M4 7am

Review 2-2

	x^2	$+2x$	-8
x	x^3	$2x^2$	$-8x$
-3	$-3x^2$	$-6x$	24

Original Function:

$$f(x)=\frac{(x-3)\cancel{(3x+1)}(x+4)(x-2)}{\cancel{(3x+1)}(x-5)(x+1)}$$

Reduced Form:

$$\boxed{f(x)=\frac{(x-3)(x+4)(x-2)}{(x-5)(x+1)}}$$

Expanded Form:

$$\boxed{f(x)=\frac{x^3-x^2-14x+24}{x^2-4x-5}}$$

C1. State the vertical asymptotes:

$x-5=0$ $\quad$ $x+1=0$

$\boxed{x=5}$ $\quad$ $\boxed{x=-1}$

C2. State the roots:

$x-3=0$ $\quad$ $x+4=0$ $\quad$ $x-2=0$

$x=3$ $\quad$ $x=-4$ $\quad$ $x=2$

$\boxed{(3,0)}$ $\quad$ $\boxed{(-4,0)}$ $\quad$ $\boxed{(2,0)}$

C3. State the removable discontinuities:

$3x+1=0$

$3x=-1$

$x=-\frac{1}{3}$

$$y=\frac{(-\frac{1}{3}-3)(-\frac{1}{3}+4)(-\frac{1}{3}-2)}{(-\frac{1}{3}-5)(-\frac{1}{3}+1)}$$

$\boxed{(-\frac{1}{3},-8.021)}$ $\quad y=-8.021$

C4. State the y-intercept:

$$y=\frac{0^3-0^2-14(0)+24}{0^2-4(0)-5}=\frac{24}{-5}$$

$\boxed{(0,\frac{-24}{5})}$ or $(0,-4.8)$

C5. State the horizontal or oblique asymptote. Use Polynomial Long Division:

$$\begin{array}{r} x+3 \\ x^2-4x-5\,\overline{)\,x^3-x^2-14x+24} \\ -x^3+4x^2+5x \\ \hline 3x^2-9x+24 \\ -3x^2+12x+15 \\ \hline 3x+39 \end{array}$$

$\boxed{y=x+3}$

C6. Find where the graph crosses the horizontal or oblique asymptote:

$3x+39=0$

$3x=-39$

$x=-13$

$y=-13+3$

$y=-10$

$\boxed{(-13,-10)}$

C7. If necessary, find a test point for each of the regions divided by the vertical asymptotes. Some of these may already exist such as removable discontinuities, roots, etc.

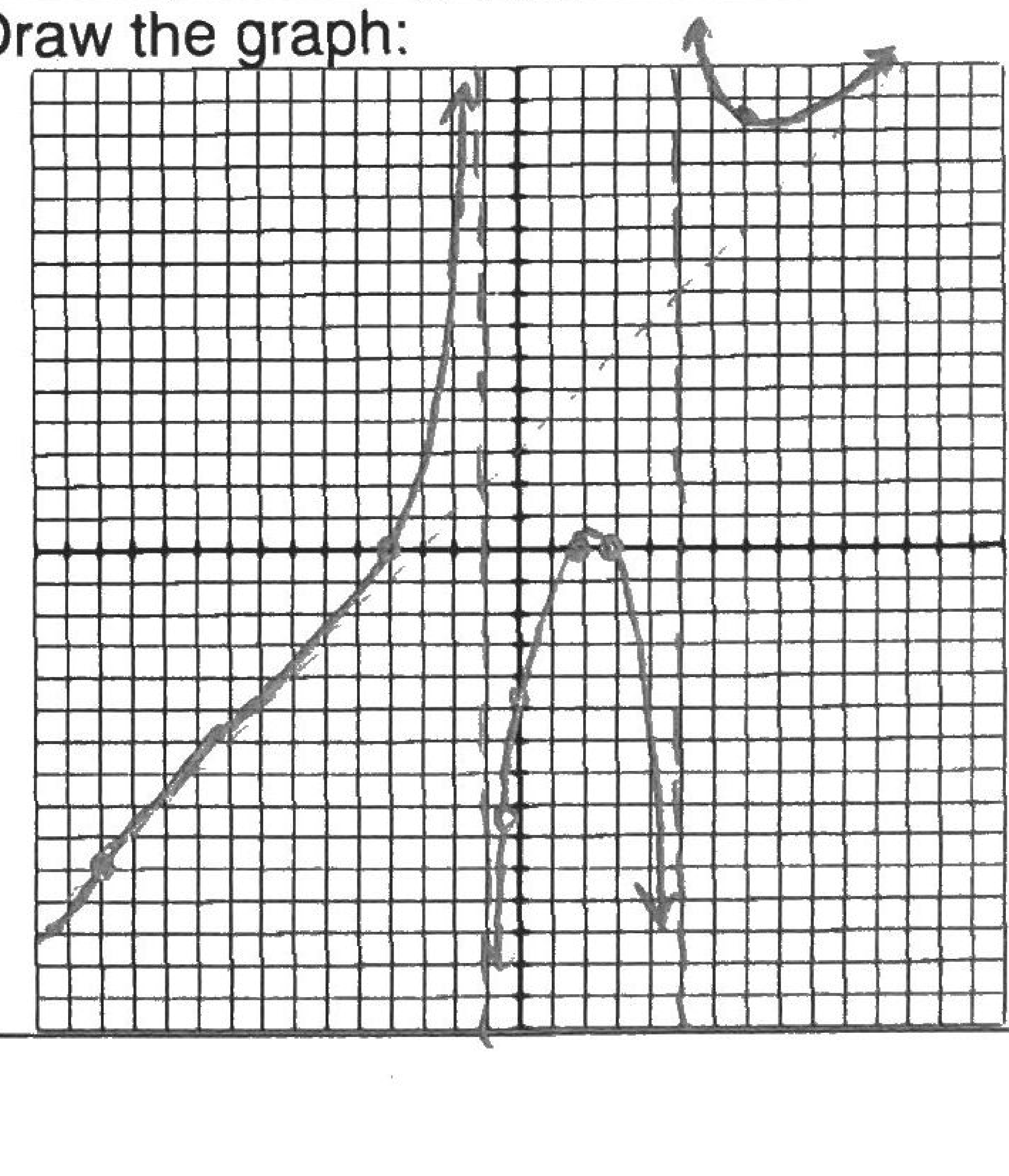
Pick $x=7$

$$y=\frac{(7-3)(7+4)(7-2)}{(7-5)(7+1)}$$

$y=13.75$

$\boxed{(7,13.75)}$

C8. Draw the graph:



C9. Given $y = \frac{2x+14}{x^2+9x+14}$ $= \frac{2(\cancel{x+7})}{(\cancel{x+7})(x+2)}$

a) Describe the end behavior of the graph.

