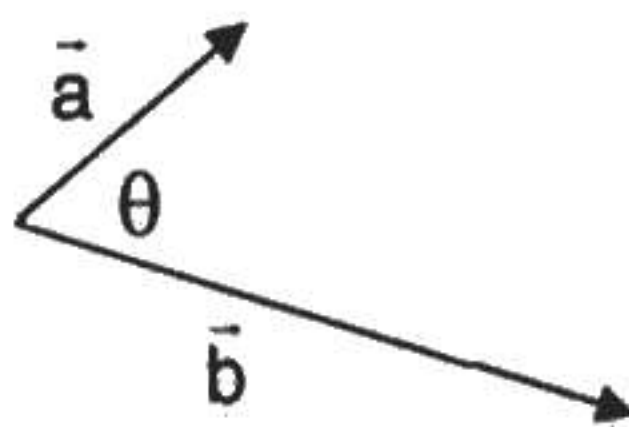


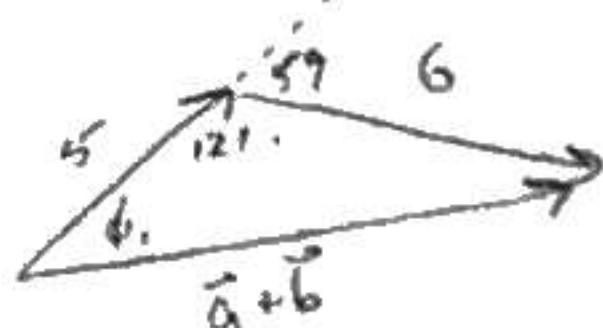
Review 4-3

M. L. Lann

For the following problems, find $|\vec{a} + \vec{b}|$, $|\vec{a} - \vec{b}|$, and the angle that each of these resultant vectors makes with \vec{a} . Draw a picture for the addition and subtraction of the two vectors.



