M'Lzann

Review 2-3

Use the properties of logarithms to expand each of the following logarithmic expressions.

Remember: $\sqrt[4]{8} = \sqrt[4]{2^3} = 2^{\frac{3}{4}}$

C1. $\log_3 4x^2 =$ $\log_3 4 + \log_3 x^2 =$ $\boxed{\log_3 4 + 2\log_3 x}$	C2. $\ln\left(\frac{3\sqrt[3]{4}}{8}\right) =$ $\ln 3 + \ln \sqrt[3]{4} - \ln 8 =$ $\ln 3 + \ln (2^2)^{1/3} - \ln 2^3 =$ $\ln 3 + \ln 2^{2/3} - \ln 2^3 =$ $\boxed{\ln 3 + \frac{2}{3}\ln 2 - 3\ln 2}$	C3. $\log_4 125 =$ $\log_4 5^3 =$ $\boxed{3\log_4 5}$

Use the properties of logarithms to write each expression as a single quantity.

C4. $\ln x + \ln(x-3) - 3\ln(x+3) =$ $\ln x + \ln(x-3) - \ln(x+3)^3 =$ $\boxed{\ln\left[\frac{x(x-3)}{(x+3)^3}\right]}$	C5. $5\ln 2 - 3\ln 4 + \frac{1}{3}\ln 8 - 2\ln 6 =$ $\ln 2^5 - \ln 4^3 + \ln 8^{1/3} \cdot \ln 6^2 =$ $\ln 32 - \ln 64 + \ln 2 - \ln 36 =$ $\ln\left(\frac{32 \cdot 2}{64 \cdot 36}\right) =$ $\boxed{\ln\left(\frac{1}{36}\right)}$

Use the properties of logarithms and exponents to find the exact value of each expression.

C6. $\log_2 8 =$ $\log_2 2^3 =$ $\boxed{3}$	C7. $\frac{2}{3}\log_2 32 =$ $\frac{2}{3}\log_2 2^5 =$ $\frac{2}{3}(5) =$ $\boxed{\frac{10}{3}}$
C8. $\ln e^3 - 4\ln e^{-2} + \log_3 9 =$ $\ln e^3 - 4\ln e^{-2} + \log_3 3^2 =$ $3 - 4(-2) + 2 =$ $3 + 8 + 2 =$ $\boxed{13}$	C9. $4^{\log_4 12} =$ $\boxed{12}$

Solve each of the following logarithmic equations.

C10. $\ln(3x+14) - \ln 5 = \ln 2x$

$$\cancel{e}^{\cancel{\ln}\left(\frac{3x+14}{5}\right)} = \cancel{e}^{\cancel{\ln}(2x)}$$

$$\frac{3x+14}{5} = 2x$$

$$3x+14 = 10x$$

$$14 = 7x$$

$$\boxed{x=2}$$

C11. $\log_{\frac{1}{2}}(3x+1)^{\frac{1}{3}} = -2$

$$(3x+1)^{1/3} = 4$$

$$3x+1 = 4^3$$

$$3x+1 = 64$$

$$3x = 63$$

$$\boxed{x=21}$$

C12. $\log_2(x+1) - \log_4 x = 1$

$$\log_2(x+1) - \frac{\log_2 x}{\log_2 4} = 1$$

$\log_2 4 = \log_2 2^2 = 2$

$$\log_2(x+1) - \frac{1}{2}\log_2 x = 1$$

$$\log_2(x+1) - \log_2(x^{\frac{1}{2}}) = 1$$

$$\cancel{2}^{\cancel{\log_2}\frac{x+1}{\sqrt{x}}} = 2^1$$

$$\frac{x+1}{\sqrt{x}} = 2$$

$$x+1 = 2\sqrt{x}$$

$$x - 2\sqrt{x} + 1 = 0$$

$$(\sqrt{x}-1)(\sqrt{x}-1) = 0$$

$$\sqrt{x} - 1 = 0$$

$$\sqrt{x} = 1$$

$$\boxed{x=1}$$

C13. $9^{3x-4} = 27^{7-2x}$

$$(3^2)^{3x-4} = (3^3)^{7-2x}$$

$$\cancel{\log_3} 3^{2(3x-4)} = \cancel{\log_3} 3^{3(7-2x)}$$

$$6x-8 = 21-6x$$

$$12x = 29$$

$$\boxed{x=\frac{29}{12}}$$

C14. $2\log 6x - \log 3x = \log 2$

$$\log(6x)^2 - \log 3x = \log 2$$

$$\log 36x^2 - \log 3x = \log 2$$

$$\cancel{10}^{\cancel{\log}\frac{36x^2}{3x}} = \cancel{10}^{\cancel{\log} 2}$$

$$12x = 2$$

$$\boxed{x=\frac{1}{6}}$$

C15. $16^{2x-5} = 4^{6-4x}$

$$(4^2)^{2x-5} = 4^{6-4x}$$

$$\cancel{\log_4} 4^{2(2x-5)} = \cancel{\log_4} 4^{6-4x}$$

$$4x-10 = 6-4x$$

$$8x = 16$$

$$\boxed{x=2}$$

C16. The population of a town is 35,000 and is increasing at the rate of 4.75% each year. Find the algebraic representation for P as a function of time. When will the population be 100,000?

