Pre-Calculus Pre-AP
1st Semester Exam Review

Name: McGrann
Date: ______________ Pd: ______

1. For each polynomial, find all of the roots (Non-Calculator).

(a) $f(x) = 2x^4 - 11x^3 - 19x^2 + 84x - 36$

$$\begin{array}{r|rrrrr} 2 & 2 & -11 & -19 & 84 & -36 \\ & & 4 & -14 & -66 & 36 \\ \hline -3 & 2 & -7 & -33 & 18 & 0 \\ & & -6 & 39 & -18 & \\ \hline & 2 & -13 & 6 & 0 & \end{array}$$

$f(x) = (x-2)(x+3)(2x^2-13x+6)$

$f(x) = (x-2)(x+3)(2x-1)(x-6)$

$\boxed{\text{roots: } (2,0),\ (-3,0),\ (\tfrac{1}{2},0),\ (6,0)}$

(b) $f(x) = -2x^4 + 13x^3 - 21x^2 + 2x + 8$

$$\begin{array}{r|rrrrr} 1 & -2 & 13 & -21 & 2 & 8 \\ & & -2 & 11 & -10 & -8 \\ \hline 2 & -2 & 11 & -10 & -8 & 0 \\ & & -4 & 14 & 8 & \\ \hline & -2 & 7 & 4 & 0 & \end{array}$$

$f(x) = (x-1)(x-2)(-2x^2+7x+4)$

$f(x) = -(x-1)(x-2)(2x^2-7x-4)$

$f(x) = -(x-1)(x-2)(2x+1)(x-4)$

roots:
$(1,0)$
$(2,0)$
$(-\tfrac{1}{2},0)$
$(4,0)$

(c) $g(x) = x^3 - 6x^2 + 13x - 10$

$$\begin{array}{r|rrrr} 2 & 1 & -6 & 13 & -10 \\ & & 2 & -8 & 10 \\ \hline & 1 & -4 & 5 & 0 \end{array}$$

$g(x) = (x-2)(x^2-4x+5)$

$x = \frac{4 \pm \sqrt{16-4(1)(5)}}{2(1)}$

$= \frac{4 \pm \sqrt{-4}}{2}$

$= \frac{4 \pm 2i}{2}$

$= 2 \pm i$

$\boxed{\text{roots: } (2,0),\ (2 \pm i, 0)}$

(d) $f(x) = 2x^4 - x^3 + 7x^2 - 4x - 4$

$$\begin{array}{r|rrrrr} 1 & 2 & -1 & 7 & -4 & -4 \\ & & 2 & 1 & 8 & 4 \\ \hline -\frac{1}{2} & 2 & 1 & 8 & 4 & 0 \\ & & -1 & 0 & -4 & \\ \hline & 2 & 0 & 8 & 0 & \\ & 1 & 0 & 4 & & \end{array}$$

$x^2+4=0$

$x^2=-4$

$x=\pm\sqrt{-4}$

$x=\pm 2i$

$f(x) = (x-1)(2x+1)(x^2+4)$

$\boxed{\text{roots: } (1,0),\ (-\tfrac{1}{2},0),\ (\pm 2i,0)}$

2. Find $f(g(x))$, $g(f(x))$, and the domain of each (Non-Calculator).

(a) $f(x) = \sqrt{x-6}$ $g(x) = \frac{1}{1-x^2}$

$\boxed{f(g(x)) = \sqrt{\frac{1}{1-x^2} - 6}}$

Domain:

$\boxed{\left(-1, -\sqrt{\tfrac{5}{6}}\right] \cup \left[\sqrt{\tfrac{5}{6}}, 1\right)}$

↑ don't worry about finding this domain ☺

$g(f(x)) = \frac{1}{1-\sqrt{x-6}^2}$

$g(f(x)) = \frac{1}{1-(x-6)}$

$\boxed{g(f(x)) = \frac{1}{7-x}}$

Domain:

$7-x \neq 0$ $x-6 \geq 0$

$x \neq 7$ $x \geq 6$

$\boxed{[6,7) \cup (7,\infty)}$

(b) $f(x) = \frac{1}{x+4}$ $g(x) = \frac{2}{1-x}$

$f(g(x)) = \frac{1(1-x)}{\left(\frac{2}{1-x}+4\right)(1-x)}$

$f(g(x)) = \frac{1-x}{2+4-4x}$

$\boxed{f(g(x)) = \frac{1-x}{6-4x}}$

Domain:

$6-4x \neq 0$ $1-x \neq 0$

$4x \neq 6$ $x \neq 1$

$x \neq \frac{3}{2}$

$\boxed{(-\infty,1) \cup (1,\tfrac{3}{2}) \cup (\tfrac{3}{2},\infty)}$

$g(f(x)) = \frac{2(x+4)}{\left(1-\frac{1}{x+4}\right)(x+4)}$

$g(f(x)) = \frac{2x+8}{x+4-1}$

$\boxed{g(f(x)) = \frac{2x+8}{x+3}}$

Domain:

$x+3 \neq 0$ $x+4 \neq 0$

$x \neq -3$ $x \neq -4$

$\boxed{(-\infty,-4) \cup (-4,-3) \cup (-3,\infty)}$

3. Add or subtract the rational expressions (Non-Calculator).

(a) $\frac{4x}{x+2}+\frac{x+3}{x-2} =$

$$\frac{4x(x-2)}{(x+2)(x-2)} + \frac{(x+3)(x+2)}{(x-2)(x+2)} =$$

$$\frac{4x^2-8x+x^2+5x+6}{(x+2)(x-2)} =$$

$$\boxed{\frac{5x^2-3x+6}{(x+2)(x-2)}}$$

(b) $\frac{9x}{2x^2-2x}+\frac{3}{x-1} =$

$$\frac{9\cancel{x}}{2\cancel{x}(x-1)} + \frac{3(2)}{(x-1)(2)} =$$

$$\frac{9+6}{2(x-1)} =$$

$$\boxed{\frac{15}{2(x-1)}}$$

(c) $\frac{8}{x^2+x-2}-\frac{2}{x^2-1} =$

$$\frac{8(x+1)}{(x+2)(x-1)(x+1)} + \frac{-2(x+2)}{(x+1)(x-1)(x+2)} =$$

$$\frac{8x+8-2x-4}{(x+2)(x-1)(x+1)} =$$

$$\boxed{\frac{6x+4}{(x+2)(x-1)(x+1)}}$$

(d) $\frac{x}{x^2-9}-\frac{6}{x^2-3x} =$

$$\frac{x(x)}{(x-3)(x+3)(x)} + \frac{-6(x+3)}{x(x-3)(x+3)} =$$

$$\boxed{\frac{x^2-6x-18}{x(x-3)(x+3)}}$$

4. Multiply or divide the rational expressions (Non-Calculator).

(a) $\frac{x^3-3x^2+2x}{2x^3+10x^2+12x}\bullet\frac{4x^2+20x+24}{x^2+2x-3} =$

$$\frac{\cancel{x}(x^2-3x+2)}{{}_1\cancel{2x}\cancel{(x^2+5x+6)}} \cdot \frac{{}^2\cancel{4}(x^2+5x+6)}{x^2+2x-3} =$$

$$\frac{(x-2)\cancel{(x-1)}}{1} \cdot \frac{2}{(x+3)\cancel{(x-1)}} =$$

$$\boxed{\frac{2(x-2)}{x+3}}$$

(b) $\frac{x^2-4}{4-4x^2}\div\frac{2-x}{5x^2-5} =$

$$\frac{\cancel{(x-2)}(x+2)}{-4\cancel{(x^2-1)}} \cdot \frac{5\cancel{(x^2-1)}}{-\cancel{(x-2)}} =$$

$$\boxed{\frac{5(x+2)}{4}}$$

(c) $\frac{2x^2+x-6}{9x^2-729}\bullet\frac{x^2-5x-36}{x-2} =$

$$\frac{2x^2+x-6}{9(x^2-81)} \cdot \frac{x^2-5x-36}{x-2} =$$

$$\frac{(2x-3)(x+2)}{9\cancel{(x-9)}(x+9)} \cdot \frac{\cancel{(x-9)}(x+4)}{(x-2)} =$$

$$\boxed{\frac{(2x-3)(x+2)(x+4)}{9(x+9)(x-2)}}$$

(d) $\frac{6x^2-13x+6}{2x^2-x-3}\div\frac{9x^2-4}{7x^2-7} =$

$$\frac{6x^2-13x+6}{2x^2-x-3} \cdot \frac{7(x^2-1)}{9x^2-4} =$$

$$\frac{\cancel{(3x-2)}\cancel{(2x-3)}}{\cancel{(2x-3)}\cancel{(x+1)}} \cdot \frac{7(x-1)\cancel{(x+1)}}{\cancel{(3x-2)}(3x+2)} =$$

$$\boxed{\frac{7(x-1)}{3x+2}}$$

5. For each rational expression, provide the requested information (Non-Calculator).

(a) $f(x)=\dfrac{(x-2)(x+7)(x-6)(2x-3)}{(8x-12)(x^2-4)(x+6)}$

Roots: $(-7,0)$, $(6,0)$

Oblique Asymptote: $y=1/4$

Vertical Asymptotes: $x=-2$, $x=-6$

Removable Discontinuities: $x=2$, $x=3/2$

$$f(x)=\frac{\cancel{(x-2)}(x+7)(x-6)\cancel{(2x-3)}}{4\cancel{(2x-3)}(x+2)\cancel{(x-2)}(x+6)}$$

$$f(x)=\frac{(x+7)(x-6)}{4(x+2)(x+6)}$$

$$f(x)=\frac{x^2+x-42}{4x^2+32x+48}$$

$$\begin{array}{r|l} & 1/4 \\ \hline 4x^2+32x+48 & x^2+x-42 \\ & -x^2-8x-12 \\ \hline & -7x-54 \end{array}$$

(b) $f(x)=\dfrac{\cancel{(x-4)}\cancel{(x+5)}(x-1)(x-3)}{(x+6)\cancel{(x-4)}(x-10)(x+2)\cancel{(x+5)}}$

Roots: $(1,0)$, $(3,0)$

Oblique Asymptote: $y=0$

Vertical Asymptotes: $x=-6$, $x=10$, $x=-2$

Removable Discontinuities: $x=4$, $x=-5$

$$f(x)=\frac{(x-1)(x-3)}{(x+6)(x-10)(x+2)}$$

	x^2	$-8x$	-20
x	x^3	$-8x^2$	$-20x$
$+6$	$6x^2$	$-48x$	-120

$$f(x)=\frac{x^2-4x+3}{x^3-2x^3-68x-120}$$

$$\begin{array}{r|l} & 0 \\ \hline x^3-2x^3-68x-120 & x^2-4x+3 \end{array}$$

(c) $f(x)=\dfrac{(x^2-1)(x+2)(x-4)(x+6)}{(x+1)(x-6)(4x^2-16)}$

$4(x^2-4)$

Roots: $(1,0)$, $(4,0)$, $(-6,0)$

Oblique Asymptote: $y=1/4x+9/4$

Vertical Asymptotes: $x=6$, $x=2$

Removable Discontinuities: $x=-1$, $x=-2$

$$f(x)=\frac{\cancel{(x+1)}(x-1)\cancel{(x+2)}(x-4)(x+6)}{4\cancel{(x+1)}(x-6)(x-2)\cancel{(x+2)}}$$

$$f(x)=\frac{(x-1)(x-4)(x+6)}{4(x-6)(x-2)}$$

	x^2	$+2x$	-24
x	x^3	$2x^2$	$-24x$
-1	$-x^2$	$-2x$	24

$$f(x)=\frac{x^3+x^2-26x+24}{4x^2-32x+48}$$

$$\begin{array}{r|l} & 1/4x+9/4 \\ \hline 4x^2-32x+48 & x^3+x^2-26x+24 \\ & -x^3+8x^2-12x \\ \hline & 9x^2-38x+24 \\ & -9x^2+72x-108 \\ \hline & 34x-84 \end{array}$$

