Review 4-4

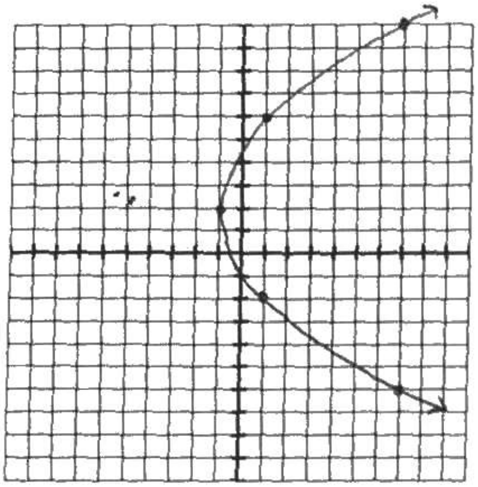
Each of the following conic sections should be represented by a Cartesian equation, a pair of parametric equations, and a graph. Given one out of the three representations, fill in the other two.

1. Cartesian: $x = \frac{1}{8}(y-2)^2 - 1$

Parametric: $x = -1 + \frac{1}{8}t^2$
$y = 2 + t$

Graph:

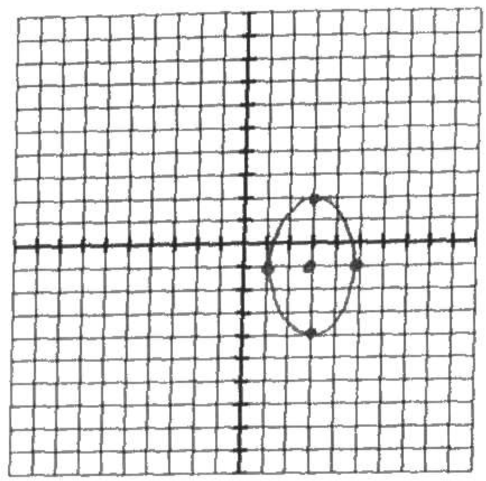
$a = \frac{1}{8}$
$= \frac{2}{16}$
$= \frac{8}{64}$



2. Cartesian: $\frac{(x-3)^2}{4} + \frac{(y+1)^2}{9} = 1$

Parametric: $x = 3 + 2\cos t$
$y = -1 + 3\sin t$

Graph:

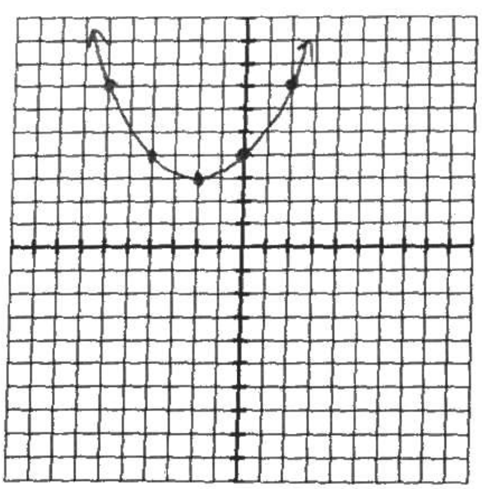


3. Cartesian: $y = \frac{1}{4}(x+2)^2 + 3$

Parametric: $x = -2 + t$
$y = 3 + \frac{1}{4}t^2$

Graph:

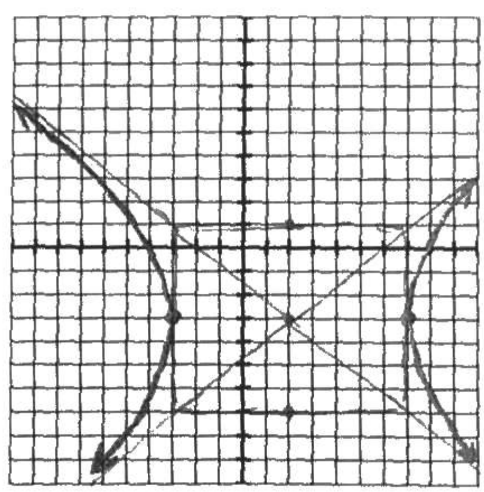
$a = \frac{1}{4}$
$= \frac{4}{16}$



4. Cartesian: $\frac{(x-2)^2}{25} - \frac{(y+3)^2}{16} = 1$

Parametric: $x = 2 + 5\sec t$
$y = -3 + 4\tan t$

Graph:



5.