$P = 35000(1 + 0.0475)^t$

$\boxed{P = 35000(1.0475)^t}$

$100{,}000 = 35000(1.0475)^t$

$\frac{20}{7} = 1.0475^t$

$\ln\left(\frac{20}{7}\right) = \ln 1.0475^t$

$\ln\left(\frac{20}{7}\right) = t \ln 1.0475$

$t = 22.622$

The population will be 100,000 in about 22.622 years

C17. The population of a town in west Texas is 23,000 and is decreasing at the rate of 1.375% each year. Find the algebraic representation for P as a function of time. Determine when the population will be 18,000.

$P = 23000(1 - 0.01375)^t$

$\boxed{P = 23000(0.98625)^t}$

$18{,}000 = 23000(0.98625)^t$

$\frac{18}{23} = 0.98625^t$

$\ln(18/23) = \ln 0.98625^t$

$\ln(18/23) = t \ln 0.98625$

$t = 17.704$

The population will be 18,000 in about 17.704 years.

C18. The half-life of a certain radioactive substance is 18 days, and there are 8 grams present initially. How many days will it be until there is less than 1 gram remaining?

$4 = 8e^{k(18)}$

$\frac{1}{2} = e^{18k}$

$\ln\left(\frac{1}{2}\right) = 18k$

$k = -0.0385$

$y = 8e^{-0.0385t}$

$1 = 8e^{-0.0385t}$

$\frac{1}{8} = e^{-0.0385t}$

$\ln(1/8) = -0.0385t$

$t = 54$

There will be 1 gram remaining in about 54 days, so there will be less than 1 gram remaining after that time.

C19. Banks which compound interest "continuously" use an exponential function to calculate the amount of money you have at any time. Suppose that you put \$1000 into a savings account and find that at the end of the year you have \$1052. The particular equation for this exponential function is $P(t) = 1000e^{0.05069t}$

a) What is the interest rate for this account?