Since the denominator grows faster than the numerator, the graph will approach a horizontal asymptote at $y=0$

b) Describe the breaks in continuity. State where they occur, what type they are, and explain how you know.

There is a removable discontinuity at $x=-7$ because $(x+7)$, which causes a divide by zero, cancels out of the equation. There is a vertical asymptote at $x=-2$ because $(x+2)$, which also causes a divide by zero, does not cancel out.

Simplify the following Rational Expressions

C10. $\frac{x-3}{x^2-x-2} - \frac{x-2}{x+1} =$

$$\frac{x-3}{(x-2)(x+1)} + \frac{-(x-2)(x-2)}{(x+1)(x-2)} =$$

$$\frac{x-3-(x^2-4x+4)}{(x-2)(x+1)} =$$

$$\frac{x-3-x^2+4x-4}{(x-2)(x+1)} =$$

$$\boxed{\frac{-x^2+5x-7}{(x-2)(x+4)}}$$

C11. $\frac{9x^3}{\sqrt{6x^2-5}} + 7x^4\sqrt{6x^2-5} =$

$$\frac{9x^3}{\sqrt{6x^2-5}} + \frac{7x^4\sqrt{6x^2-5}}{1} \cdot \frac{\sqrt{6x^2-5}}{\sqrt{6x^2-5}} =$$

$$\frac{9x^3+7x^4(6x^2-5)}{\sqrt{6x^2-5}} =$$

$$\boxed{\frac{9x^3+42x^6-35x^4}{\sqrt{6x^2-5}}}$$

C12. $\frac{5x^2+3x-2}{1-x^2} \bullet \frac{2x-2}{20x-8} =$

$$\frac{5x^2+3x-2}{-(x^2-1)} \cdot \frac{2x-2}{20x-8} =$$

$$\frac{(\cancel{5x-2})(\cancel{x+1})}{-(\cancel{x-1})(\cancel{x+1})} \cdot \frac{2(\cancel{x-1})}{4(\cancel{5x-2})} =$$

$$\boxed{-\frac{1}{2}}$$

C13. $\frac{(4-\sqrt{9x-1})(4+\sqrt{9x-1})}{(17-9x)(4+\sqrt{9x-1})} =$

$$\frac{16-(9x-1)}{(17-9x)(4+\sqrt{9x-1})} =$$

$$\frac{16-9x+1}{(17-9x)(4+\sqrt{9x-1})} =$$

$$\frac{\cancel{17-9x}}{(\cancel{17-9x})(4+\sqrt{9x-1})} = \boxed{\frac{1}{4+\sqrt{9x-1}}}$$

Solve for x:

C14. $\frac{x-1}{x} - \frac{5}{x+2} = \frac{3}{x^2+2x}$

$\frac{(x-1)(x+2)}{\cancel{x(x+2)}} + \frac{-5(x)}{\cancel{(x+2)(x)}} = \frac{3}{\cancel{x(x+2)}}$

$(x-1)(x+2) - 5x = 3$

$x^2 + x - 2 - 5x = 3$

$x^2 - 4x - 5 = 0$

$(x-5)(x+1) = 0$

$x-5=0 \qquad x+1=0$

$\boxed{x=5} \qquad \boxed{x=-1}$

Excluded Values:

$x=0$

$x+2=0$

$x=-2$

C15. $\left(6\sqrt{9x-1} + \frac{2x}{\sqrt{9x-1}} = 0\right)\sqrt{9x-1}$

$6(9x-1) + 2x = 0$

$54x - 6 + 2x = 0$

$56x = 6$

$x = \frac{3}{28}$

Since $\frac{3}{28} < \frac{1}{9}$,

$\boxed{\text{No Solution}}$

Excluded Values

$9x-1 \le 0$

$9x \le 1$

$x \le \frac{1}{9}$

Find the interval(s) for which the following are true:

C16. $\frac{2x-6}{1-x} < 2$

Excluded Value:

$1-x=0$

$x=1$

$\frac{2x-6}{1-x} - 2 < 0$

$2x-6-2(1-x) = 0$

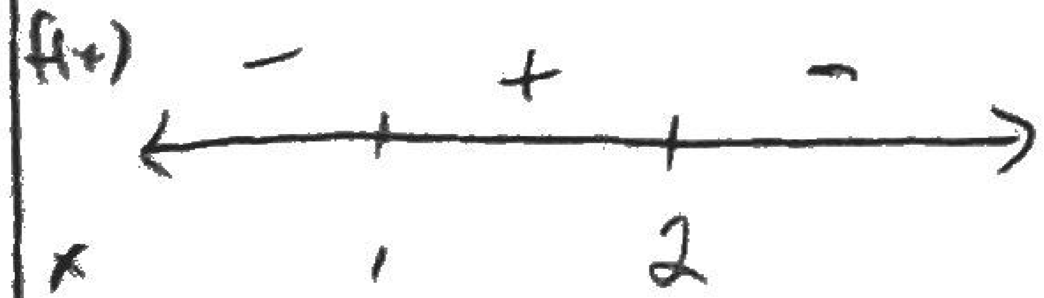
$2x-6-2+2x = 0$

$4x-8=0$

$4x=8$

$x=2$

$\boxed{(-\infty,1)\cup(2,\infty)}$



C17. $\frac{6}{x+3} \ge 1$

Excluded Value:

$x+3=0$

$x=-3$

$\frac{6}{x+3} - 1 \ge 0$

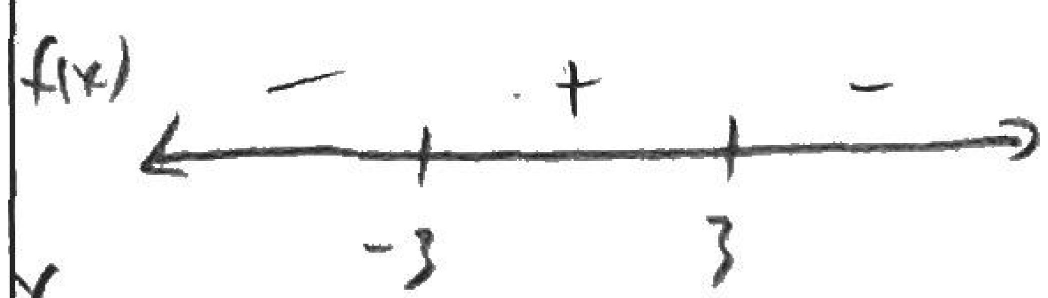
$6 - 1(x+3) = 0$

$6 - x - 3 = 0$

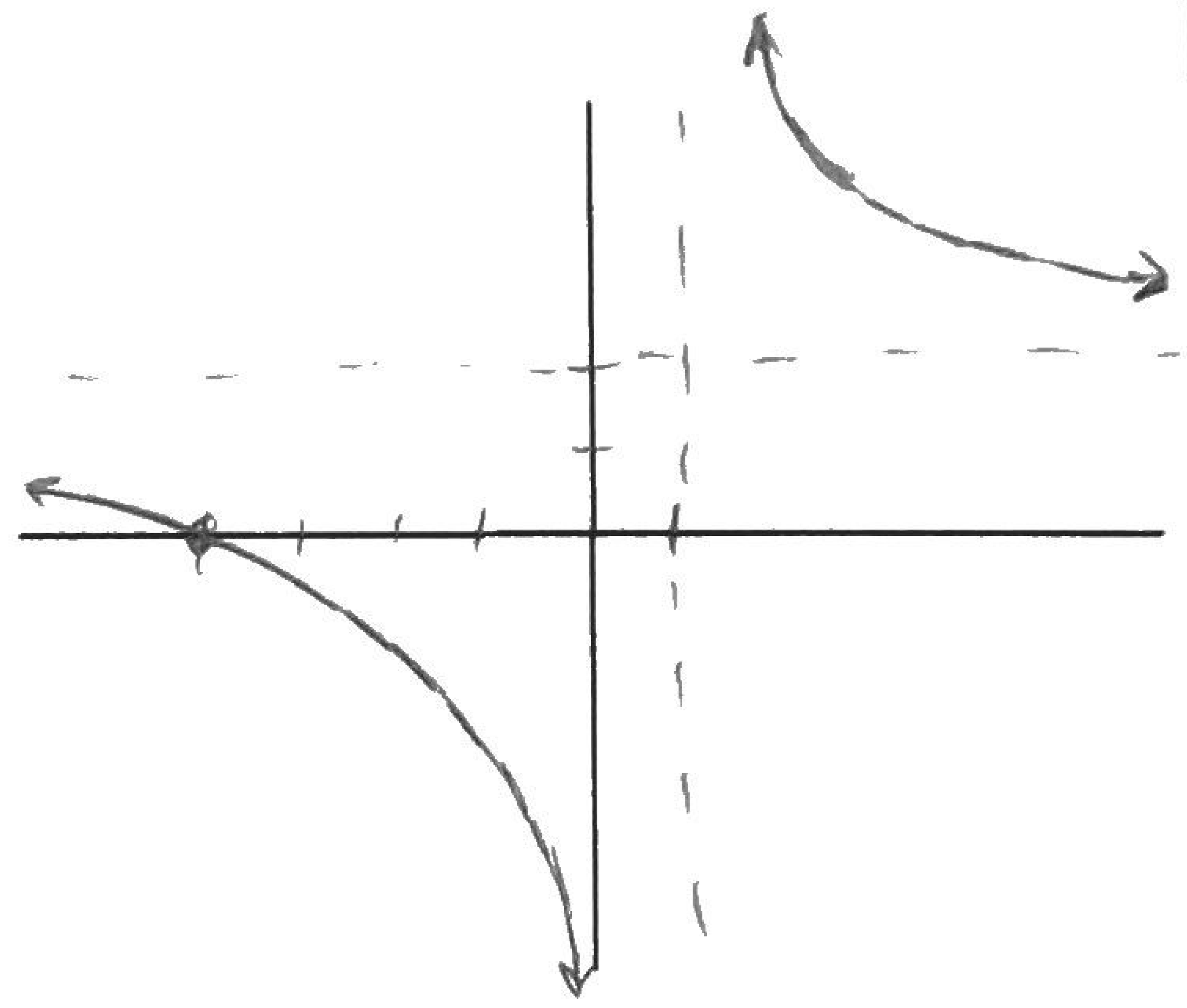
$-x+3=0$

$x=3$

$\boxed{(-3,3]}$



C18. Given the following conditions, sketch a graph of a function $f(x)$ and state the domain, range, and equation:



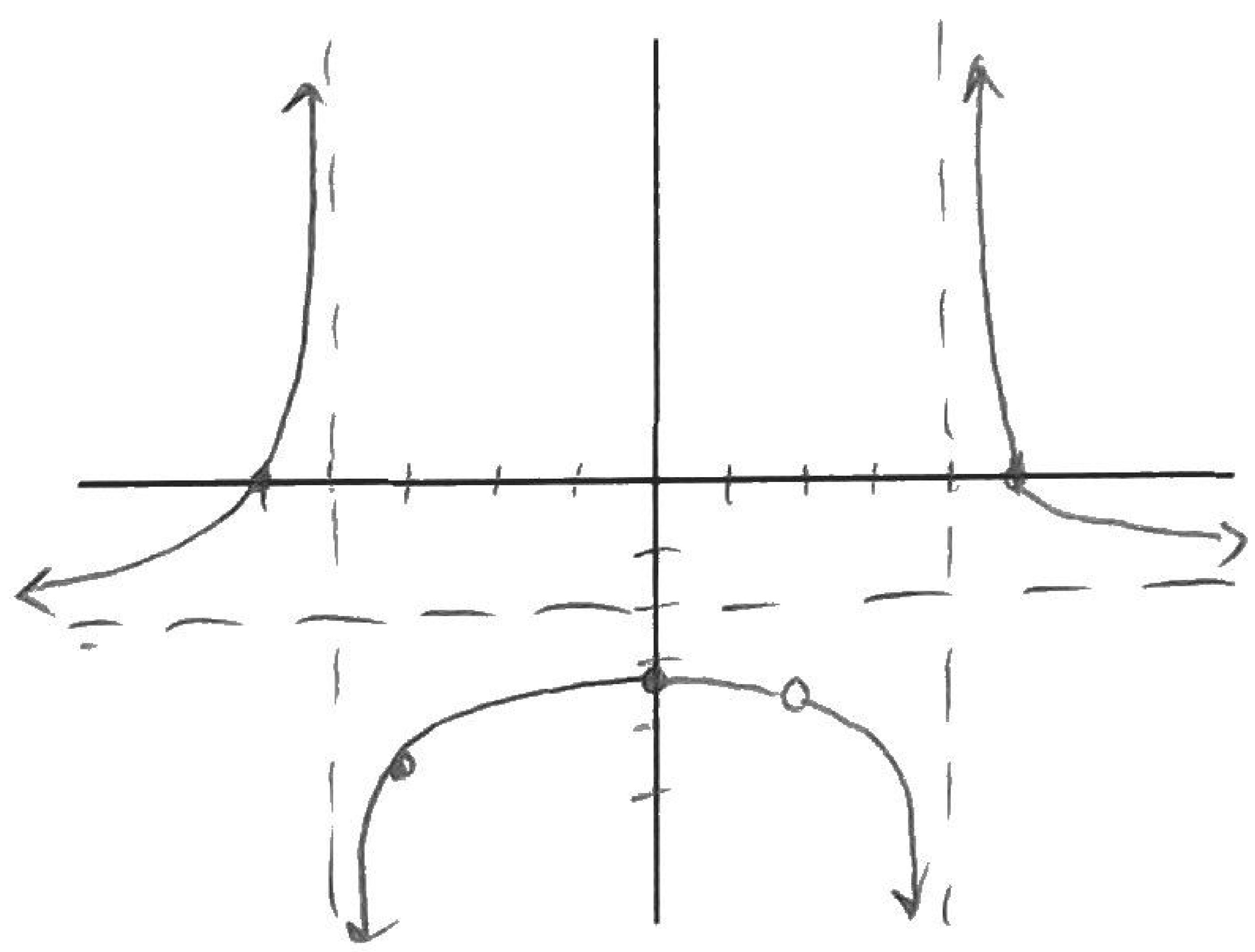
- ☺ $f(x)$ is decreasing on $(-\infty, 1)$ and $(1, \infty)$
- ☺ $f(x)$ has a horizontal asymptote at $y = 2$
- ☺ $f(x)$ has a vertical asymptote at $x = 1$
- ☺ $f(-4) = 0$

Equation: $f(x) = \dfrac{2(x+4)}{x-1}$

Domain: $(-\infty, 1) \cup (1, \infty)$

Range: $(-\infty, 2) \cup (2, \infty)$

C19. Given the following conditions, sketch a graph of a function $f(x)$ and state the domain, range, and equation:



- ☺ $f(x)$ has a horizontal asymptote at $y = -2$
- ☺ $f(x)$ has a vertical asymptotes at $x = -4$ and $x = 4$
- ☺ $f(x)$ has a removable discontinuity at $(2, -3.5)$
- ☺ $f(x)$ has a y-intercept at -3.125
- ☺ $f(x)$ has roots at -5 and 5
- ☺ $f(-3) = -4.571$

Equation: $f(x) = \dfrac{-2(x+5)(x-5)(x-2)}{(x+4)(x-4)(x-2)}$

Domain: $(-\infty, -4) \cup (-4, 2) \cup (2, \infty)$

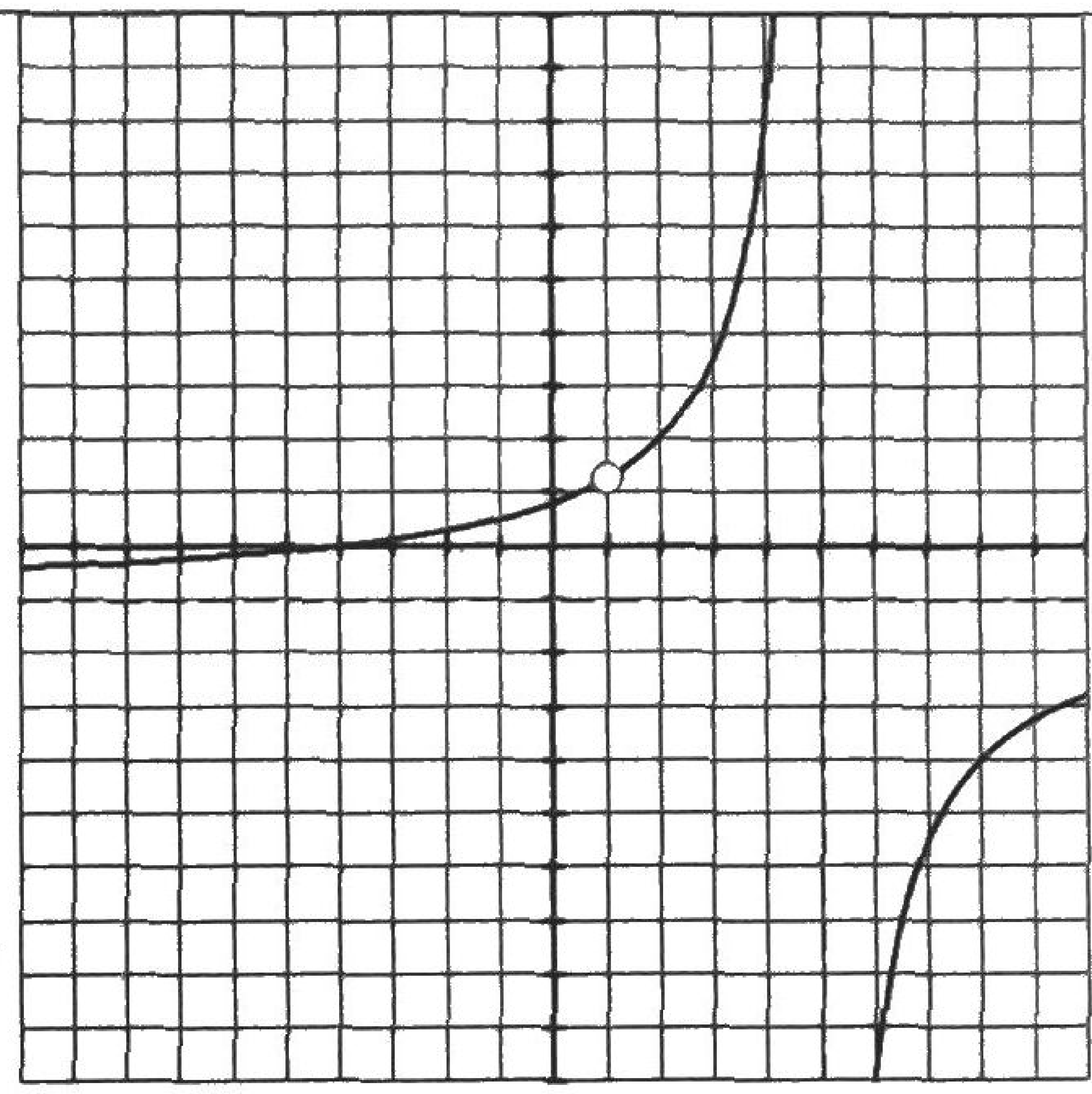
Range: $(-\infty, -3.125] \cup (-2, \infty)$