Cartesian: $y = \frac{1}{16}(x+4)^2 - 2$

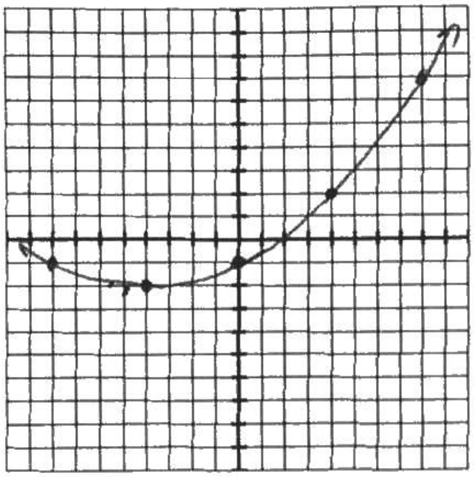
Parametric: $x = -4 + t$
$y = -2 + \frac{1}{16}t^2$

Graph:

$a = \frac{1}{16}$

$= \frac{4}{64}$

$= \frac{9}{144}$

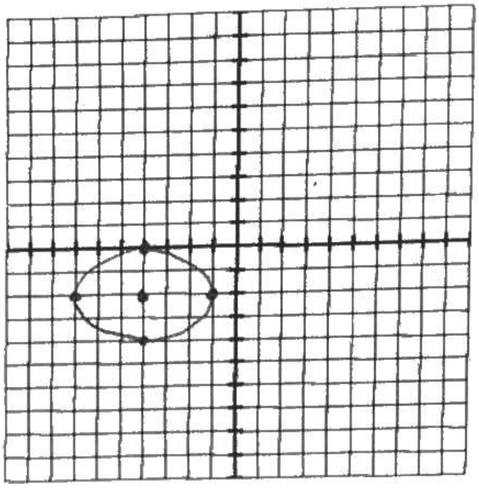


6.

Cartesian: $\frac{(x+4)^2}{9} + \frac{(y+2)^2}{4} = 1$

Parametric: $x = -4 + 3\cos t$
$y = -2 + 2\sin t$

Graph:

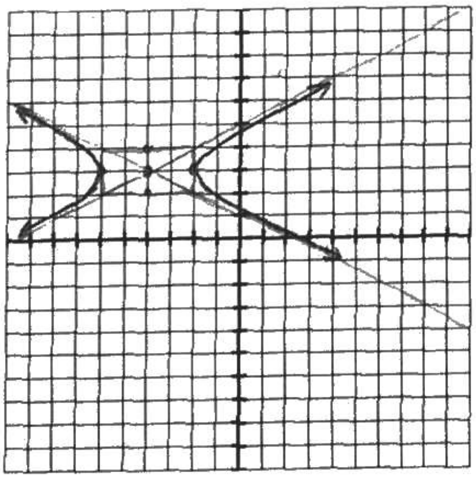


7.

Cartesian: $\frac{(x+4)^2}{4} - (y-3)^2 = 1$

Parametric: $x = -4 + 2\sec t$
$y = 3 + \tan t$

Graph:

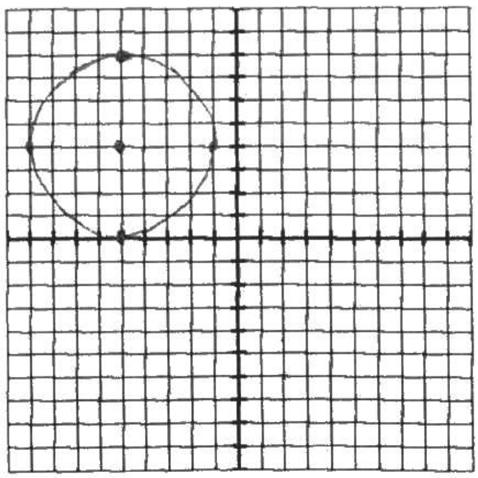


8.