1. $|\vec{a}| = 5, |\vec{b}| = 6, \theta = 59^\circ$



$$\cos \phi_1 = \frac{5^2 + 9.587^2 - 6^2}{2(5)(9.587)}$$

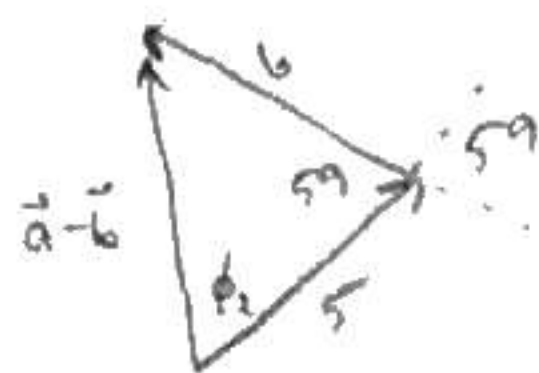
$$\cos \phi_1 = 0.844$$

$$\phi_1 = 32.444^\circ$$

$$|\vec{a} + \vec{b}|^2 = 5^2 + 6^2 - 2(5)(6) \cos 121$$

$$|\vec{a} + \vec{b}|^2 = 91.902$$

$$|\vec{a} + \vec{b}| = 9.587$$



$$|\vec{a} - \vec{b}|^2 = 5^2 + 6^2 - 2(5)(6) \cos 59$$

$$|\vec{a} - \vec{b}|^2 = 30.098$$

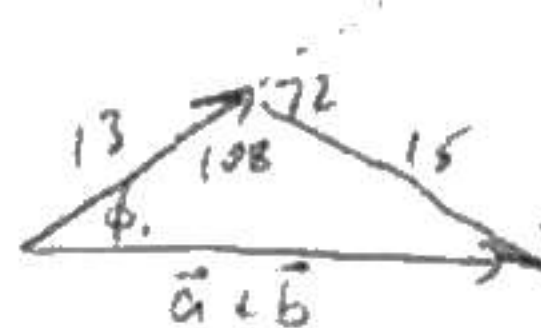
$$|\vec{a} - \vec{b}| = 5.486$$

$$\cos \phi_2 = \frac{5^2 + 5.486^2 - 6^2}{2(5)(5.486)}$$

$$\cos \phi_2 = 0.348$$

$$\phi_2 = 69.628^\circ$$

2. $|\vec{a}| = 13, |\vec{b}| = 15, \theta = 72^\circ$



$$\cos \phi_1 = \frac{13^2 + 22.683^2 - 15^2}{2(13)(22.683)}$$

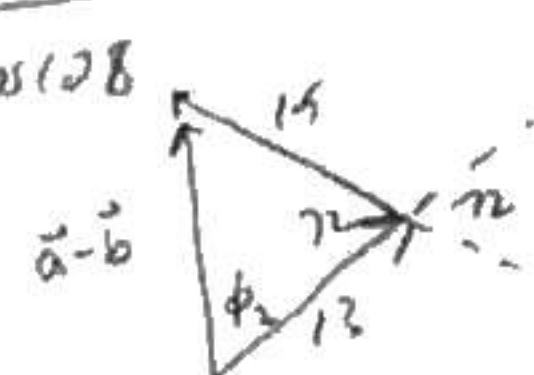
$$\cos \phi_1 = 0.777$$

$$\phi_1 = 38.971^\circ$$

$$|\vec{a} + \vec{b}|^2 = 13^2 + 15^2 - 2(13)(15) \cos 108$$

$$|\vec{a} + \vec{b}|^2 = 514.517$$

$$|\vec{a} + \vec{b}| = 22.683$$



$$|\vec{a} - \vec{b}|^2 = 13^2 + 15^2 - 2(13)(15) \cos 72$$

$$|\vec{a} - \vec{b}|^2 = 273.483$$

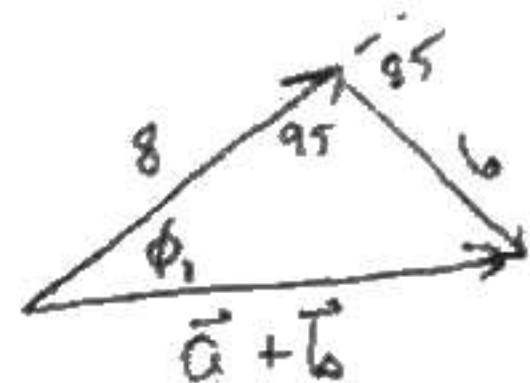
$$|\vec{a} - \vec{b}| = 16.537$$

$$\cos \phi_2 = \frac{13^2 + 16.537^2 - 15^2}{2(13)(16.537)}$$

$$\cos \phi_2 = 0.506$$

$$\phi_2 = 59.615^\circ$$

3. $|\vec{a}| = 8, |\vec{b}| = 6, \theta = 85^\circ$



$$\cos \phi_1 = \frac{8^2 + 10.410^2 - 6^2}{2(8)(10.410)}$$

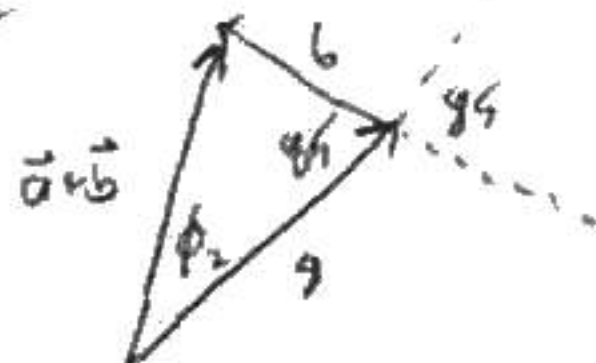
$$\cos \phi_1 = 0.819$$

$$\phi_1 = 35.042^\circ$$

$$|\vec{a} + \vec{b}|^2 = 8^2 + 6^2 - 2(8)(6) \cos 95$$