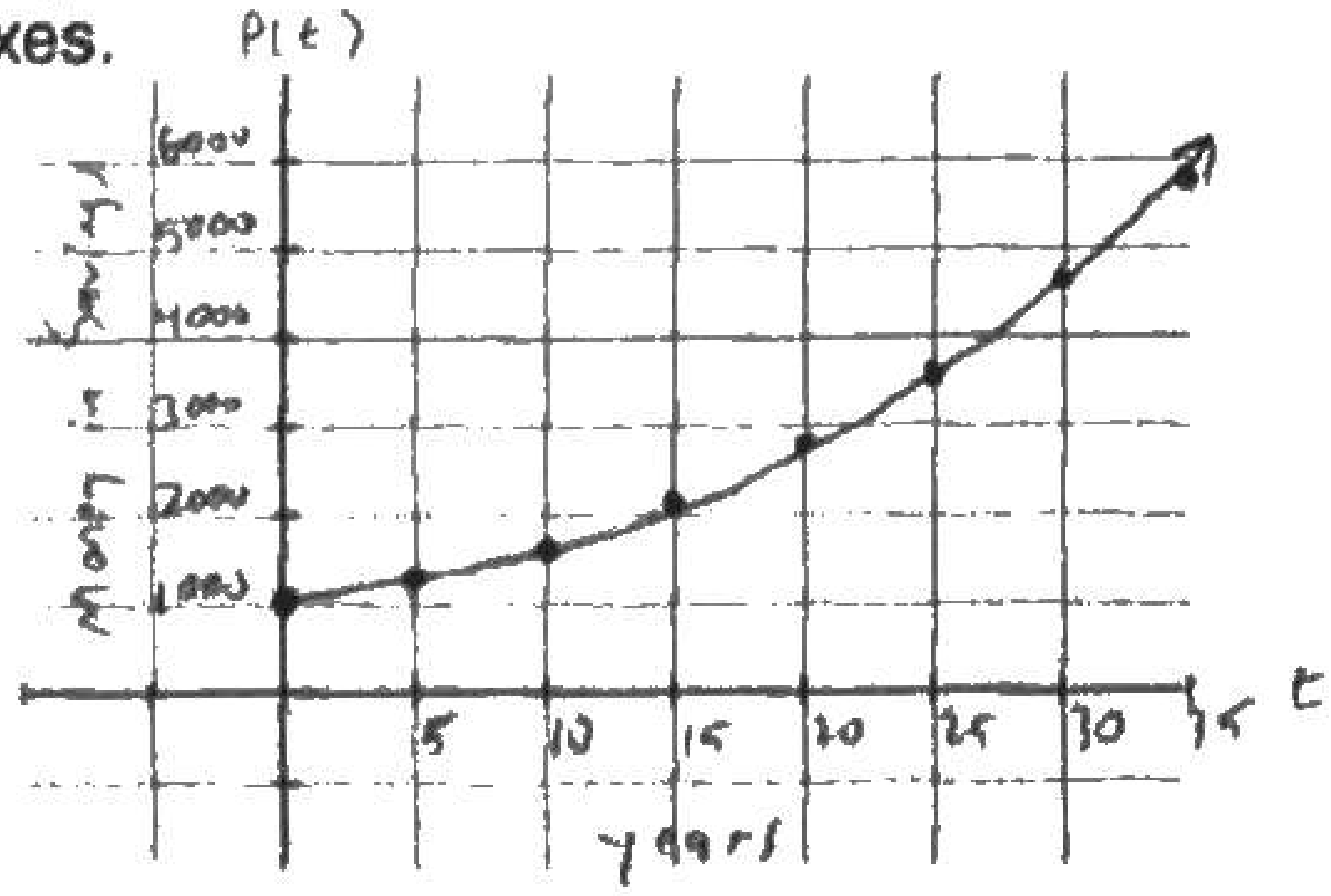
$e^{0.05069} = 1.05200 = 105.200\%$

$\boxed{5.200\%}$

b) Predict the amount you will have 10 years after investing the \$1000.

$P(10) = 1000e^{0.05069(10)} = \boxed{\$1660.14}$

c) Graph the function. Be sure to scale and label the axes.



d) How long will it take to double your initial investment?

$2000 = 1000e^{0.05069t}$

$2 = e^{0.05069t}$

$\ln 2 = 0.05069t$

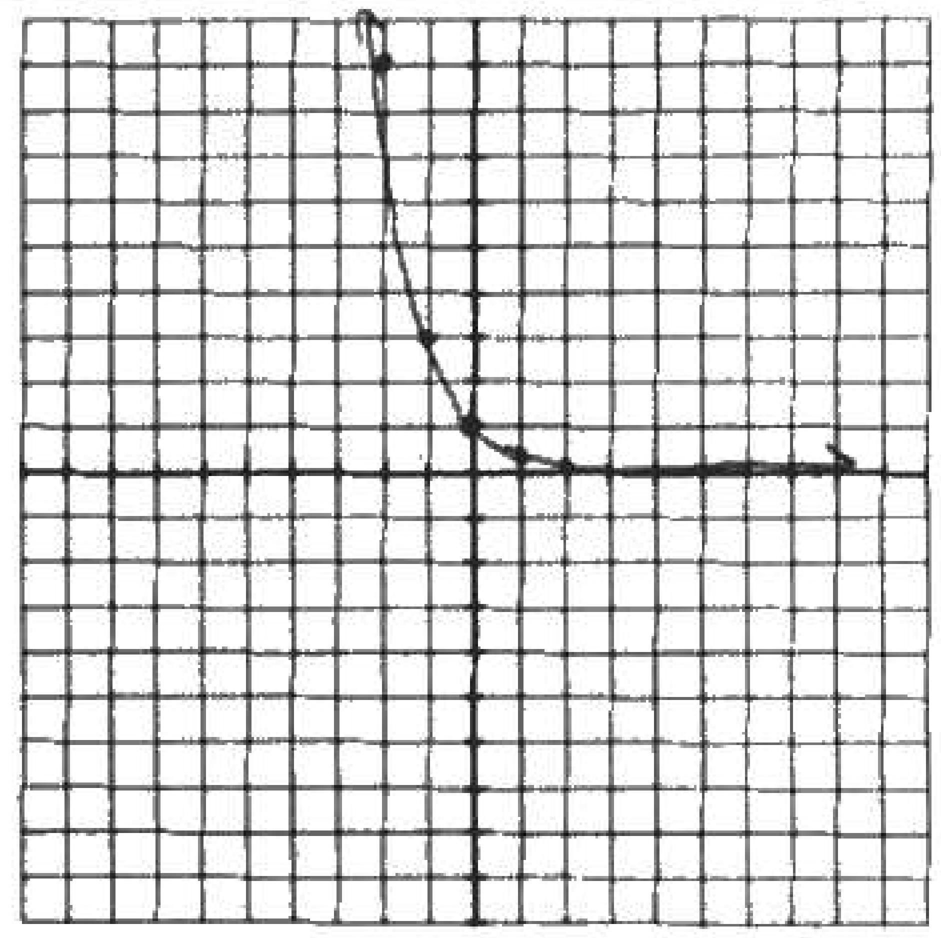
$t = 13.674$

Your investment will double in 13.674 years.