Cartesian: $\frac{(x+5)^2}{16} + \frac{(y-4)^2}{16} = 1$

Parametric: $x = -5 + 4\cos t$
$y = 4 + 4\sin t$

Graph:

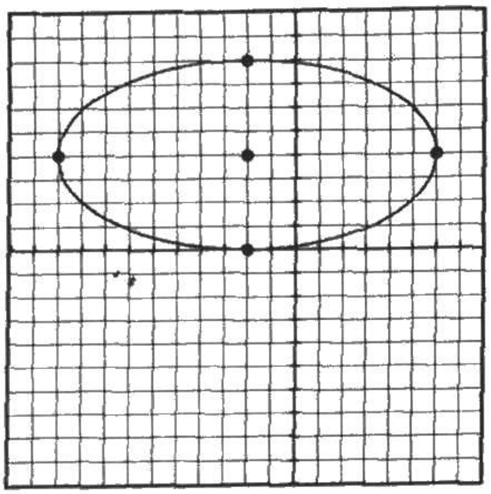


9.

Cartesian: $\dfrac{(x+2)^2}{64} + \dfrac{(y-4)^2}{16} = 1$

Parametric: $x = -2 + 8\cos t$
$y = 4 + 4\sin t$

Graph:



10.

Cartesian: $x = -\frac{2}{5}(y+2)^2 - 8$

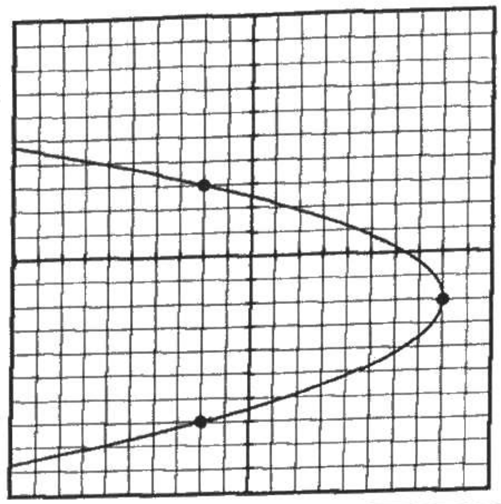
Parametric: $x = -8 - \frac{2}{5}t^2$
$y = -2 + t$

Graph:

$a = \frac{-10}{5^2}$

$a = \frac{-10}{25}$

$a = -\frac{2}{5}$

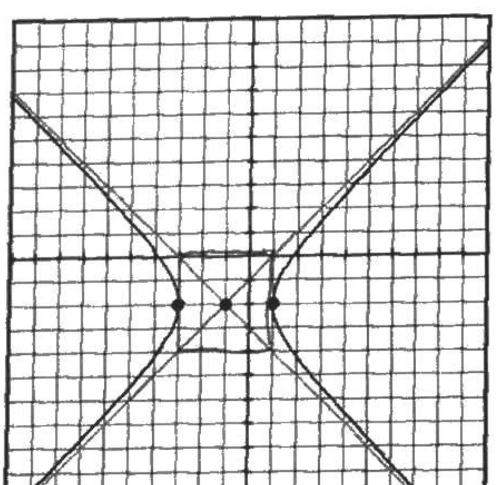


11.

Cartesian: $\dfrac{(x+1)^2}{4} - \dfrac{(y+2)^2}{4} = 1$

Parametric: $x = -1 + 2\sec t$
$y = -2 + 2\tan t$

Graph:

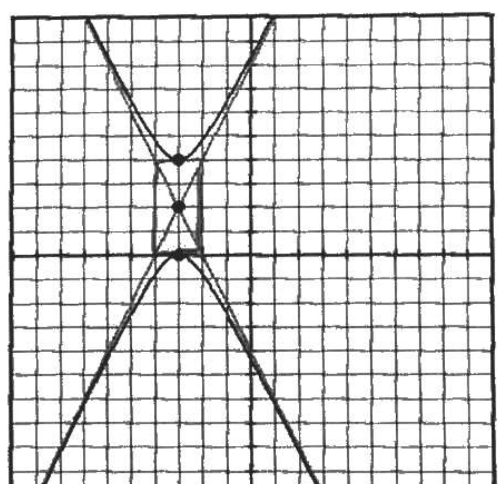


12.

