M McGann

Review 2-3

Find the rectangular coordinates for the following points:

1NC. $\left(-2, \frac{7\pi}{6}\right)$

$x = -2\cos\frac{7\pi}{6}$ $\quad y = -2\sin\frac{7\pi}{6}$

$= -2\left(-\sqrt{3}/2\right)$ $\quad = -2\left(-\frac{1}{2}\right)$

$= \sqrt{3}$ $\quad = 1$

$(\sqrt{3}, 1)$

2NC. $(4, -45°)$

$x = 4\cos(-45)$ $\quad y = 4\sin(-45)$

$= 4\left(\frac{\sqrt{2}}{2}\right)$ $\quad = 4\left(-\sqrt{2}/2\right)$

$= 2\sqrt{2}$ $\quad = -2\sqrt{2}$

$(2\sqrt{2}, -2\sqrt{2})$

Find polar coordinates for the following points:

3NC. $(0, -5)$

$r^2 = 0^2 + (-5)^2$

$r^2 = 25$

$r = 5$

$\theta = \frac{3\pi}{2}$

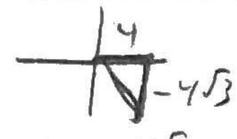
$\left(5, \frac{3\pi}{2}\right)$

4NC. $\left(4, -4\sqrt{3}\right)$

$r^2 = 4^2 + (-4\sqrt{3})^2$

$r^2 = 64$

$r = 8$



$\tan\theta = -\frac{4\sqrt{3}}{4}$

$\tan\theta = -\sqrt{3}$

$\theta = 5\pi/3$

$\left(8, \frac{5\pi}{3}\right)$

Transform the following polar equations into rectangular equations:

5NC. $r = \dfrac{6}{2\cos\theta - 3\sin\theta}$

$2r\cos\theta - 3r\sin\theta = 6$

$2x - 3y = 6$

6NC. $3r = \sin\theta$

$3r^2 = r\sin\theta$

$3(x^2 + y^2) = y$

$3x^2 + 3y^2 = y$

Transform the following rectangular equations into polar equations:

7NC. $x^2 + y^2 - 64 = 0$

$(r\cos\theta)^2 + (r\sin\theta)^2 - 64 = 0$

$r^2\cos^2\theta + r^2\sin^2\theta = 64$

$r^2(\cos^2\theta + \sin^2\theta) = 64$

$r^2 = 64$

$r = 8$

8NC. $3xy = 2$

$3(r\cos\theta)(r\sin\theta) = 2$

$3r^2\cos\theta\sin\theta = 2$

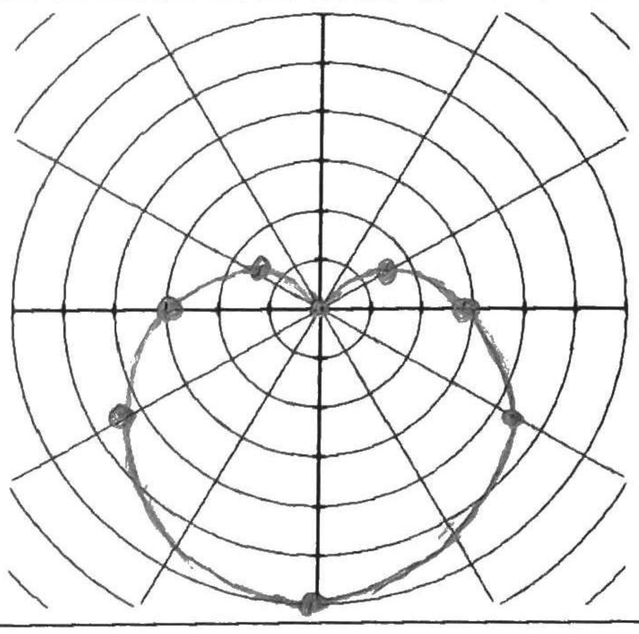
$r^2 = \dfrac{2}{3\cos\theta\sin\theta}$

$r = \sqrt{\dfrac{2}{3\cos\theta\sin\theta}}$

Fill in the table and graph the following polar functions:

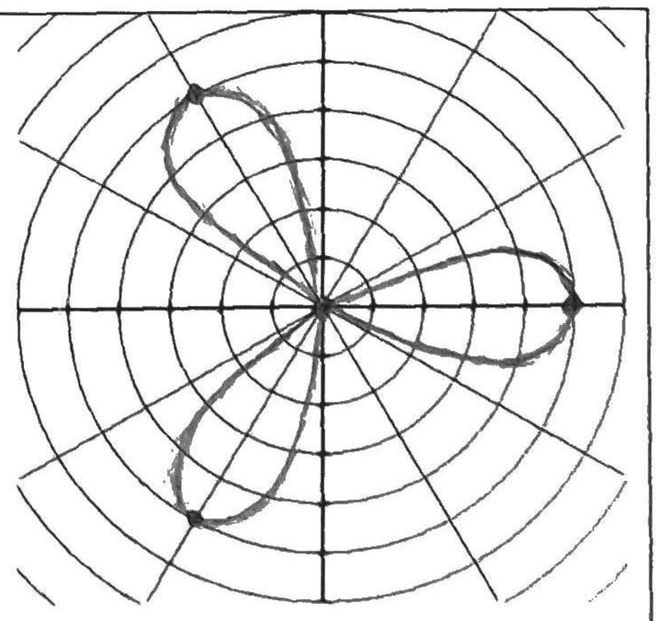
9NC. $r = -3 - 3\sin\theta$

θ	r
$-\pi/2$	$-3-3(-1) = 0$
$-\pi/6$	$-3-3\left(-\frac{1}{2}\right) = -1.5$
0	$-3-3(0) = -3$
$\pi/6$	$-3-3\left(\frac{1}{2}\right) = -4.5$
$\pi/2$	$-3-3(1) = -6$



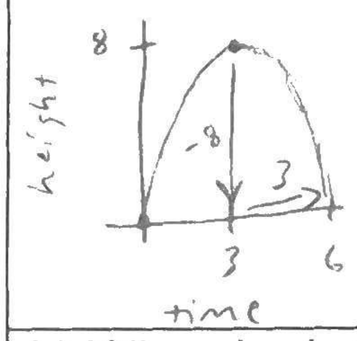
10NC. $r = 5\cos 3\theta$

θ	r
0	$5(1) = 5$
$\pi/6$	$5(0) = 0$
$\pi/3$	$5(-1) = -5$
$\pi/2$	$5(0) = 0$
$2\pi/3$	$5(1) = 5$



11NC*. Zach is a programmer working on the new Super Mario Brother's game for Nintendo. The game designer tells Zach that when Mario jumps, his maximum height needs to be 8 "feet" above the ground, and he needs to stay in the air for 6 seconds. He is moving forward at 3 "feet" per second.

a) Write the parametric equations describing Mario's height and distance as functions of time:



$a = -\frac{8}{3^2}$

$a = -\frac{8}{9}$

$x_m = 3t$

$y_m = -\frac{8}{9}(t-3)^2 + 8$

b) When he jumps, a fireball is launched form 10 feet behind Mario at a height of 6 feet (assume that the fireball has no mass, therefore gravity has no effect). It has a vertical velocity of -3 feet per second and a horizontal velocity of 15 feet per second. Write the parametric equations which describe the fireball's height and distance as functions of time.

$x_F = 15t - 10$

$y_F = -3t + 6$

c) Will Mario get hit by the fireball? Why or why not?

$3t = 15t - 10$

$-12t = -10$

$t = 0.833$

$x_m = 3(0.833) = 2.5$

$y_m = -\frac{8}{9}(0.833 - 3)^2 + 8 = 3.827$

$x_F = 15(0.833) - 10 = 2.5$

$y_F = -3(0.833) + 6 = 3.5$

No. At $t = 0.833$ seconds, when Mario and the fireball are both at $x = 2.5$, the fireball is 0.327 "feet" below Mario's feet.

d) An albatross is chasing Mario while flying at a height of 12 feet, a horizontal speed of -5 feet per second, and a vertical speed of -3 feet per second. The albatross is holding a bomb in its talons and drops the bomb when it is 11 feet in front of Mario. Write the parametric equations which describe the bomb's height and distance as functions of time. Bombs have mass, so gravity does have an effect.

$x_B = -5t + 11$

$y_B = -\frac{8}{9}t^2 - 3t + 12$

e) Will Mario get hit by the bomb? Why or why not?

