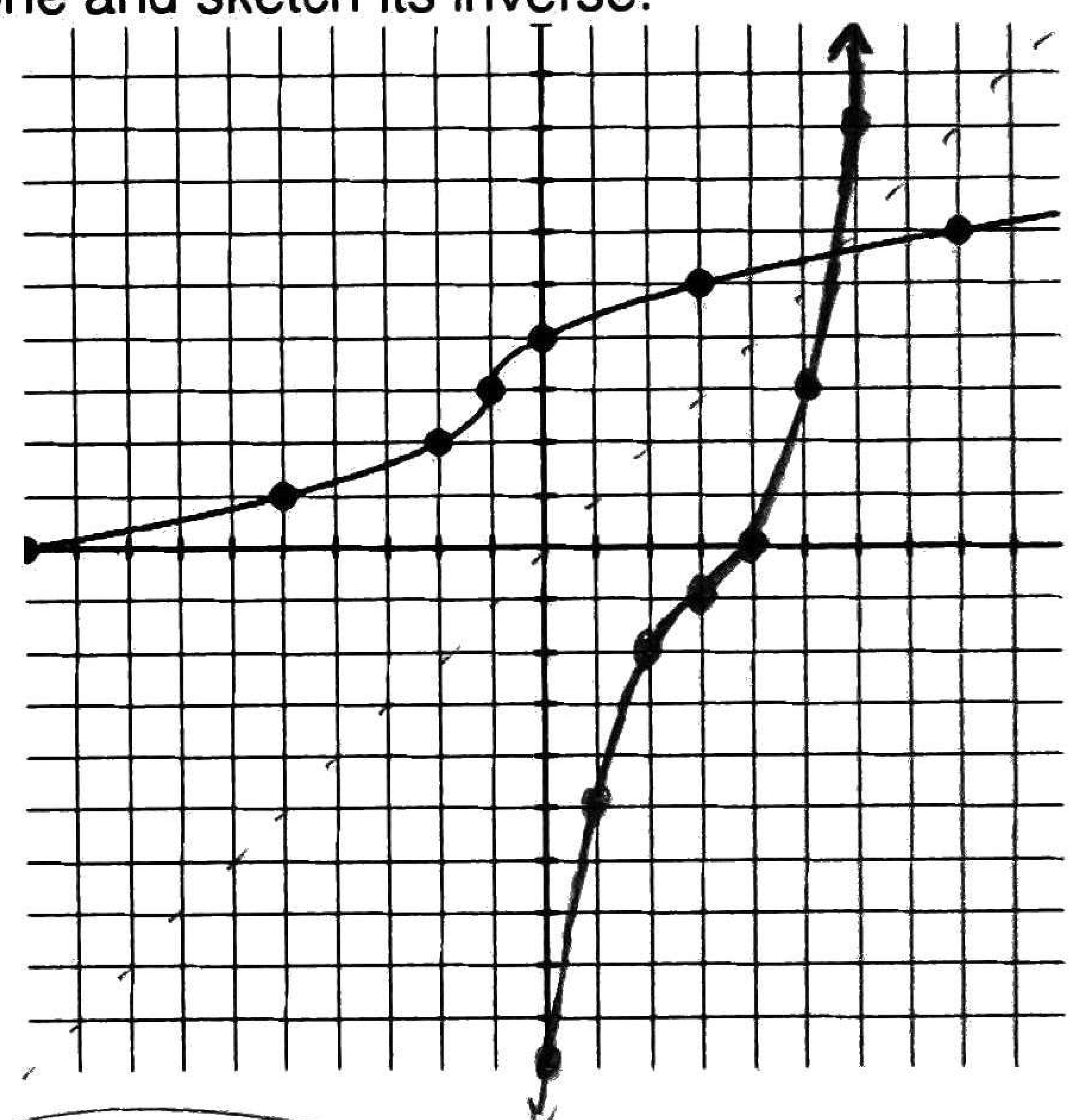
$\boxed{\text{Domain: } [-\tfrac{1}{3}, 1) \cup (1, \infty)}$

NC15. Determine if the following function is one-to-one and sketch its inverse:



$\boxed{\text{Not one-to-one}}$

NC16. Determine if the following function is one-to-one and sketch its inverse:



$\boxed{\text{One-to-one}}$

NC17. Find the inverse of the following function and find the domain and range of both the function and its inverse:

$$f(x) = 2\sqrt{x+3} - 5$$

Domain: $[-3, \infty)$
Range: $[-5, \infty)$

$x = 2\sqrt{y+3} - 5$

$x+5 = 2\sqrt{y+3}$

$\frac{x+5}{2} = \sqrt{y+3}$

$\left(\frac{x+5}{2}\right)^2 = y+3$

Domain: $[-5, \infty)$
Range: $[-3, \infty)$

$$\boxed{f^{-1}(x) = \left(\frac{x+5}{2}\right)^2 - 3}$$

NC18. Find the inverse of the following function and find the domain and range of both the function and its inverse:

$$f(x) = \frac{x-8}{x+6}$$

$(y+6)\,x = \frac{y-8}{\cancel{y+6}}\,\cancel{(y+6)}$

Domain: $(-\infty, -6) \cup (-6, \infty)$
Range: $(-\infty, 1) \cup (1, \infty)$

$xy + 6x = y - 8$

$xy - y = -6x - 8$

$y(x-1) = -6x - 8$

$$\boxed{f^{-1}(x) = \frac{-6x-8}{x-1}}$$

Domain: $(-\infty, 1) \cup (1, \infty)$
Range: $(-\infty, -6) \cup (-6, \infty)$