$$|\vec{a} + \vec{b}|^2 = 108.367$$

$$|\vec{a} + \vec{b}| = 10.410$$



$$|\vec{a} - \vec{b}|^2 = 8^2 + 6^2 - 2(8)(6) \cos 85$$

$$|\vec{a} - \vec{b}| = 9.573$$

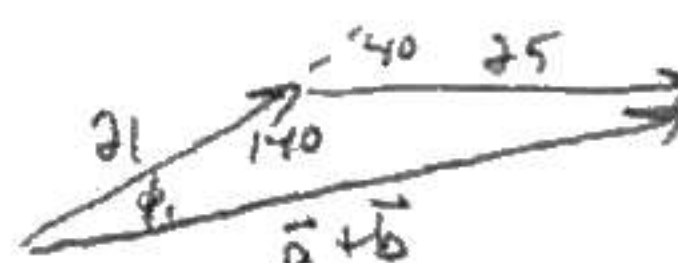
$$|\vec{a} - \vec{b}| = 9.573$$

$$\cos \phi_2 = \frac{8^2 + 9.573^2 - 6^2}{2(8)(9.573)}$$

$$\cos \phi_2 = 0.781$$

$$\phi_2 = 38.639^\circ$$

4. $|\vec{a}| = 21, |\vec{b}| = 25, \theta = 40^\circ$



$$\cos \phi_1 = \frac{21^2 + 43.248^2 - 25^2}{2(21)(43.248)}$$

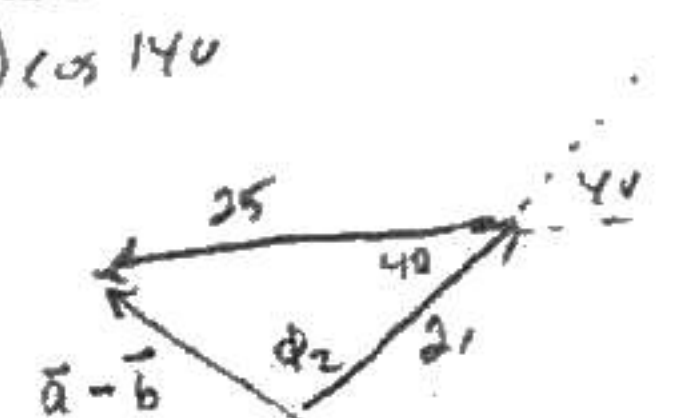
$$\cos \phi_1 = 0.928$$

$$\phi_1 = 21.813^\circ$$

$$|\vec{a} + \vec{b}|^2 = 21^2 + 25^2 - 2(21)(25) \cos 140$$

$$|\vec{a} + \vec{b}|^2 = 1870.347$$

$$|\vec{a} + \vec{b}| = 43.248$$



$$|\vec{a} - \vec{b}|^2 = 21^2 + 25^2 - 2(21)(25) \cos 40$$

$$|\vec{a} - \vec{b}|^2 = 261.653$$

$$|\vec{a} - \vec{b}| = 16.176$$

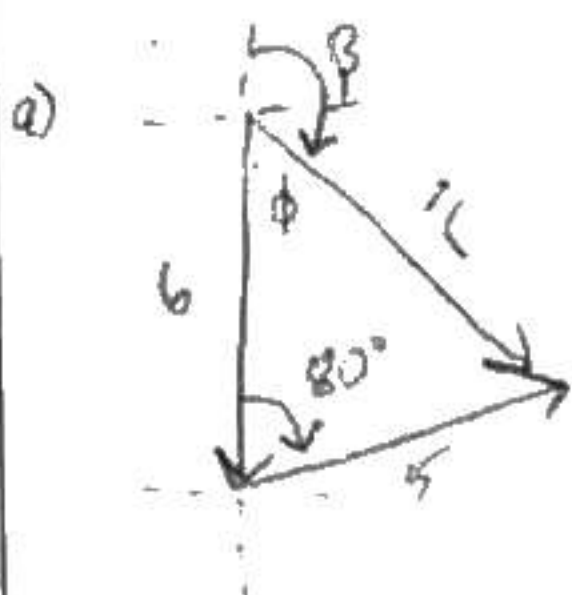
$$\cos \phi_2 = \frac{21^2 + 16.176^2 - 25^2}{2(21)(16.176)}$$

$$\cos \phi_2 = 0.114$$

$$\phi_2 = 83.437^\circ$$

For the following problems, a) sketch a diagram, b) find the magnitude of the resultant vector, c) find the bearing of the resultant vector, and d) find the bearing from the end point back to the starting point.

5. You move 6 miles south (180° bearing) followed by 5 miles along a bearing of 80° .



b) $|\vec{r}|^2 = 6^2 + 5^2 - 2(6)(5)\cos 80$

$|\vec{r}|^2 = 50.581$

$|\vec{r}| = 7.112 \text{ miles}$

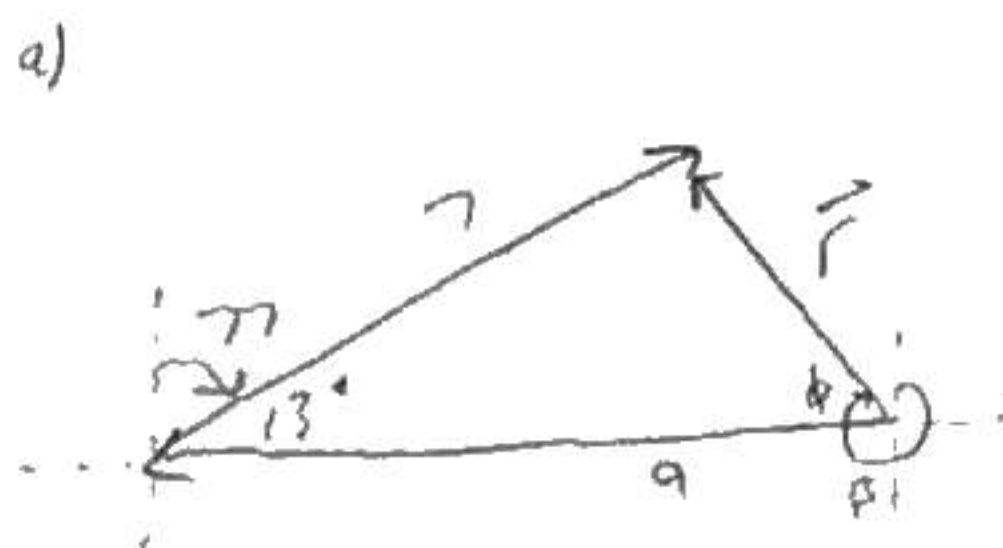
c) $\cos \phi = \frac{6^2 + 7.112^2 - 5^2}{2(6)(7.112)}$ $\beta = 180 - 43.817$