6. Write the equation of the function and identify the Domain & Range (Non-Calculator).

(a) Equation:

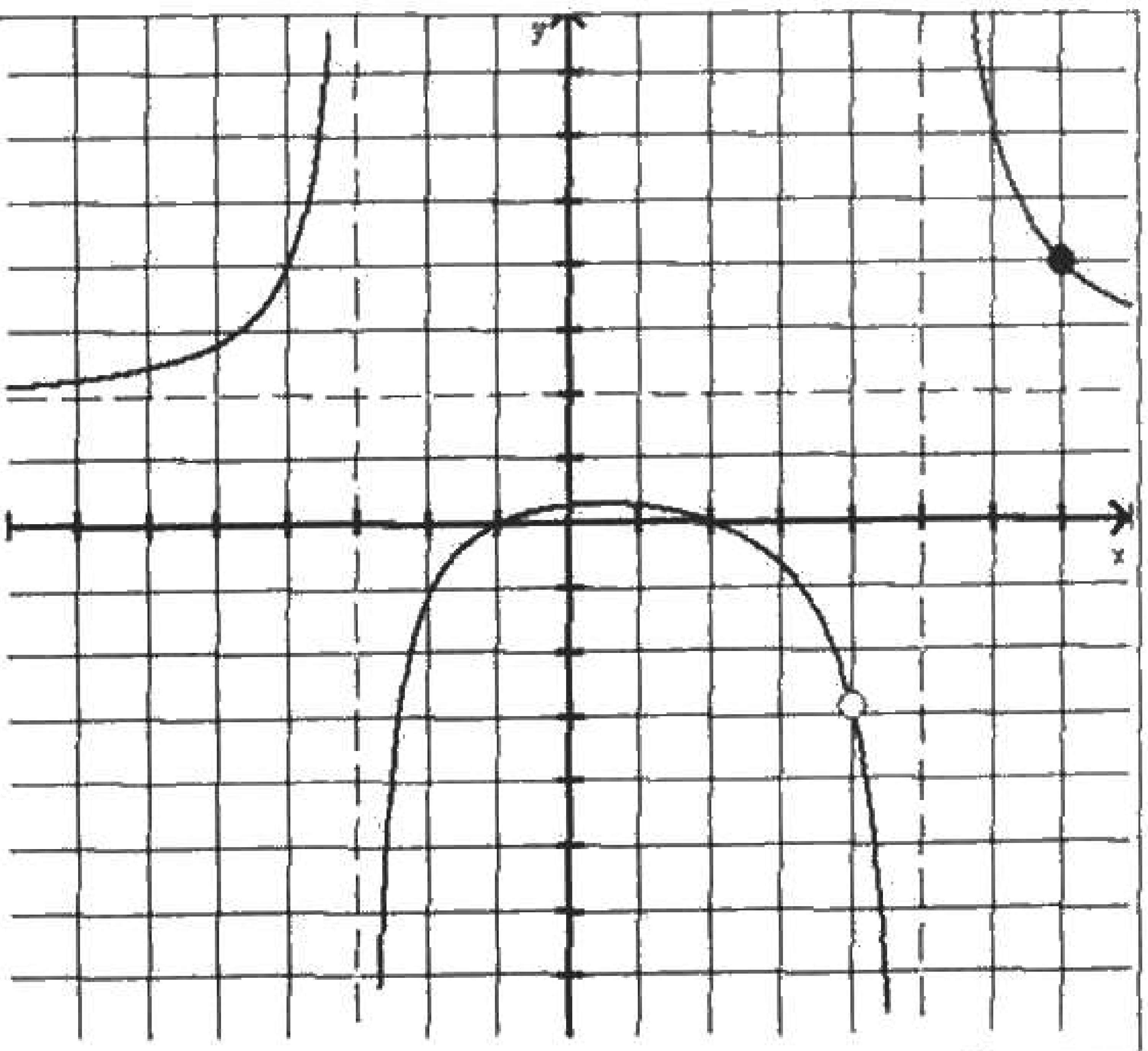
$$f(x) = \frac{2(x+1)(x-2)(x-4)}{(x+3)(x-5)(x-4)}$$

Domain: $(-\infty, -3) \cup (-3, 4) \cup (4, 5) \cup (5, \infty)$

Range: $(-\infty, 0.4] \cup (2, \infty)$

↑

Approximation



(b) Equation:

$$f(x) = \frac{a(x+2)(x-3)(x-5)}{(x+4)(x+1)(x-3)(x-5)}$$

$$\frac{1}{2} = \frac{a(0+2)}{(0+4)(0+1)}$$

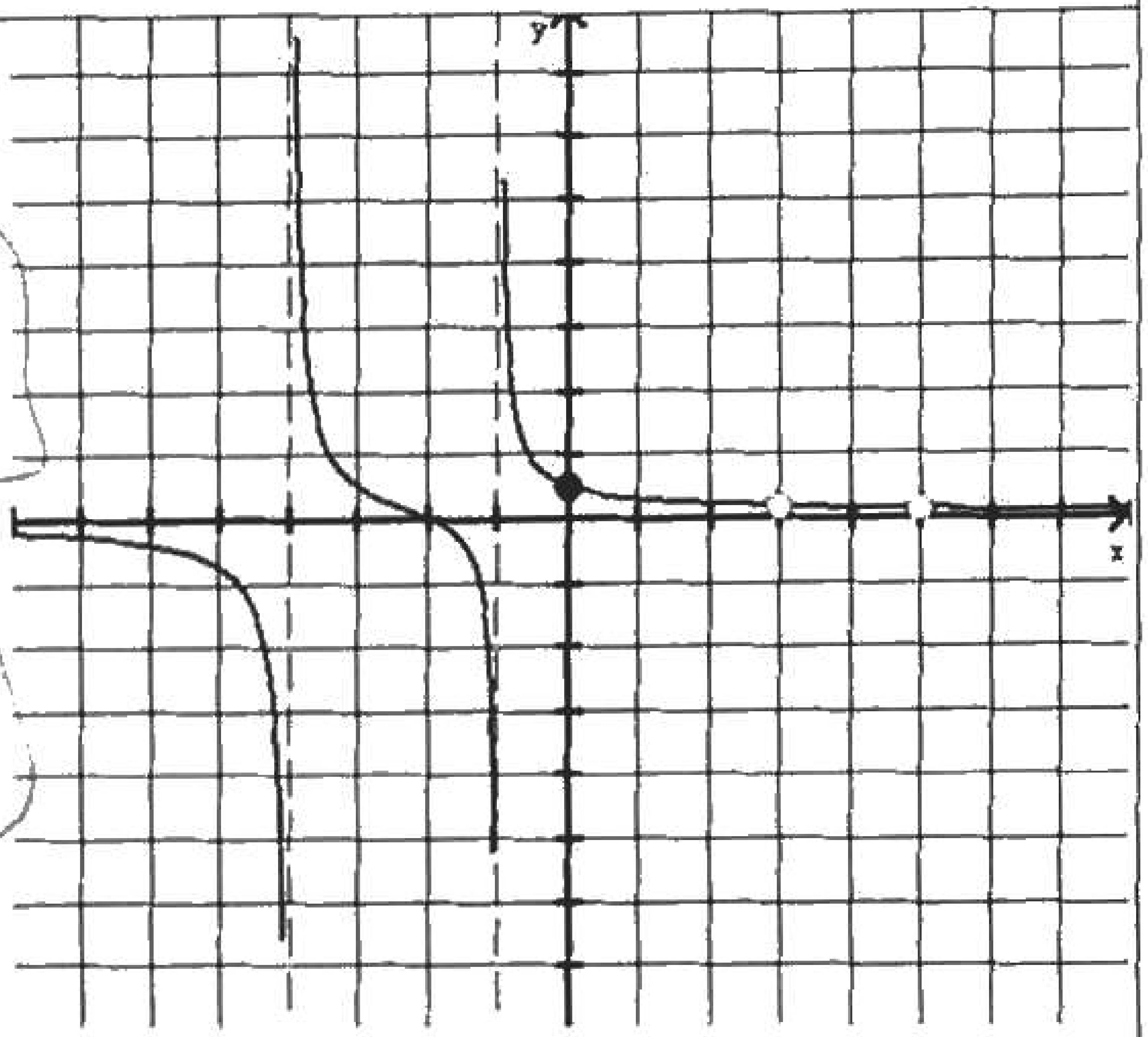
$$\frac{1}{2} = \frac{1}{2}a$$

$$a = 1$$

$$f(x) = \frac{(x+2)(x-3)(x-5)}{(x+4)(x+1)(x-3)(x-5)}$$

Domain: $(-\infty, -4) \cup (-4, -1) \cup (-1, 3) \cup (3, 5) \cup (5, \infty)$

Range: $(-\infty, \infty)$



(c) Equation:

$$f(x) = \frac{a(x)(x-3)(x+4)}{(x+2)(x+4)}$$

$$1 = \frac{a(-1)(-1-3)}{(-1+2)}$$

$$1 = 4a$$

$$a = \frac{1}{4}$$

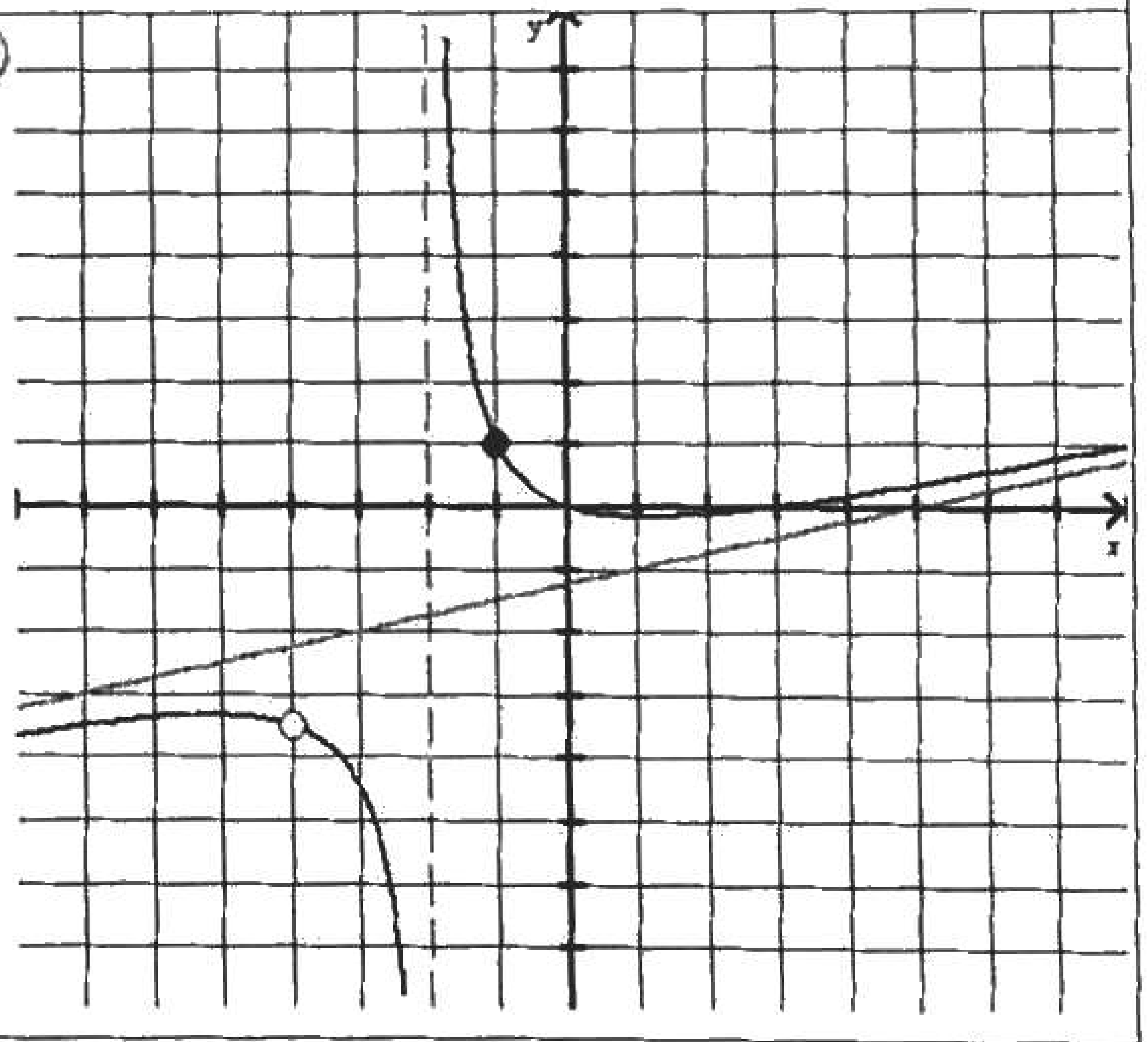
$$f(x) = \frac{x(x-3)(x+4)}{4(x+2)(x+4)}$$