Graph each function:

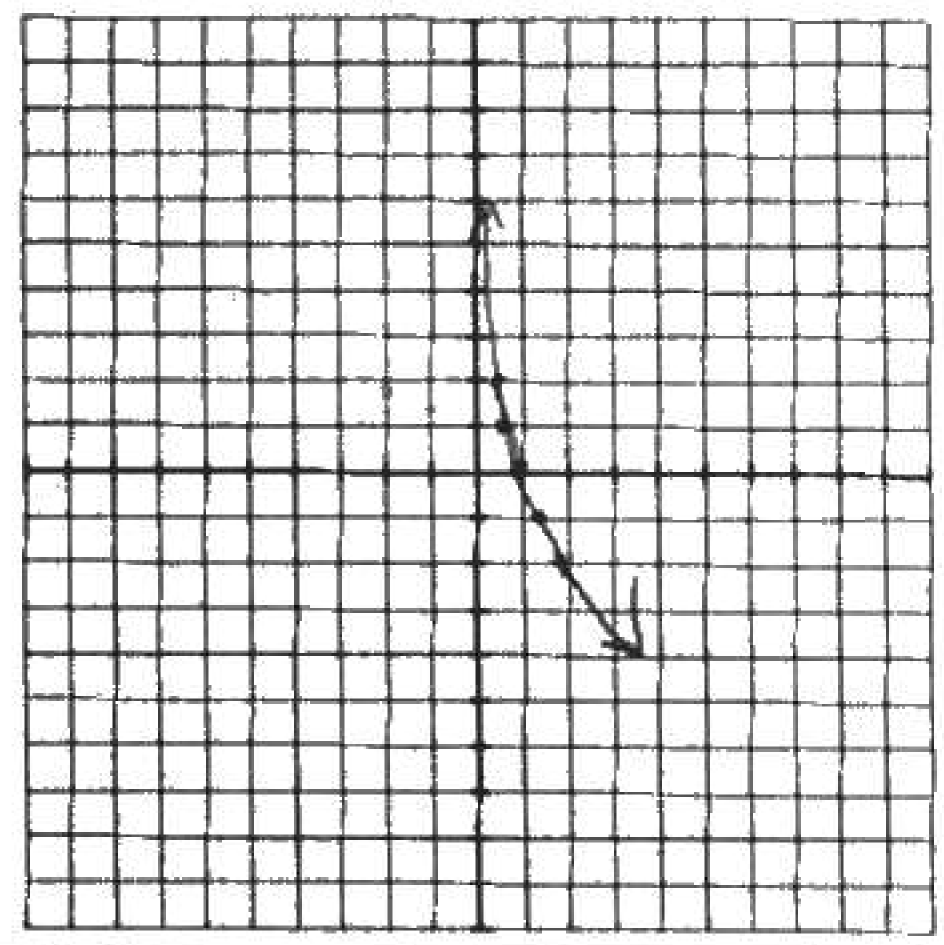
C20. $f(x) = \left(\frac{1}{3}\right)^x$

x	y
-2	$\left(\frac{1}{3}\right)^{-2} = 3^2 = 9$
-1	$\left(\frac{1}{3}\right)^{-1} = 3^1 = 3$
0	$\left(\frac{1}{3}\right)^0 = 1$
1	$\left(\frac{1}{3}\right)^1 = \frac{1}{3}$
2	$\left(\frac{1}{3}\right)^2 = \frac{1}{9}$