$3t = -5t + 11$

$8t = 11$

$t = 1.375$

$x_m = 3(1.375) = 4.125$

$y_m = -\frac{8}{9}(1.375 - 3)^2 + 8 = 5.653$

$x_B = -5(1.375) + 11 = 4.125$

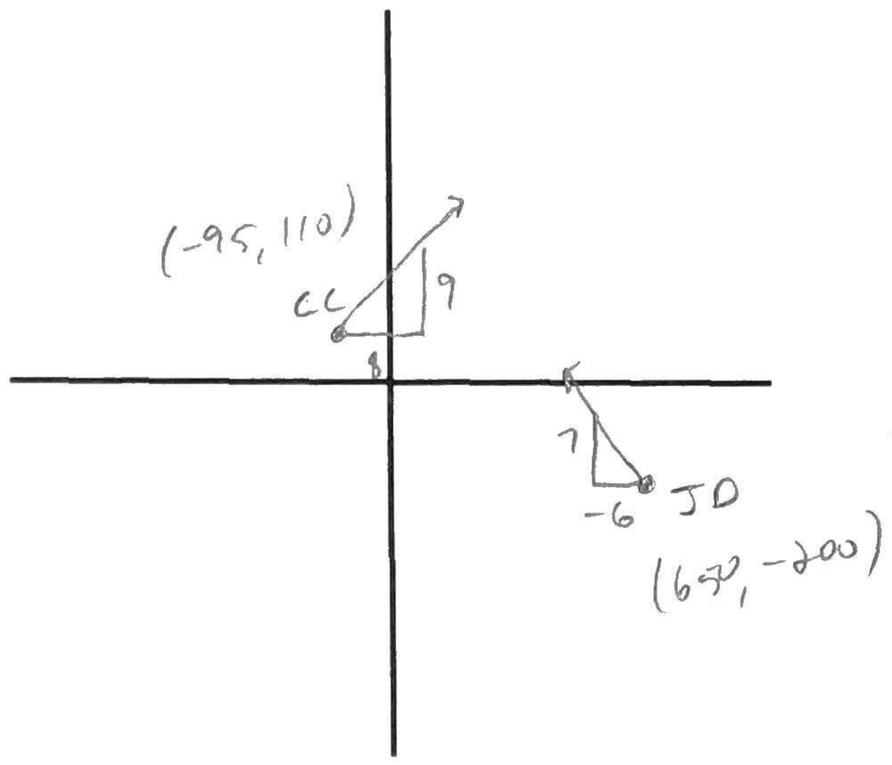
$y_B = -\frac{8}{9}(1.375)^2 - 3(1.375) + 12 = 6.194$

Yes. At $t = 1.375$ seconds, when Mario and the bomb are both at $x = 4.125$, the bomb is only 0.541 "feet" above Mario's feet.

** You may find a need for a calculator on these problems but, with numbers that work better, be prepared for similar problems on a non-calculator test.*

12NC*. Christopher Cross and John Denver were out in their sailboats on Canyon Lake. Suddenly, a huge thunderstorm rolled in and Canyon Lake turned into a churning cauldron of waves. Both captains turned on their transponders, so the authorities were able to monitor the boats' positions from shore. When the transponders were turned on, John Denver was located 650 meters east and 200 meters south of the nearest navigational buoy. Christopher Cross was 110 meters north and 95 meters west of the same buoy. After one minute of tracking the two sailboats, John Denver moved 6 meters west and 7 meters north. Christopher Cross moved 8 meters east and 9 meters north.

a) Graph the initial locations of the two sailboats, assuming the buoy is located at the origin. Also indicate the movement of the two sailboats.



b) Write a set of parametric equations for each sailboat.

$x_{JD} = -6t + 650$

$y_{JD} = 7t - 200$

$x_{CC} = 8t - 95$

$y_{CC} = 9t + 110$

c) Determine whether the sailboats will collide. Justify your answer ~~verbally~~ and algebraically.

$-6t + 650 = 8t - 95$

$-14t = -745$

$t = 53.214$

$x_{JD} = -6(53.214) + 650 = 330.714$

$y_{JD} = 7(53.214) - 200 = 172.5$

$x_{CC} = 8(53.214) - 95 = 330.714$

$y_{CC} = 9(53.214) + 110 = 588.929$

They do not collide because at $t = 53.214$ minutes, when both are at $x = 330.714$, the y-coordinates are off by 416.429 meters

* *You may find a need for a calculator on these problems but, with numbers that work better, be prepared for similar problems on a non-calculator test.*

Find two different sets of parametric equations for each rectangular equation.

13NC. $y = -8x + 3$

$y = -8x + 3$

$8x + y = 3$

$8x = 3 - y$

$x = \frac{3-y}{8}$

$x = t$

$y = -8t + 3$

$x = \frac{3-t}{8}$

$y = t$

14NC. $y = \sqrt{7 - x^2}$

$y = \sqrt{7 - x^2}$

$y^2 = 7 - x^2$

$x^2 + y^2 = 7$

$x^2 = 7 - y^2$

$x = \pm\sqrt{7 - y^2}$

$x = t$

$y = \sqrt{7 - t^2}$

$x = \pm\sqrt{7 - t^2}$

$y = t$

Rewrite each set of parametric equations as a rectangular equation in slope intercept form.

15NC. $x = 2t - 6$, $y = 4t^2 + 9$

$x = 2t - 6$

$2t = x + 6$

$t = \frac{x+6}{2}$

$y = 4\left(\frac{x+6}{2}\right)^2 + 9$

$y = 4\left(\frac{x^2+12x+36}{4}\right) + 9$

$y = x^2 + 12x + 36 + 9$

$y = x^2 + 12x + 45$

16NC. $x = e^t - 2$, $y = \sqrt{t-5}$

$x = e^t - 2$

$e^t = x + 2$

$t = \ln(x+2)$

$y = \sqrt{\ln(x+2) - 5}$