For each graph write the equation of the function, identify the domain and range, and evaluate the indicated limits.

C20. Equation:

$$y = \frac{-(x+4)(x-1)}{(x-5)(x-1)}$$

$$\frac{-(1+4)}{(1-5)} = 1.25$$



Domain: $(-\infty, 1) \cup (1, 5) \cup (5, \infty)$

Range: $(-\infty, -1) \cup (-1, 1.25) \cup (1.25, \infty)$

$\lim\limits_{x\to 1} f(x) = 1.25$ $\lim\limits_{x\to\infty} f(x) = -1$ $\lim\limits_{x\to 5^-} f(x) = \infty$ $\lim\limits_{x\to 5^+} f(x) = -\infty$

C17. Equation:

$$f(x) = \frac{a(x+4)(x-2)(x-3)}{(x+2)(x-4)}$$

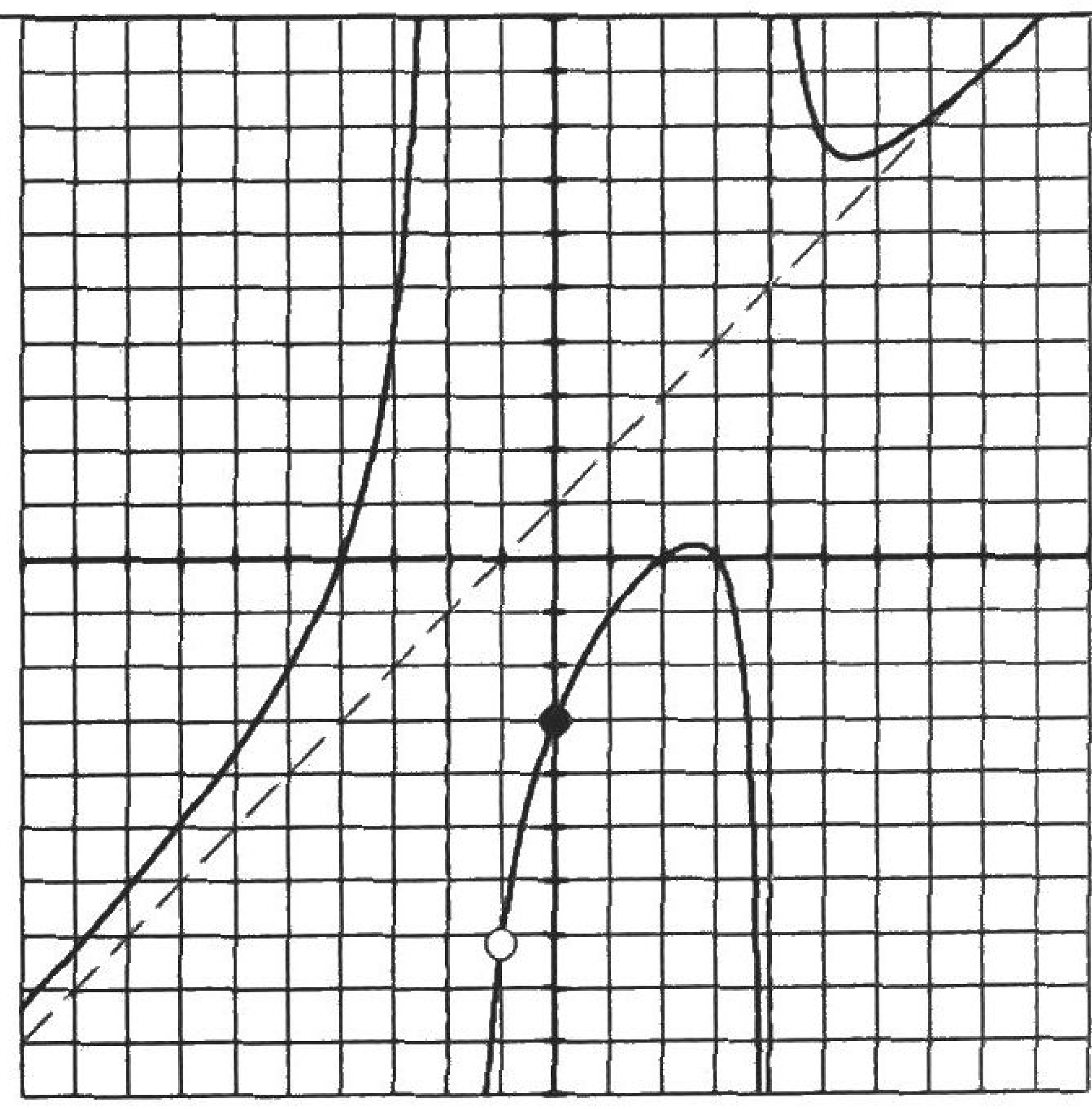
$$f(x) = \frac{(x+4)(x-2)(x-3)(x+1)}{(x+2)(x-4)(x+1)}$$

$$-3 = \frac{a(0-4)(0-2)(0-3)}{(0+2)(0-4)}$$

$$\frac{(-1+4)(-1-2)(-1-3)}{(-1+2)(-1-4)} = -7.2$$

$-3 = -3a$

$a = 1$



Domain: $(-\infty, -2) \cup (-2, -1) \cup (-1, 4) \cup (4, \infty)$

Range: $(-\infty, \infty)$

$\lim\limits_{x\to -\infty} f(x) = -\infty$ $\lim\limits_{x\to -1} f(x) = -7.2$ $\lim\limits_{x\to -2^-} f(x) = \infty$ $\lim\limits_{x\to -2^+} f(x) = -\infty$ $\lim\limits_{x\to 4^-} f(x) =$ $\lim\limits_{x\to 4^+} f(x) =$