Domain: $(-\infty, -4) \cup (-4, -2) \cup (-2, \infty)$

Range: $(-\infty, -3.3] \cup [-0.2, \infty)$

↑ ↗

Approximations



7. Use the properties of logarithms to expand the following expressions (Non-Calculator).

(a) $\ln xyz^2 = \ln x + \ln y + \ln z^2$ $= \boxed{\ln x + \ln y + 2\ln z}$	(b) $\log 4x^2y = \log 2^2 + \log x^2 + \log y$ $= \boxed{2\log 2 + 2\log x + \log y}$
(c) $\ln\sqrt[3]{\frac{x}{y}} = \ln\left(\frac{x}{y}\right)^{1/3}$ $= \frac{1}{3}\ln\left(\frac{x}{y}\right)$ $= \boxed{\frac{1}{3}(\ln x - \ln y)}$	(d) $\log_2 \frac{\sqrt{x}y^4}{z^4} = \log_2 x^{1/2} + \log_2 y^4 - \log_2 z^4$ $= \boxed{\frac{1}{2}\log_2 x + 4\log_2 y - 4\log_2 z}$

8. Condense the expression using the properties of logarithms (Non-Calculator).

(a) $\ln x - 3\ln(x+1) = \ln x - \ln(x+1)^3$ $= \boxed{\ln\left[\frac{y}{(x+1)^3}\right]}$	(b) $3\log_3 x + 4\log_3 y - 4\log_3 z =$ $\log_3 x^3 + \log_3 y^4 - \log_3 z^4 =$ $\boxed{\log_3\left(\frac{x^3y^4}{z^4}\right)}$
(c) $\ln x - 4[\ln(x+2) + \ln(x-2)] =$ $\ln x - 4\ln[(x+2)(x-2)] =$ $\ln x - 4\ln(x^2-4) =$ $\ln x - \ln(x^2-4)^4 =$ $\boxed{\ln\left[\frac{x}{(x^2-4)^4}\right]}$	(d) $2[3\ln x - \ln(x+1) - \ln(x-1)] =$ $6\ln x - 2\ln(x+1) - 2\ln(x-1) =$ $\ln x^6 - \ln(x+1)^2 - \ln(x-1)^2 =$ $\boxed{\ln\left[\frac{x^6}{(x+1)^2(x-1)^2}\right]}$

9. Solve each equation (Non-Calculator).

(a) $7 + 3\ln x = 5$ $3\ln x = -2$ $\underset{e}{\cancel{\ln}} x = \underset{e}{-\frac{2}{3}}$ $x = e^{-2/3}$ $\boxed{x = \frac{1}{\sqrt[3]{e^2}}}$	(b) $\underset{2}{\cancel{\log_2}}(2x-3) = \underset{2}{\cancel{\log_2}}(x+4)$ $2x - 3 = x + 4$ $\boxed{x = 7}$
(c) $\log_3 x + \log_3(x-8) = 2$ $\underset{3}{\cancel{\log_3}}[x(x-8)] = \underset{3}{2}$ $x^2 - 8x = 9$ $x^2 - 8x - 9 = 0$ $(x-9)(x+1) = 0$ $x = 9, -1$ $\boxed{x = 9}$	(d) $\log_5\left(\frac{1}{625}\right) = x$ $\cancel{\log_x}(x^{-4}) = x$ $\boxed{x = -4}$

10. For each function, find the inverse and the domain and range of both the function and its inverse (Non-Calculator):

(a) $f(x) = \frac{5x-2}{x+4}$

domain: $(-\infty,-4)\cup(-4,\infty)$
range: $(-\infty,5)\cup(5,\infty)$

$x = \frac{5y-2}{y+4}$

$x(y+4) = 5y-2$

$xy+4x = 5y-2$

$xy-5y = -4x-2$

$y(x-5) = -4x-2$

$y = \frac{-4x-2}{x-5}$

$\boxed{f^{-1}(x) = -\frac{4x+2}{x-5}}$

domain: $(-\infty,5)\cup(5,\infty)$
range: $(-\infty,-4)\cup(-4,\infty)$

(b) $f(x) = \frac{3x-7}{2x+9}$

domain: $(-\infty,-\frac{9}{2})\cup(-\frac{9}{2},\infty)$
range: $(-\infty,\frac{3}{2})\cup(\frac{3}{2},\infty)$