Cartesian: $\dfrac{(y-2)^2}{4} - (x+3)^2 = 1$

Parametric: $x = -3 + \tan t$
$y = 2 + 2\sec t$

Graph:



13. An unknown comet is approaching Earth, and its orbit is focused at the Sun. Luckily, the comet does not strike the Earth, so scientists begin making observations to determine if our planet might be in danger of the comet returning one day. They find that the closest the comet comes to the sun is 50 million miles, and it is 90 million miles away at a point perpendicular to the perigee.

a) Find the eccentricity.

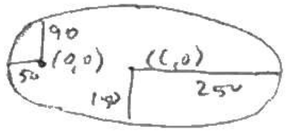
$e = 1 + \frac{90 - 2(50)}{50}$ $e = 0.8$

b) Will the comet ever return? Why or why not?

Yes. Since $0<e<1$, the orbit is elliptical.

c) Sketch a diagram of this situation. Assume the Sun is at the origin.

$a = \frac{-50^2}{90-2(50)} = 250$



$c = 250 - 50 = 200$

$250^2 = b^2 + 200^2$

$b = 150$

d) Find the coordinates of the center.

$(200, 0)$

e) Write the Cartesian and parametric equations ~~that describes this~~ conic section.

$\frac{(x-200)^2}{62500} + \frac{y^2}{22500} = 1$

$x = 200 + 250\cos t$
$y = 150\sin t$

14. Jupiter has an elliptical orbit around the sun. Scientists have found that the closest the Jupiter comes to the sun is 80 million miles, and its furthest point is 120 million miles.

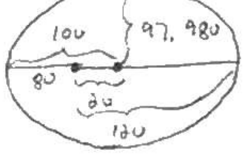
a) Find a and c.

$a = \frac{80 + 120}{2}$ $a = 100$ $c = 100 - 80$ $c = 20$

b) Find the eccentricity.

$e = \frac{20}{100}$ $e = 0.2$

c) Sketch a diagram of this situation. Assume the Sun is at the origin.



$100^2 = b^2 + 20^2$

$b^2 = 9600$

$b = 97.980$

d) Find the coordinates of the center.

$(20, 0)$

e) Write the Cartesian and parametric equations that describes this conic section.

$\frac{(x-20)^2}{10000} + \frac{y^2}{9600} = 1$

$x = 20 + 100\cos t$
$y = 97.980\sin t$

15. An unknown comet is approaching Earth, and its orbit is focused at the Sun. Luckily, the comet does not strike the Earth, so scientists begin making observations to determine if our planet might be in danger of the comet returning one day. They find that the closet the comet comes to the sun is 20 million miles, and it is 70 million miles away at a point perpendicular to the perigee.

a) Find the eccentricity.

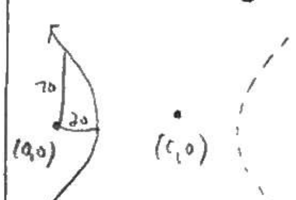
$e = 1 + \frac{70 - 2(20)}{20}$ $e = 1.5$

b) Will the comet ever return? Why or why not?

No. Since $e>1$, the orbit is hyperbolic.

c) Sketch a diagram of this situation. Assume the Sun is at the origin.

$a = \frac{20^2}{70-2(20)} = 13.333$



$c = 13.333 + 20 = 33.333$

$33.333^2 = 13.333^2 + b^2$

$b = 30.551$

d) Find ~~the coordinates~~ of the center.

$(33.333, 0)$

e) Write the ~~Cartesian~~ and parametric equations that describes this conic section.

$\frac{(x-33.333)^2}{177.778} - \frac{y^2}{933.333} = 1$

$x = 33.333 + 13.333\sec t$
$y = 30.551\tan t$

Transform the given rectangular equation into the a) standard form b) parametric form, then c) determine the shape of the conic section, and d) find the *eccentricity*.