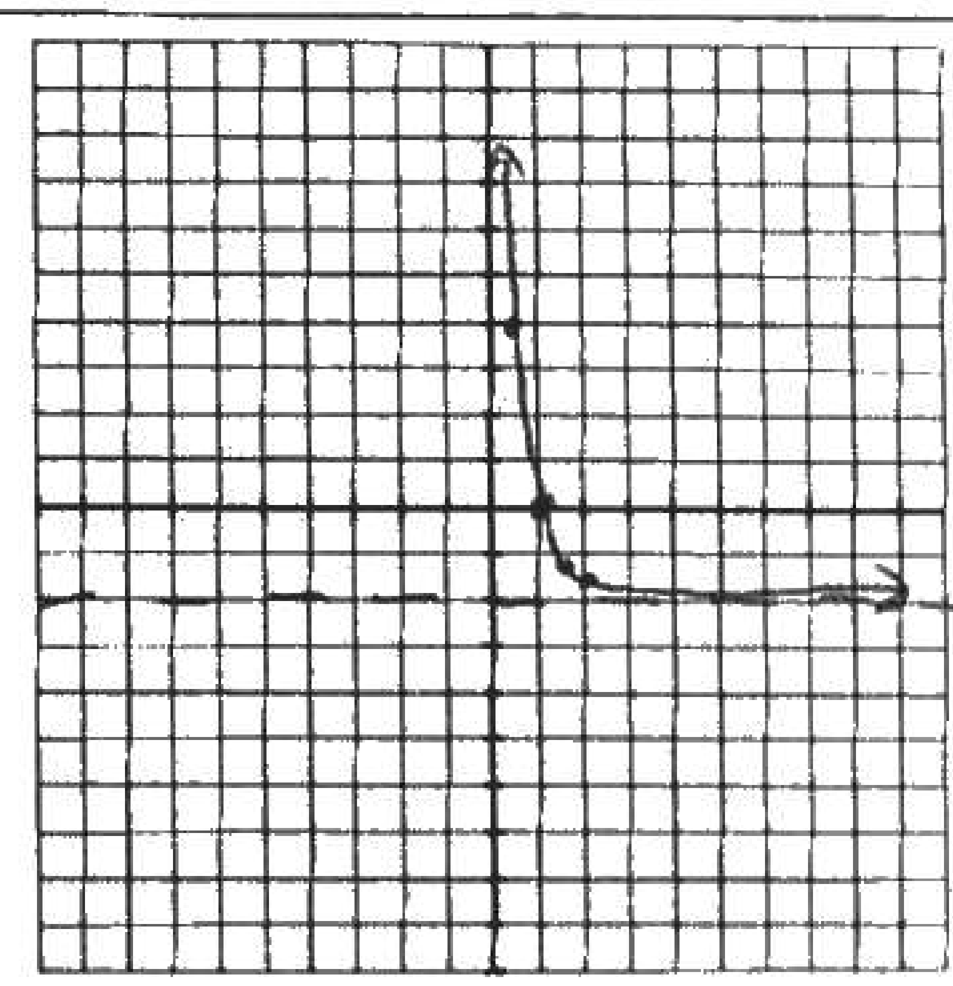
C21. $f(x) = \log_{\frac{3}{4}} x$

x	y
$\left(\frac{3}{4}\right)^{-2} = \left(\frac{4}{3}\right)^2 = \frac{16}{9}$	-2
$\left(\frac{3}{4}\right)^{-1} = \left(\frac{4}{3}\right)^1 = \frac{4}{3}$	-1
$\left(\frac{3}{4}\right)^0 = 1$	0
$\left(\frac{3}{4}\right)^1 = \frac{3}{4}$	1
$\left(\frac{3}{4}\right)^2 = \frac{9}{16}$	2



C22. $g(x) = 2\left(\frac{1}{3}\right)^{2(x-1)} - 2$ → Asymptote: $y = -2$

x	y
$-2\left(\frac{1}{2}\right)+1 = 0$	$9(2)-2 = 16$
$-1\left(\frac{1}{2}\right)+1 = \frac{1}{2}$	$3(2)-2 = 4$
$0\left(\frac{1}{2}\right)+1 = 1$	$1(2)-2 = 0$
$1\left(\frac{1}{2}\right)+1 = \frac{3}{2}$	$\frac{1}{3}(2)-2 = -\frac{4}{3}$
$2\left(\frac{1}{2}\right)+1 = 2$	$\frac{1}{9}(2)-2 = -\frac{16}{9}$



C23. $g(x) = 3\log_{\frac{3}{4}}\left[\frac{1}{2}(x-1)\right] - 4$ → Asymptote: $x = 1$

x	y
$\frac{16}{9}(2)+1 = \frac{41}{9}$	$-2(3)-4 = -10$
$\frac{4}{3}(2)+1 = \frac{11}{3}$	$-1(3)-4 = -7$
$1(2)+1 = 3$	$0(3)-4 = -4$
$\frac{3}{4}(2)+1 = \frac{5}{2}$	$1(3)-4 = -1$
$\frac{9}{16}(2)+1 = \frac{17}{8}$	$2(3)-4 = 2$