$x = \frac{3y-7}{2y+9}$

$x(2y+9) = 3y-7$

$2xy+9x = 3y-7$

$2xy-3y = -9x-7$

$y(2x-3) = -9x-7$

$y = \frac{-9x-7}{2x-3}$

$\boxed{f^{-1}(x) = -\frac{9x-7}{2x-3}}$

domain: $(-\infty,\frac{3}{2})\cup(\frac{3}{2},\infty)$
range: $(-\infty,-\frac{9}{2})\cup(-\frac{9}{2},\infty)$

11. Determine if each function is even, odd, or neither (Non-Calculator):

(a) $f(x) = \frac{x^2-x}{x^4}$

$f(-x) = \frac{(-x)^2-(-x)}{(-x)^4}$

$= \frac{x^2+x}{x^4}$

Neither

(b) $f(x) = \frac{x^3-x}{x^4}$

$f(-x) = \frac{(-x)^3-(-x)}{(-x)^4}$

$= \frac{-x^3+x}{x^4}$

$= -\frac{x^3-x}{x^4}$

Odd

(c) $f(x) = \frac{x^3-x}{x^5}$

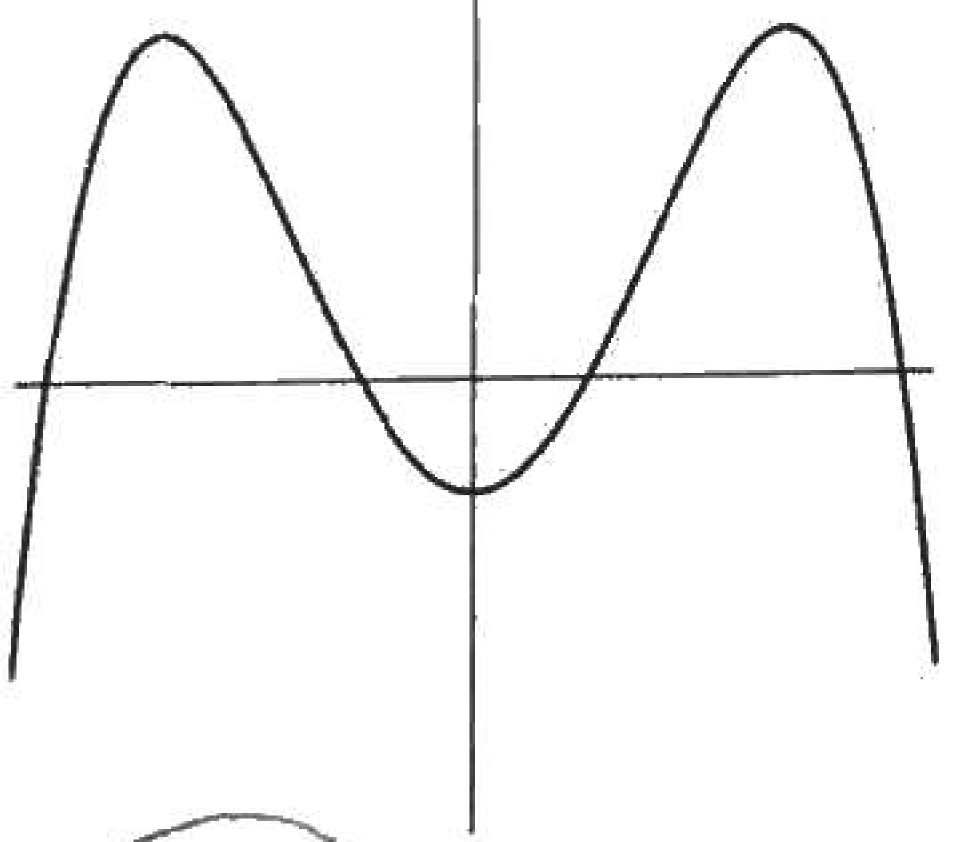
$f(-x) = \frac{(-x)^3-(-x)}{(-x)^5}$

$= \frac{-x^3+x}{-x^5}$

$= \frac{x^3-x}{x^5}$

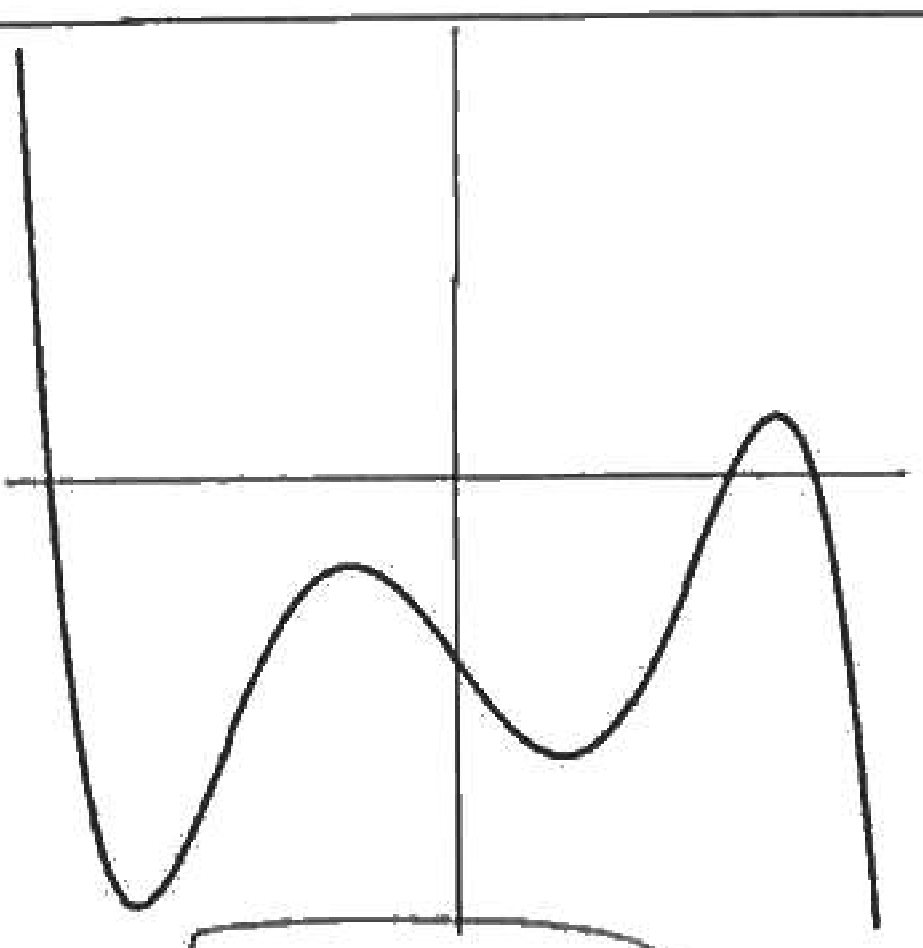
Even

(d)