16. $x^2+8x=4y-8$

a) $(x^2+8x+16)=4y-8+16$

$(x+4)^2=4y+8$

$4y=(x+4)^2-8$

$\boxed{y=\frac{1}{4}(x+4)^2-2}$

b) $x=-4+t$

$y=-2+\frac{1}{4}t^2$

c) parabola

d) $e=1$

17. $y^2+2y-x=0$

a) $(y^2+2y+1)-x=1$

$(y+1)^2=1+x$

$\boxed{x=(y+1)^2-1}$

b) $x=-1+t^2$

$y=-1+t$

c) parabola

d) $e=1$

18. $x^2+6x-4y+1=0$

a) $(x^2+6x+9)-4y=-1+9$

$(x+3)^2-4y=8$

$(x+3)^2-8=4y$

$\boxed{y=\frac{1}{4}(x+3)^2-2}$

b) $x=-3+t$

$y=-2+\frac{1}{4}t^2$

c) parabola

d) $e=1$

19. $9x^2+4y^2-18x+16y-11=0$

a) $9(x^2-2x+1)+4(y^2+4y+4)=11+9(1)+4(4)$

$\frac{9(x-1)^2}{36}+\frac{4(y+2)^2}{36}=\frac{36}{36}$

$\boxed{\frac{(x-1)^2}{4}+\frac{(y+2)^2}{9}=1}$

b) $x=1+2\cos t$

$y=-2+3\sin t$

c) ellipse

d) $a=3$

$b=2$

$3^2=2^2+c^2$

$c=\sqrt{5}$

$e=\frac{c}{a}$

$e=\frac{\sqrt{5}}{3}$

20. $x^2+9y^2+6x-18y+9=0$

a) $(x^2+6x+9)+9(y^2-2y+1)=-9+9+9(1)$

$\frac{(x+3)^2}{9}+\frac{9(y-1)^2}{9}=\frac{9}{9}$

$\boxed{\frac{(x+3)^2}{9}+(y-1)^2=1}$

b) $x=-3+3\cos t$

$y=1+\sin t$

c) ellipse

d) $a=3$

$b=1$

$3^2=1^2+c^2$

$c=\sqrt{8}=2\sqrt{2}$

$e=\frac{c}{a}$

$e=\frac{2\sqrt{2}}{3}$

21. $y^2-4x^2-16x-2y-19=0$

a) $(y^2-2y+1)-4(x^2+4x+4)=19+1-4(4)$

$\frac{(y-1)^2}{4}-\frac{4(x+2)^2}{4}=\frac{4}{4}$

$\boxed{\frac{(y-1)^2}{4}-(x+2)^2=1}$

b) $x=-2+\tan t$

$y=1+2\sec t$

c) hyperbola

d) $a=2$

$b=1$

$c^2=2^2+1^2$

$c=\sqrt{5}$

$e=\frac{c}{a}$

$e=\frac{\sqrt{5}}{2}$

22. $2y^2-x^2+2x+8y-1=0$

a) $2(y^2+4y+4)-(x^2-2x+1)=1+2(4)-1$

$\frac{2(y+2)^2}{8}-\frac{(x-1)^2}{8}=\frac{8}{8}$

$\boxed{\frac{(y+2)^2}{4}-\frac{(x-1)^2}{8}=1}$

b) $x=1+2\sqrt{2}\tan t$

$y=-2+2\sec t$

c) hyperbola

d) $a=2$

$b=2\sqrt{2}$

$c^2=2^2+8$

$c=\sqrt{12}=2\sqrt{3}$

$e=\frac{c}{a}$

$e=\frac{2\sqrt{3}}{2}$

$e=\sqrt{3}$

23. $4x^2-y^2-24x-4y+16=0$

a) $4(x^2-6x+9)-(y^2+4y+4)=-16+4(9)-4$

$\frac{4(x-3)^2}{16}-\frac{(y+2)^2}{16}=\frac{16}{16}$

$\boxed{\frac{(x-3)^2}{4}-\frac{(y+2)^2}{16}=1}$

b) $x=3+2\sec t$

$y=-2+4\tan t$

c) hyperbola

d) $a=2$

$b=4$

$c^2=2^2+4^2$

$c=\sqrt{20}=2\sqrt{5}$

$e=\frac{c}{a}$

$e=\frac{2\sqrt{5}}{2}$

$e=\sqrt{5}$

Graph each of the following conic sections. Be sure to label the coordinates of the center, the vertices and the foci.