$\cos \phi = 0.722$

$\phi = 43.87^\circ$

d) $136.183 + 180 = 316.183^\circ$

6. You move 9 kilometers west (270° bearing) followed by 7 kilometers along a bearing of 77° .



b) $|\vec{r}|^2 = 9^2 + 7^2 - 2(9)(7)\cos 13$

$|\vec{r}|^2 = 7.229$

$|\vec{r}| = 2.689 \text{ kilometers}$

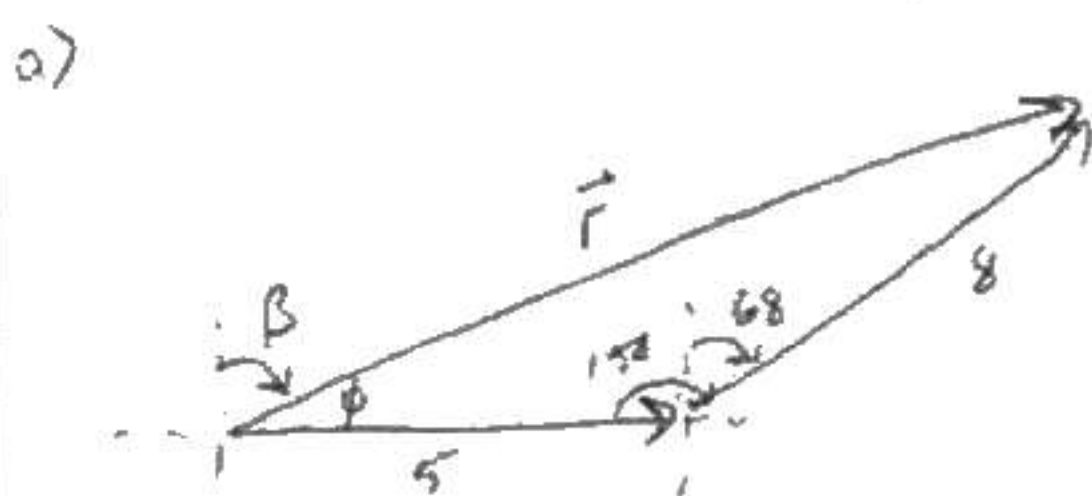
c) $\cos \phi = \frac{9^2 + 2.689^2 - 7^2}{2(9)(2.689)}$ $\beta = 270 + 35.849$