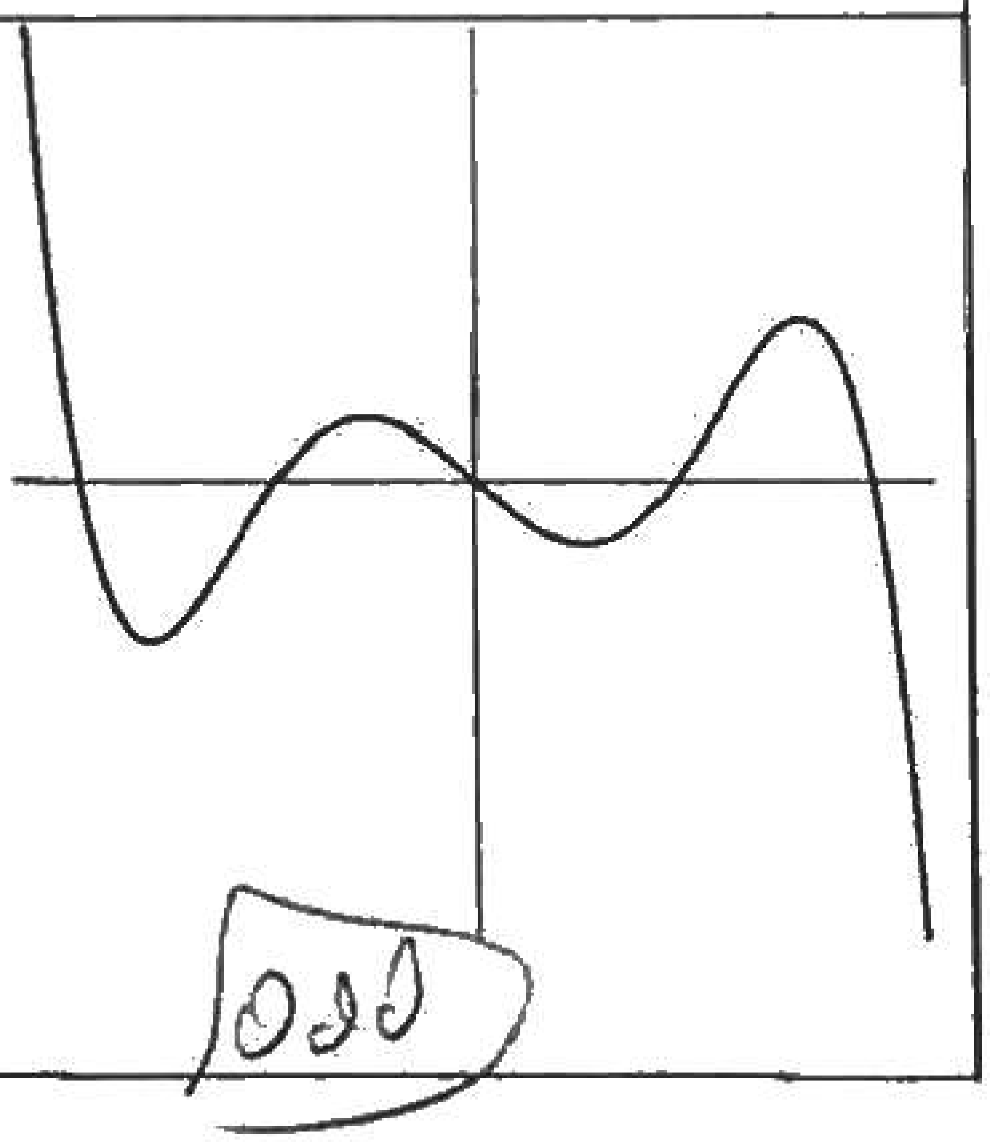
Even

(e)



Neither

(f)