24. $x = \frac{1}{8}(y+3)^2 + 4$

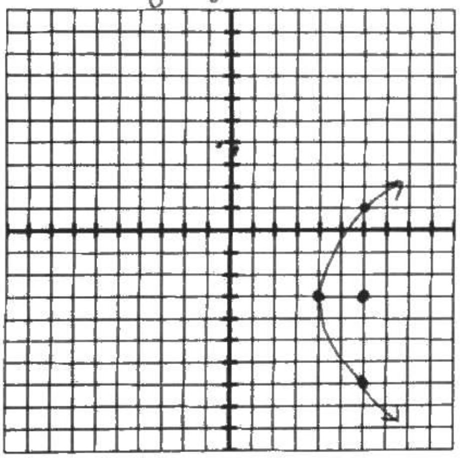
$a = \frac{1}{8} = \frac{2}{16}$

Vertex: $\boxed{(4,-3)}$

Focus: $\frac{1}{4\delta} = \frac{1}{8}$ $\quad \boxed{(6,-3)}$

$4\delta = 8$

$\delta = 2$



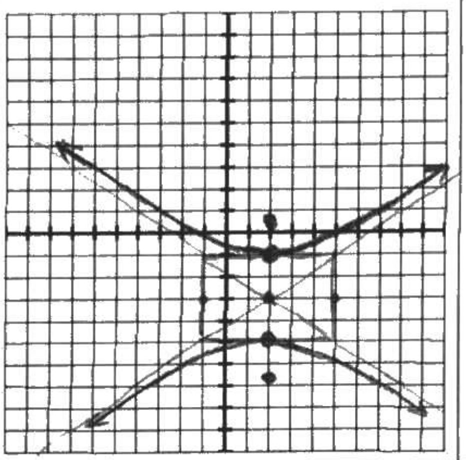
25. $\frac{(y+3)^2}{4} - \frac{(x-2)^2}{9} = 1$

Center: $\boxed{(2,-3)}$

Vertices: $\boxed{(2,-1),\ (2,-5)}$

$a=2 \quad b=3 \quad c^2=2^2+3^2 \quad c=3.606$

Foci: $\boxed{(2,\ 0.606),\ (2,-6.606)}$



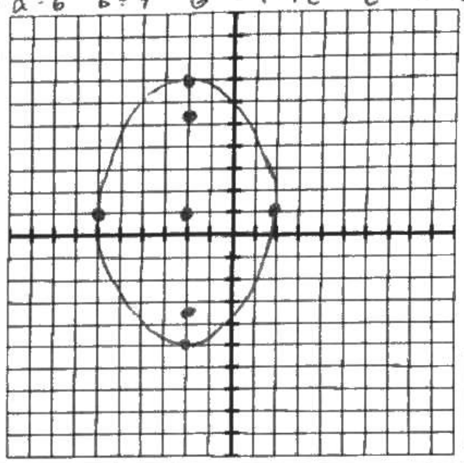
26. $\frac{(x+2)^2}{16} + \frac{(y-1)^2}{36} = 1$

Center: $\boxed{(-2,1)}$

Vertices: $\boxed{(2,1),\ (-6,1),\ (-2,7),\ (-2,-5)}$

Foci: $\boxed{(-2,5.472),\ (-2,-3.472)}$

$a=6 \quad b=4 \quad 6^2=4^2+c^2 \quad c=4.472$



27. Compare and contrast the eccentricity of all the conic sections. Why are the e values 1, >1, <1, or 0? (Hint: look at the a and c values of each conic section)

For an ellipse, the eccentricity is less than 1 because a is greater than c and $e = \frac{c}{a}$. The closer the foci get to the center, the closer c gets to zero. Therefore, if the ellipse is a circle, the eccentricity is zero since the foci are at the center and c=0. For a hyperbola, the eccentricity is greater than 1 because c is greater than a and $e = \frac{c}{a}$. A parabola doesn't have a and c values like an ellipse or a hyperbola, but it has an eccentricity of exactly one because a parabola is defined to be the set of all points that are the same distance from the focus that they are from the directrix. Therefore, the ratio of these values is always one.