$\cos \phi = 0.811$

$\phi = 35.849^\circ$

d) $305.849 - 180 = 125.849^\circ$

7. You move 5 kilometers east (90° bearing) followed by 8 kilometers along a bearing of 68° .



b) $|\vec{r}|^2 = 5^2 + 8^2 - 2(5)(8)\cos 158$

$|\vec{r}|^2 = 163.175$

$|\vec{r}| = 12.774 \text{ kilometers}$

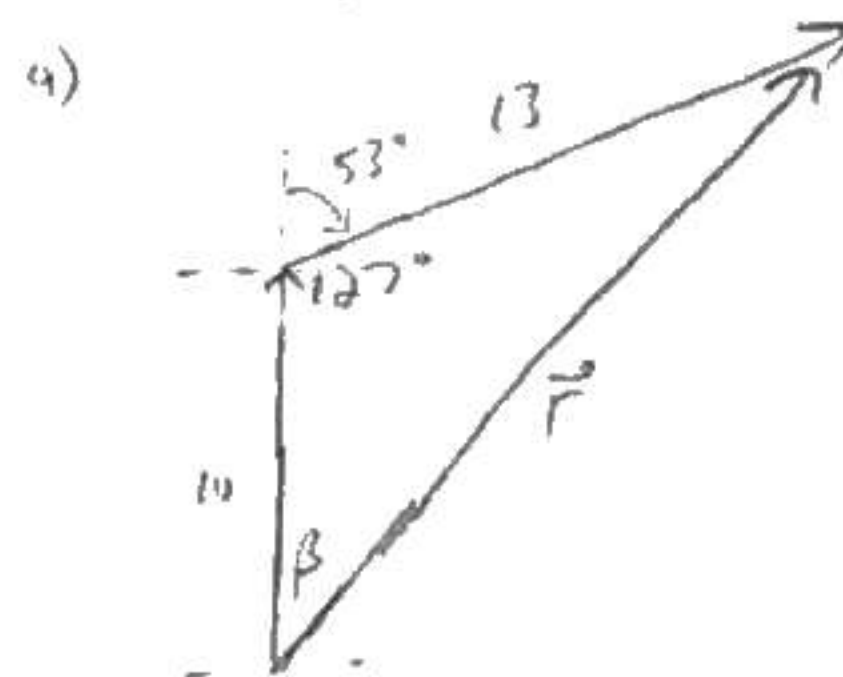
c) $\cos \phi = \frac{5^2 + 12.774^2 - 8^2}{2(5)(12.774)}$ $\beta = 90 - 13.568$

$\cos \phi = 0.972$

$\phi = 13.568^\circ$

d) $76.432 + 180 = 256.432^\circ$

8. You move 10 miles north (0° bearing) followed by 13 miles along a bearing of 53° .



b) $|\vec{r}|^2 = 10^2 + 13^2 - 2(10)(13)\cos 127$

$|\vec{r}|^2 = 425.472$

$|\vec{r}| = 20.627 \text{ miles}$

c) $\cos \beta = \frac{10^2 + 20.627^2 - 13^2}{2(10)(20.627)}$

$\cos \beta = 0.864$

$\beta = 30.221^\circ$

d) $30.221 + 180 =$

210.221°

$$\theta = 90 - 128 = -38^\circ$$

$$\theta = 90 - 289 = -199^\circ$$

9. A ship sails for 56 miles on a bearing of 128° , then turns and sails on a bearing of 289° for 23 more miles. Find its displacement (resultant vector), as a distance and bearing.

$$\vec{a} = 56 \cos(-38) \vec{i} + 56 \sin(-38) \vec{j}$$

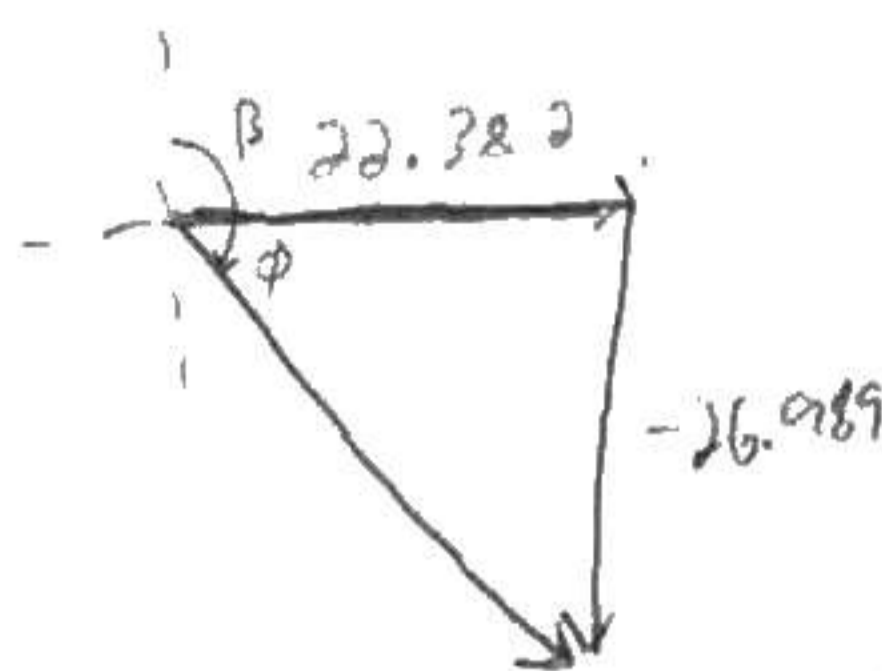
$$+ \vec{b} = 23 \cos(-199) \vec{i} + 23 \sin(-199) \vec{j}$$

$$\vec{r} = 22.382 \vec{i} - 26.989 \vec{j}$$

$$|\vec{r}|^2 = 22.382^2 + 26.989^2$$

$$|\vec{r}|^2 = 1229.344$$

$$|\vec{r}| = 35.062 \text{ miles}$$



$$\tan \phi = \frac{26.989}{22.382}$$

$$\phi = 50.331^\circ$$

$$\beta = 90 + 50.331$$

$$\beta = 140.331^\circ$$

$$\theta = 90 - 320 = -230^\circ$$

10. An airplane flies 307 miles along a bearing of 320° , then turns and goes 158 miles along a bearing of 86° . Find its displacement (resultant vector), as a distance and bearing.