C18. Equation:

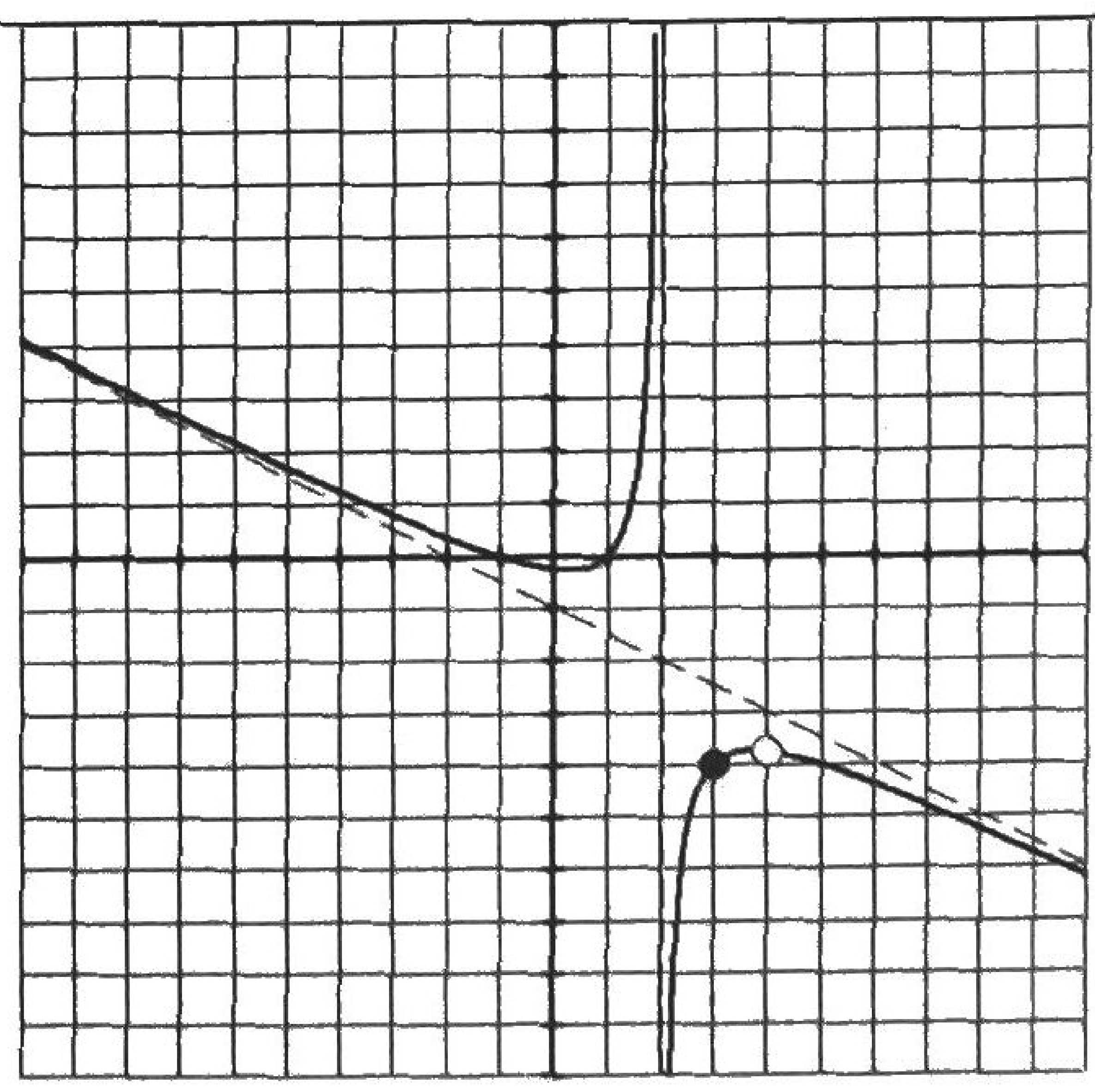
$$f(x) = \frac{a(x+1)(x-1)}{x-2}$$

$$f(x) = -\frac{(x+1)(x-1)(x-4)}{2(x-2)(x-4)}$$

$$-4 = \frac{a(3+1)(3-1)}{3-2}$$

$-4 = 8a$

$a = -\frac{1}{2}$



Domain: $(-\infty, 2) \cup (2, 4) \cup (4, \infty)$

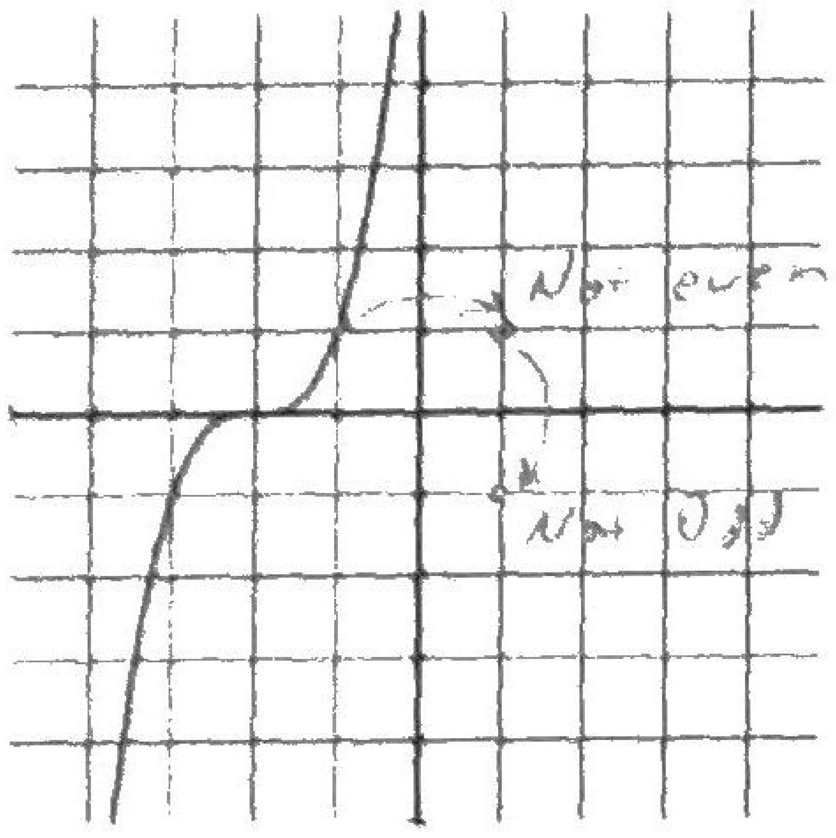
Range: $(-\infty, -3.732] \cup [-0.268, \infty)$

