Review 3-1

McGann

Assume that this year, n = 1, the average annual salary for women in the United States is $21,000 and that is increasing by 5.5% per year.

1. Write an equation that describes the average annual salary for women. $\boxed{a_n = 21,000(1.055)^{n-1}}$	2. How much will a woman be making 24 years from now (year n = 25)? $a_{25} = 21,000(1.055)^{25-1}$ $\boxed{a_{25} = \$75,906.39}$
3. During which year will women's average annual salary be $50,000 or more? $50,000 = 21,000(1.055)^{n-1}$ $50/21 = 1.055^{n-1}$ $\ln(50/21) = \ln 1.055^{n-1}$ $\ln(50/21) = (n-1)\ln 1.055$ $16.203 = n-1$ $n = 17.203$ During the 18th year	4. What is the total an average woman would make over the first 25 years? $S_{25} = 21000\left(\frac{1-1.055^{25}}{1-1.055}\right)$ $\boxed{S_{25} = \$1,704,204.35}$

Assume that the average annual salary for men is $28,000 and that it is increasing by $1,500 per year.

5. Write an equation that describes the average annual salary for men. $\boxed{a_n = 28,000 + 1,500(n-1)}$	6. How much will a man be making 24 years from now (year n = 25)? $a_{25} = 28,000 + 1,500(25-1)$ $\boxed{a_{25} = \$64,000.00}$
7. During which year will men's average annual salary be $50,000 or more? $50,000 = 28,000 + 1,500(n-1)$ $22,000 = 1,500(n-1)$ $14.667 = n-1$ $n = 15.667$ During the 16th year	8. What is the total an average man would make over the first 25 years? $S_{25} = \left(\frac{64000+28000}{2}\right)(25)$ $\boxed{S_{25} = \$1,150,000}$

An arithmetic series has $a_1 = 12$ and $a_2 = 21$. $d = 21 - 12 = 9$

9. Find the recursive and explicit formula for the sequence. $\boxed{\text{recursive: } a_1 = 12,\ a_n = a_{n-1} + 9}$ $\boxed{\text{explicit: } a_n = 12 + 9(n-1)}$	10. Find the 400^{th} term. $a_{400} = 12 + 9(400-1)$ $\boxed{a_{400} = 3603}$
11. Find the sum of the first 400 terms. $S_{400} = \left(\frac{12+3603}{2}\right)(400)$ $\boxed{S_{400} = 723{,}000}$	12. Find n if $a_n = 210$. $210 = 12 + 9(n-1)$ $198 = 9(n-1)$ $22 = n-1$ $\boxed{n = 23}$
13. Find n if $S_n = 12{,}087$. $12087 = \left(\frac{12+a_n}{2}\right)n$ $12087 = \left(\frac{12+12+9(n-1)}{2}\right)n$ $24174 = (24 + 9n - 9)n$ $24174 = (15 + 9n)n$ $24174 = 15n + 9n^2$	$0 = 9n^2 + 15n - 24174$ $0 = 3n^2 + 5n - 8058$ $0 = (3n+158)(n-51)$ $n = -158/3,\ 51$ $\boxed{n = 51}$

14. How many rows are in the corner section of a stadium containing 3087 seats if the first row has 12 seats and each successive row has 3 additional seats.

$3087 = \left(\frac{12+a_n}{2}\right)n$

$3087 = \left(\frac{12+12+3(n-1)}{2}\right)n$

$6174 = (24 + 3n - 3)n$

$6174 = (21 + 3n)n$

$6174 = 21n + 3n^2$

$0 = 3n^2 + 21n - 6174$

$0 = n^2 + 7n - 2058$

$0 = (n-42)(n+49)$

$n = 42,\ -49$

$\boxed{n = 42}$

15. How many rows are in the corner section of a stadium containing 2040 seats if the first row has 10 seats and each successive row has 4 additional seats.