$$\theta = 90 - 86 = 4^\circ$$

$$\vec{a} = 307 \cos(-230) \vec{i} + 307 \sin(-230) \vec{j}$$

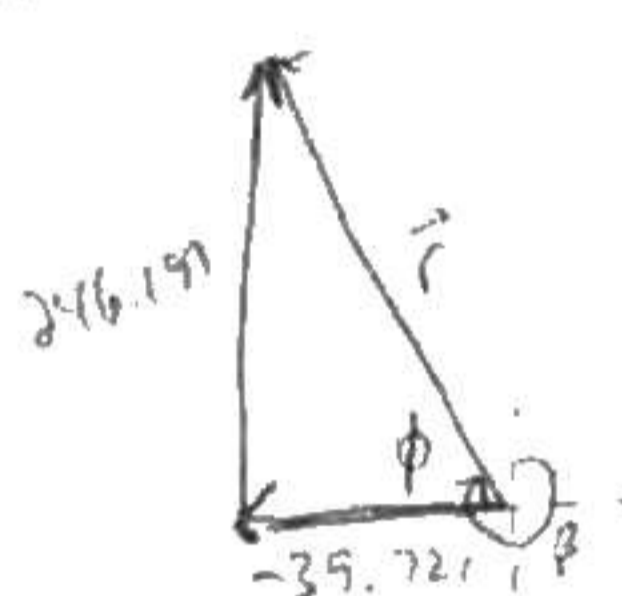
$$+ \vec{b} = 158 \cos(4) \vec{i} + 158 \sin(4) \vec{j}$$

$$\vec{r} = -39.721 \vec{i} + 246.197 \vec{j}$$

$$|\vec{r}|^2 = 39.721^2 + 246.197^2$$

$$|\vec{r}|^2 = 62190.777$$

$$|\vec{r}| = 249.381$$



$$\tan \phi = \frac{246.197}{39.721}$$

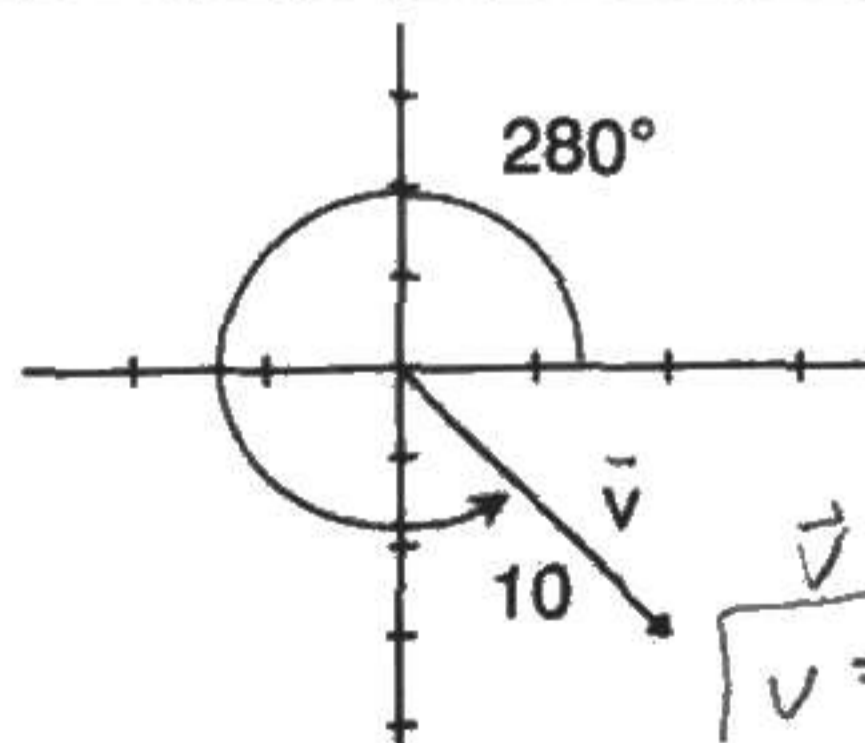
$$\phi = 80.835^\circ$$

$$\beta = 270 + 80.835$$

$$\beta = 350.835^\circ$$

Resolve vector into horizontal and vertical components:

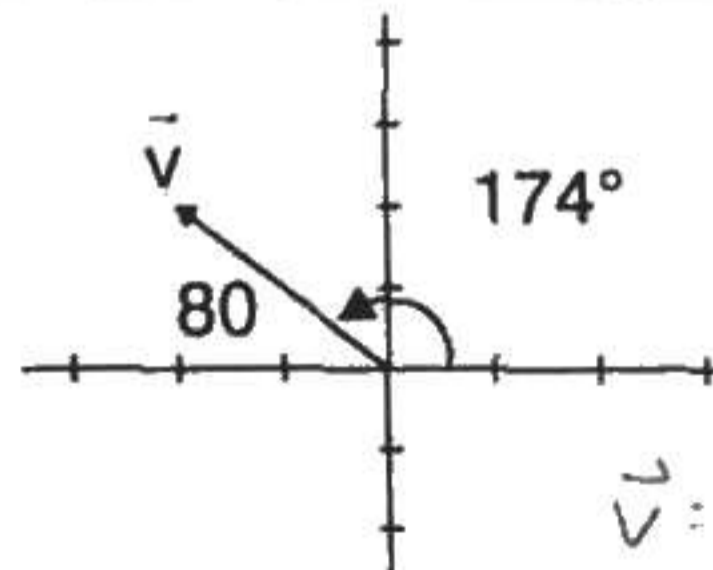
11.



$$\vec{v} = 10 \cos 280 \vec{i} + 10 \sin 280 \vec{j}$$

$$\vec{v} = 1.736 \vec{i} - 9.848 \vec{j}$$

12.



$$\vec{v} = 80 \cos 174 \vec{i} + 80 \sin 174 \vec{j}$$

$$\vec{v} = -79.562 \vec{i} + 8.362 \vec{j}$$

13. $|\vec{v}| = 25$ at 11°

$$\vec{v} = 25 \cos 11 \vec{i} + 25 \sin 11 \vec{j}$$

$$\vec{v} = 24.541 \vec{i} + 4.770 \vec{j}$$

14. $|\vec{v}| = 46$ at 247°

$$\vec{v} = 46 \cos 247 \vec{i} + 46 \sin 247 \vec{j}$$

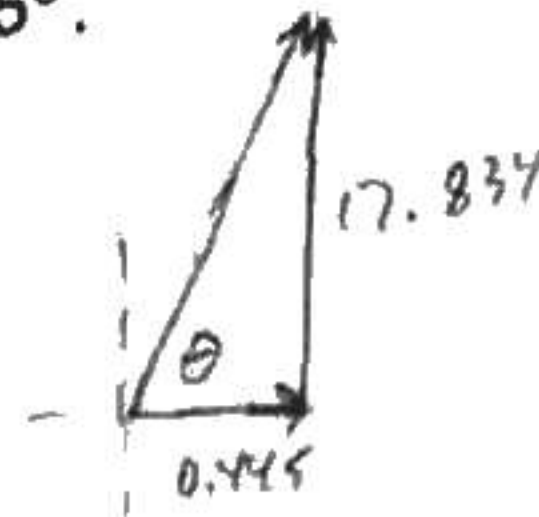
$$\vec{v} = -17.974 \vec{i} - 42.343 \vec{j}$$

If $\vec{v} = \vec{r} + \vec{s}$, resolve \vec{v} into a) as a sum of two components and b) as a magnitude and direction.

15. If $|\vec{r}| = 8$ units, $\theta = 32^\circ$ and $|\vec{s}| = 15$ units, $\theta = 115^\circ$.

$$\begin{aligned} \text{a) } \vec{r} &= 8\cos 32^\circ \vec{i} + 8\sin 32^\circ \vec{j} \\ + \vec{s} &= 15\cos 115^\circ \vec{i} + 15\sin 115^\circ \vec{j} \end{aligned}$$