$$-\frac{(4+1)(4-1)}{2(4-2)} = -3.75$$

$\lim\limits_{x\to -\infty} f(x) = \infty$ $\lim\limits_{x\to\infty} f(x) = -\infty$ $\lim\limits_{x\to 4} f(x) = -3.75$ $\lim\limits_{x\to 3} f(x) = -4$ $\lim\limits_{x\to 2^-} f(x) = \infty$ $\lim\limits_{x\to 2^+} f(x) = -\infty$

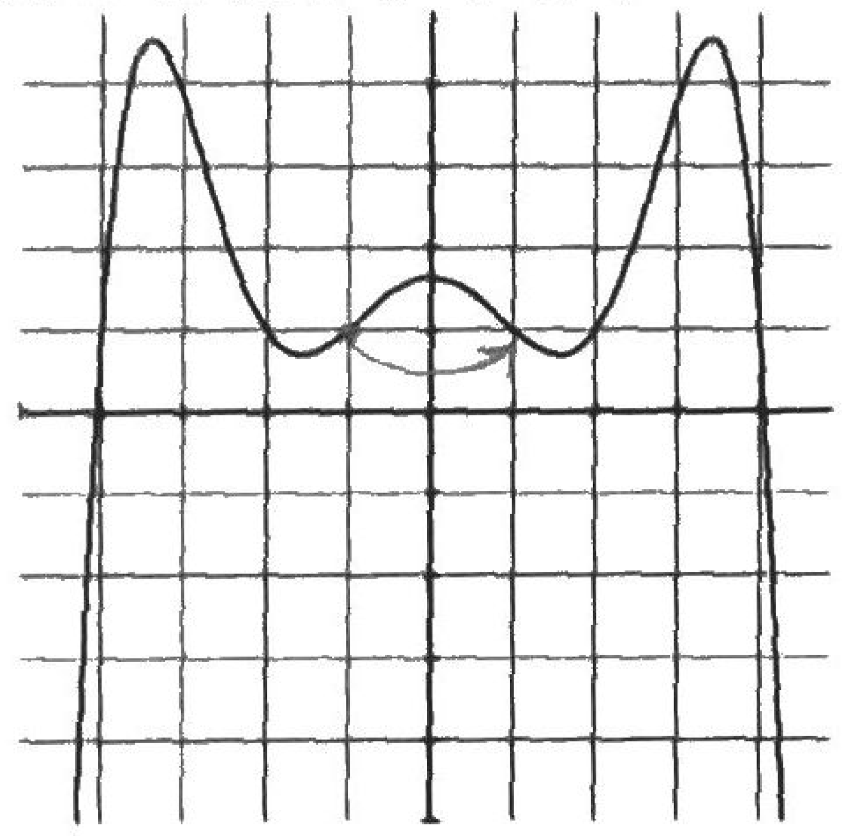
Determine if the following functions are even, odd, or neither:

C19.



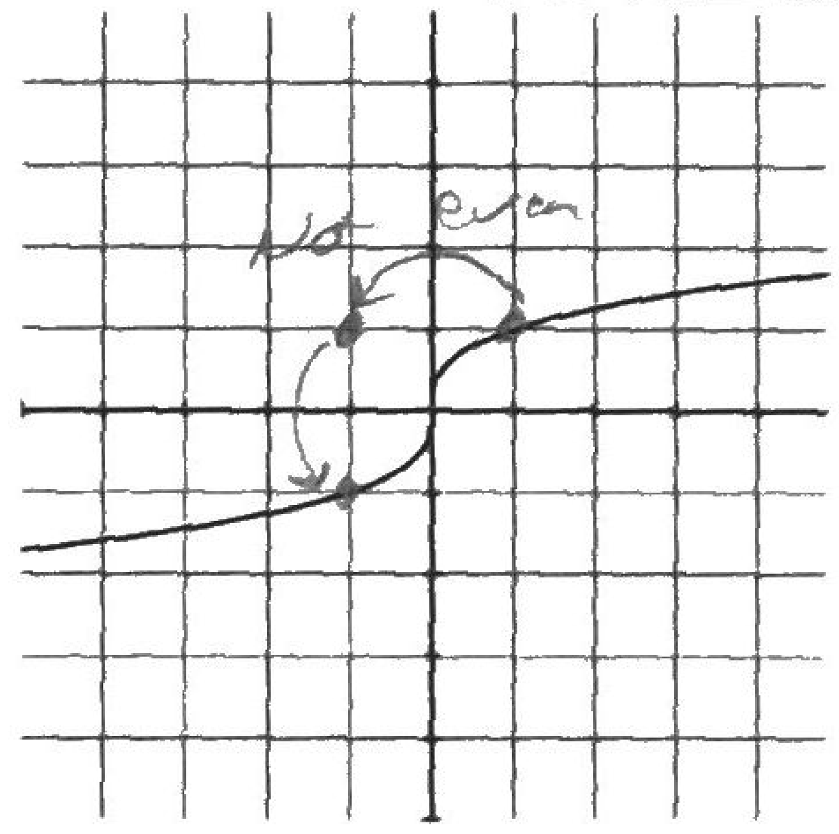
Neither

C20.



Even

C21.