$2040 = \left(\frac{10+a_n}{2}\right)n$

$2040 = \left(\frac{10+10+4(n-1)}{2}\right)n$

$2040 = (10 + 2n - 2)n$

$2040 = (8 + 2n)n$

$2040 = 8n + 2n^2$

$0 = 2n^2 + 8n - 2040$

$0 = n^2 + 4n - 1020$

$0 = (n-30)(n+34)$

$n = 30,\ -34$

$\boxed{n = 30}$

A geometric series has $a_1 = 100$ and $a_2 = 95$. $r = \frac{95}{100} = 0.95$

16. Find the recursive and explicit formula for the sequence.

$\boxed{\text{recursive: } a_1 = 100, \quad a_n = 0.95a_{n-1}}$

$\boxed{\text{explicit: } a_n = 100(0.95)^{n-1}}$

17. Find the 20th term.

$a_{20} = 100(0.95)^{20-1}$

$\boxed{a_{20} = 37.735}$

18. Find n if $a_n = 56.88$

$56.88 = 100(0.95)^{n-1}$

$0.5688 = 0.95^{n-1}$

$\ln 0.5688 = \ln 0.95^{n-1}$

$\ln 0.5688 = (n-1)\ln 0.95$

$11 = n-1$

$n = 12$

19. Find S_3, the third partial sum, by writing and adding the first three terms.

$a_1 = 100 \quad a_2 = 95 \quad a_3 = 90.25$

$S_3 = 100 + 95 + 90.25$

$\boxed{S_3 = 285.25}$

20. Find S_3 algebraically, using the formula to show that the answer is the same as that in problem 19.

$S_3 = 100\left(\frac{1-0.95^3}{1-0.95}\right)$

$\boxed{S_3 = 285.25}$

21. Find n if $S_n = 1024.65$

$1024.65 = 100\left(\frac{1-0.95^n}{1-0.95}\right)$

$10.2465 = \frac{1-0.95^n}{0.05}$

$0.512325 = 1-0.95^n$

$-0.487675 = -0.95^n$

$0.487675 = 0.95^n$

$\ln 0.487675 = \ln 0.95^n$

$\ln 0.487675 = n\ln 0.95$

$\boxed{n = 14}$

Determine the first four terms of the following sequences. Then, determine a_n as an explicit function of n and determine the 8th term:

22. $a_1 = -4$; $a_n = a_{n-1} + 1$

$\boxed{-4, -3, -2, -1}$

$a_8 = -4 + 1(8-1)$

$\boxed{a_8 = 3}$

$\boxed{a_n = -4 + 1(n-1)}$

23. $a_1 = 2$; $a_n = 3a_{n-1}$

$\boxed{2, 6, 18, 54}$

$\boxed{a_n = 2(3)^{n-1}}$

$a_8 = 2(3)^{8-1}$

$\boxed{a_8 = 4374}$

24. Find the first term and the common difference of the arithmetic sequence in which the 5th term is 33 and the 10th term is 68.

$a_5 = a_1 + d(5-1) \qquad a_{10} = a_1 + d(10-1)$

$33 = a_1 + 4d \qquad 68 = a_1 + 9d$

$68 = a_1 + 9d$

$-33 = -a_1 - 4d$

$35 = 5d$

$\boxed{d = 7}$

$33 = a_1 + 4(7)$

$33 = a_1 + 28$

$\boxed{a_1 = 5}$

25. Find the first term and the common difference of the arithmetic sequence in which the 4th term is 3 and the 20th term is 35.

$a_4 = a_1 + d(4-1) \qquad a_{20} = a_1 + d(20-1)$

$3 = a_1 + 3d \qquad 35 = a_1 + 19d$

$35 = a_1 + 19d$

$-3 = -a_1 - 3d$

$32 = 16d$

$\boxed{d = 2}$

$3 = a_1 + 3(2)$

$3 = a_1 + 6$

$\boxed{a_1 = -3}$

Write the following series in sigma notation.

26. $4+12+36+\ldots+324$ $a_1 = 4, \quad r = 3$ $\boxed{\sum_{k=1}^{5} 4(3)^{k-1}}$ $324 = 4(3)^{n-1}$ $81 = 3^{n-1}$ $3^4 = 3^{n-1}$ $4 = n-1$ $n = 5$	27. $-16-9-2+\ldots+103$ $a_1 = -16 \quad d = 7$ $\boxed{\sum_{k=1}^{18} \left[-16+7(k-1)\right]}$ $-16+7(n-1) = 103$ $7(n-1) = 119$ $n-1 = 17$ $n = 18$

Determine if the following series are convergent or divergent and state why. If it is convergent, find the infinite sum.

28. $625+250+100+\ldots$ $a_1 = 625 \quad r = \frac{250}{625} = \frac{100}{250} = \frac{2}{5}$ Converges because $\lvert r \rvert < 1$ $S = \frac{625}{1-2/5} = \frac{625}{3/5} = 625\left(\frac{5}{3}\right) = \boxed{\frac{3125}{3}}$	29. $28+22+16+\ldots$ $a_1 = 28 \quad d = -6$ Diverges because it is Arithmetic

Find the following binomial expansions:

30. Expand: $(2x-y)^6$ $1(2x)^6(-y)^0 + 6(2x)^5(-y)^1 + 15(2x)^4(-y)^2 + 20(2x)^3(-y)^3 +$ $15(2x)^2(-y)^4 + 6(2x)^1(-y)^5 + 1(2x)^0(-y)^6$ $\boxed{64x^6 - 192x^5y + 240x^4y^2 - 160x^3y^3 + 60x^2y^4 - 12xy^5 + y^6}$	31. Find the y^{20} -term: $(x^3-y^5)^7$ $35(x^3)^3(-y^5)^4$ ${}_7C_4 = \frac{7!}{4!3!}$ $\boxed{35x^9y^{20}}$ ${}_7C_4 = 35$

Evaluate the following sums:

32. $\sum_{k=1}^{50}(3k-1)^2 =$ $\sum_{k=1}^{50}(9k^2-6k+1) =$ $9\left[\frac{50(50+1)(100+1)}{6}\right] - 6\left[\frac{50(50+1)}{2}\right] + 50 =$ $\boxed{378,725}$	33. $\sum_{k=1}^{4} 3^{k+2} =$ $3^3+3^4+3^5+3^6 =$ $\boxed{1080}$	34. $\sum_{k=1}^{100}(3k^2-1) =$ $3\left[\frac{100(100+1)(200+1)}{6}\right] - 100 =$ $\boxed{1,014,950}$