$$\vec{v} = 0.445 \vec{i} + 17.834 \vec{j}$$



$$\tan \theta = \frac{17.834}{0.445}$$

$$\theta = 88.570^\circ$$

$$\text{b) } |\vec{v}|^2 = 0.445^2 + 17.834^2$$

$$|\vec{v}|^2 = 318.249$$

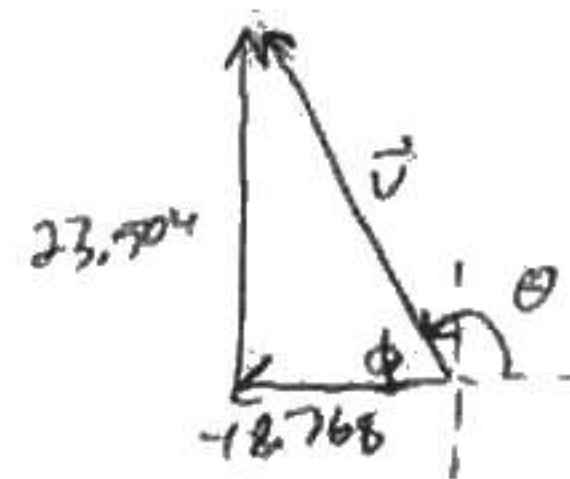
$$|\vec{v}| = 17.840$$

$$\theta = 88.570^\circ$$

16. If $|\vec{r}| = 16$ units, $\theta = 87^\circ$ and $|\vec{s}| = 21$ units, $\theta = 159^\circ$.

$$\begin{aligned} \text{a) } \vec{r} &= 16\cos 87^\circ \vec{i} + 16\sin 87^\circ \vec{j} \\ + \vec{s} &= 21\cos 159^\circ \vec{i} + 21\sin 159^\circ \vec{j} \end{aligned}$$

$$\vec{v} = -18.768 \vec{i} + 23.504 \vec{j}$$



$$\tan \phi = \frac{23.504}{18.768}$$

$$\phi = 51.393^\circ$$

$$\text{b) } |\vec{v}|^2 = 18.768^2 + 23.504^2$$

$$|\vec{v}|^2 = 904.659$$

$$|\vec{v}| = 30.078$$

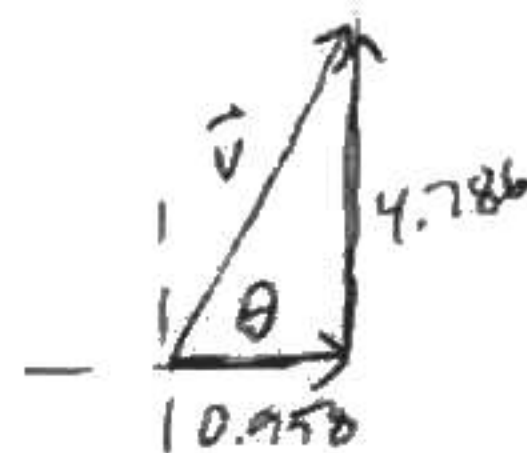
$$\theta = 180 - 51.393^\circ$$

$$\theta = 128.607^\circ$$

17. If $|\vec{r}| = 2$ units, $\theta = 357^\circ$ and $|\vec{s}| = 5$ units, $\theta = 102^\circ$.

$$\begin{aligned} \text{a) } \vec{r} &= 2\cos 357^\circ \vec{i} + 2\sin 357^\circ \vec{j} \\ + \vec{s} &= 5\cos 102^\circ \vec{i} + 5\sin 102^\circ \vec{j} \end{aligned}$$

$$\vec{v} = 0.958 \vec{i} + 4.786 \vec{j}$$



$$\tan \theta = \frac{4.786}{0.958}$$

$$\theta = 78.684^\circ$$

$$\text{b) } |\vec{v}|^2 = 0.958^2 + 4.786^2$$

$$|\vec{v}|^2 = 23.824$$

$$|\vec{v}| = 4.881$$

$$\theta = 78.684^\circ$$

Definitions: Give a definition and an example of each of the following.

18. Magnitude: The length of the line segment.

Example: speed

19. Direction: The angle at which the line segment is pointing.

Example: compass heading

20. Vector: Anything with both magnitude and direction - a directed line segment. Example: velocity, which includes both speed and direction.

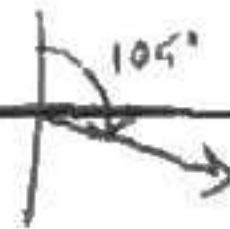
21. Scalar: A one-dimensional number that multiplies the magnitude of a vector. Example: If car A is moving 3 times faster than car B, the scalar multiple is 3.

22. Vector Addition: The vector that goes from the beginning of vector \vec{a} to the end of vector \vec{b} when \vec{b} is placed at the end of \vec{a} .
Example: An airplane's resultant velocity is the addition of its actual velocity and the wind's velocity.

23. Vector Subtraction: The vector that goes from the beginning of vector \vec{a} to the end of vector $-\vec{b}$ when $-\vec{b}$ is placed at the end of \vec{a} .
Example: Wind velocity can be found by subtracting an airplane's actual velocity from its resultant velocity.

24. Bearing: direction measured from 0° north going clockwise

Example: A bearing of 105° is



25. Vector Components: A vector can be split into horizontal, \vec{i} , and vertical, \vec{j} , components. Example: a vector with magnitude 5 and direction 124° can be written $5\cos(124)\vec{i} + 5\sin(124)\vec{j} = -2.796\vec{i} + 4.145\vec{j}$ which means 2.796 units left and 4.145 units up.