Odd

C22. $f(x) = 4x - 5x^2$

$f(-x) = 4(-x) - 5(-x)^2$

$= -4x - 5x^2$

changed sign

did not change sign

Neither

C23. $f(x) = \dfrac{|x|}{x^3}$

$f(-x) = \dfrac{|-x|}{(-x)^3}$

$= \dfrac{|x|}{-x^3}$

stayed positive

changed sign, so the entire fraction changed sign

Odd

C24. $f(x) = \dfrac{\sqrt[3]{x}}{x - x^7}$

$f(-x) = \dfrac{\sqrt[3]{-x}}{-x-(-x)^7}$

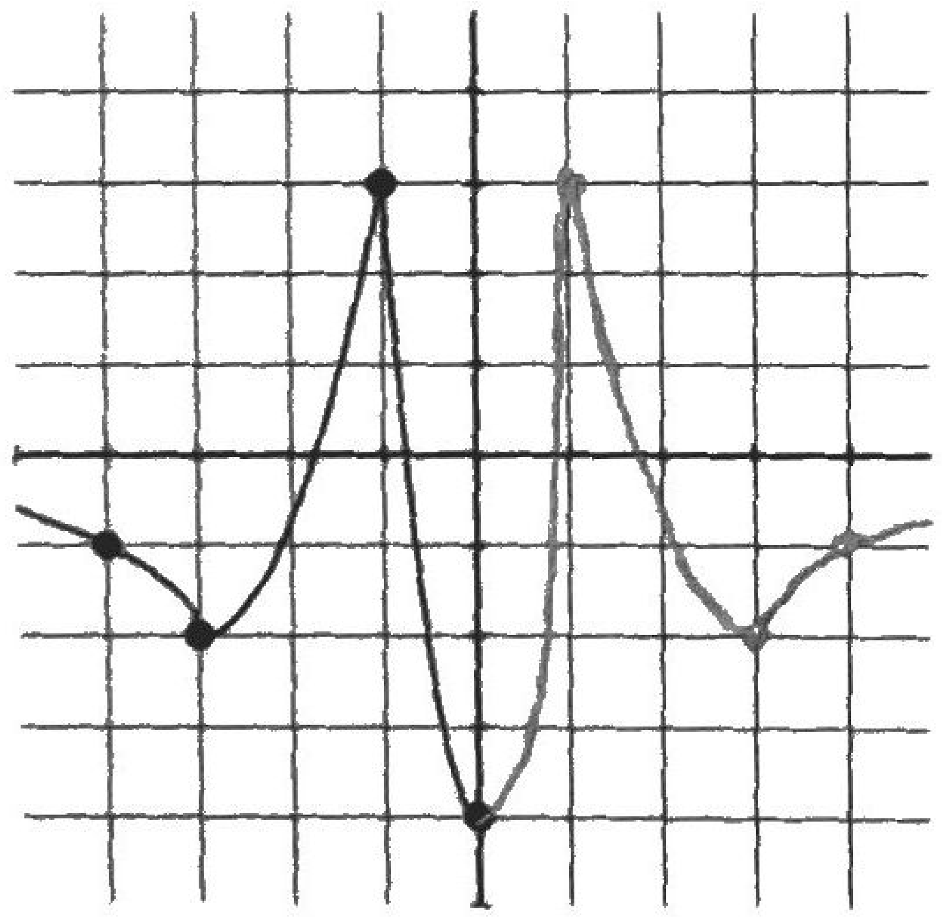
$= \dfrac{-\sqrt[3]{x}}{-x+x^7}$

The numerator changed sign, but so did the entire denominator, so the whole fraction stayed the same sign.

Even

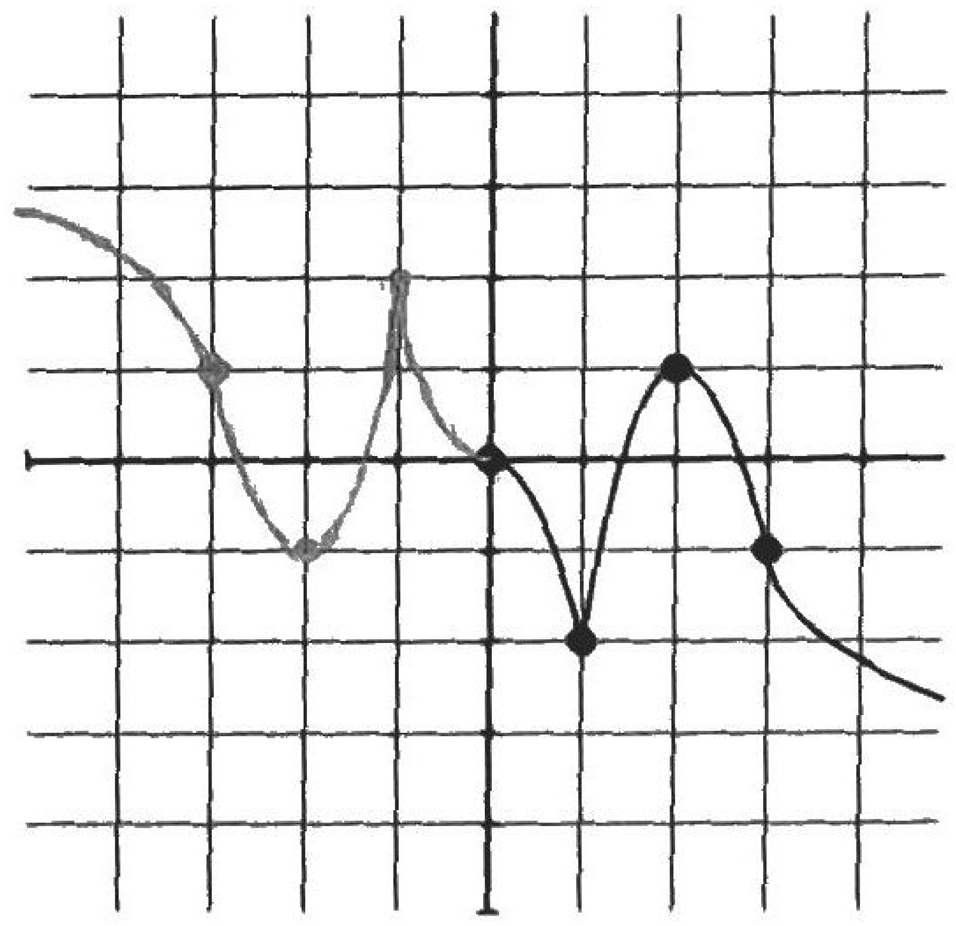
Complete the following graphs:

C25. Make the function even:



x	y
-4	-1
-3	-2
-1	3
0	-4
1	3
3	-2
4	-1

C26. Make the function odd:



x	y
-3	1
-2	-1
-1	2
0	0
1	-2
2	1
3	-1