Odd

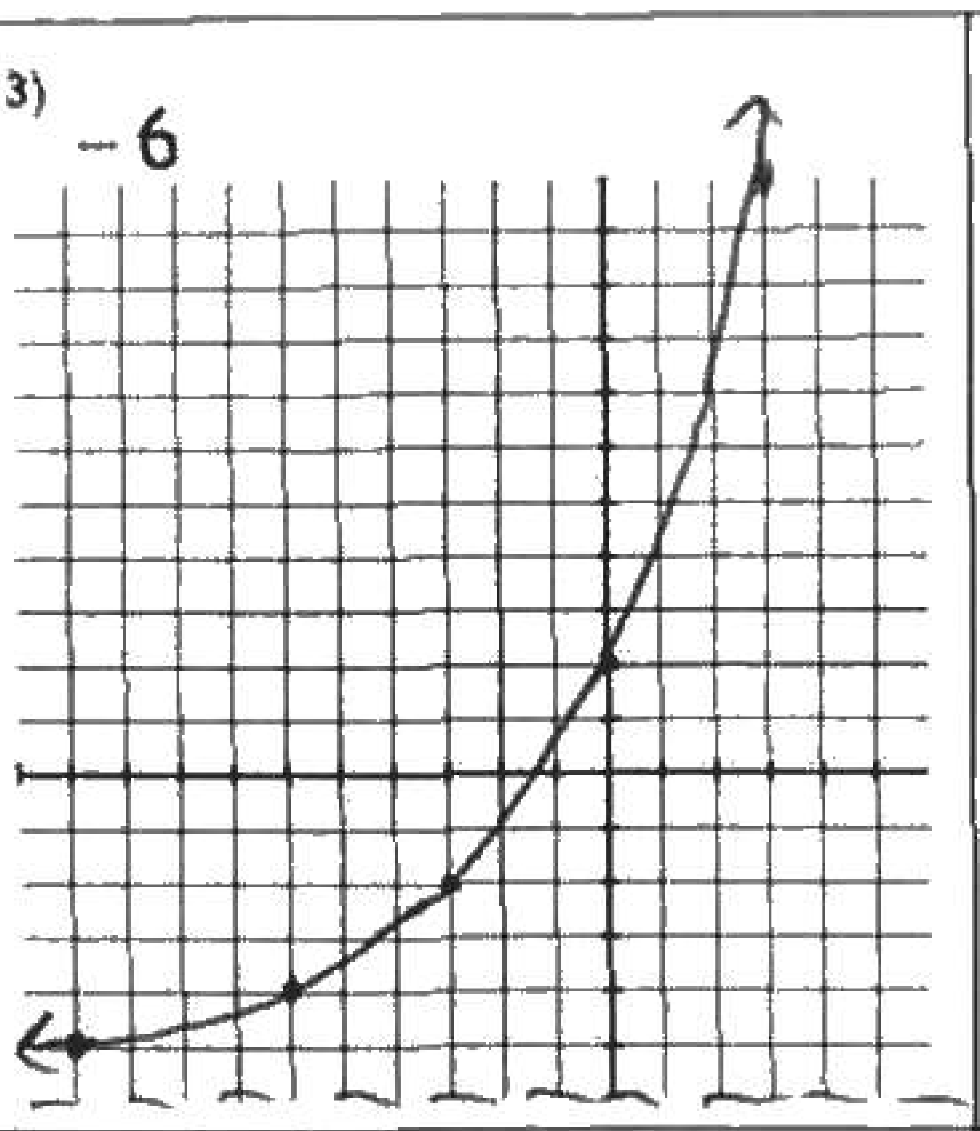
12. Graph the following equations (Non-Calculator):

(a) $f(x) = 4 \cdot 2^{\frac{1}{3}(x+3)} - 6$

X	Y
-2(3)-3 = -9	1/4(4)-6 = -5
-1(3)-3 = -6	1/2(4)-6 = -4
0(3)-3 = -3	1(4)-6 = -2
1(3)-3 = 0	2(4)-6 = 2
2(3)-3 = 3	4(4)-6 = 10

Asymptote: y=0(4)-6

$\boxed{y=-6}$

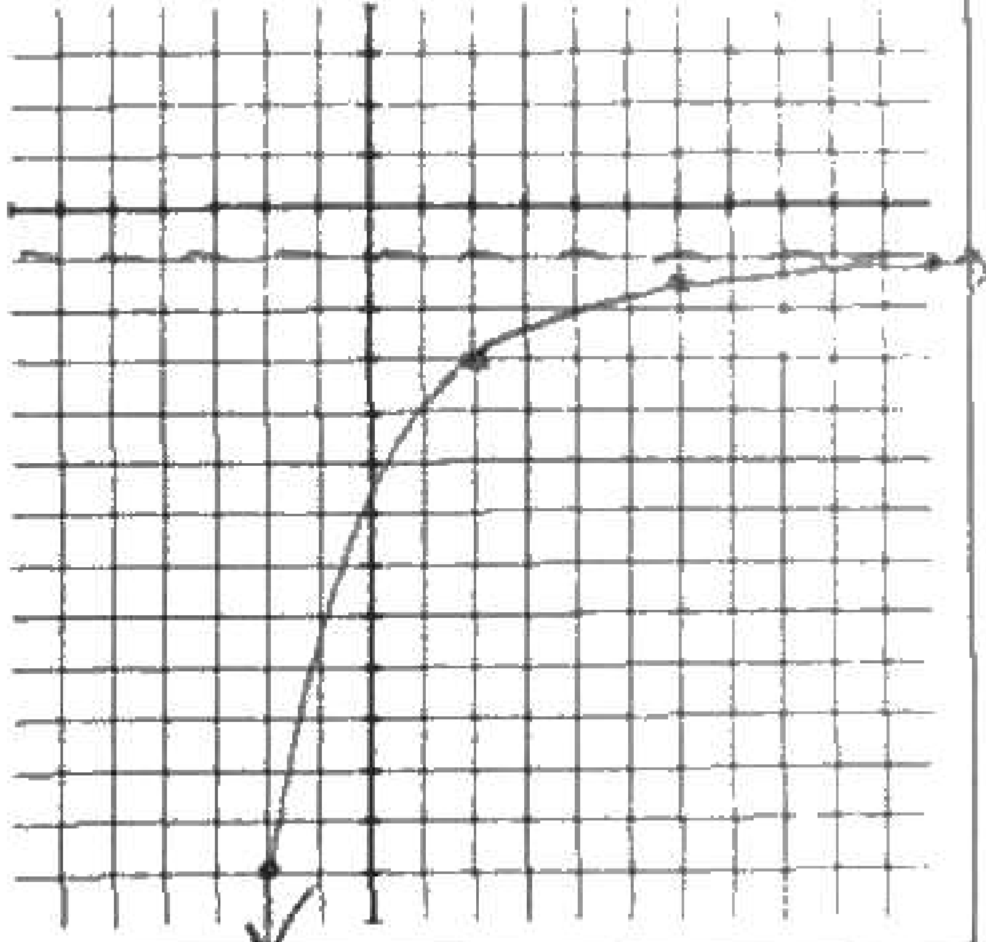


(b) $f(x) = -2 \cdot 4^{-\frac{1}{4}(x-2)} - 1$

X	Y
-2(-4)+2 = 10	1/16(-2)-1 = -1 1/8
-1(-4)+2 = 6	1/4(-2)-1 = -1 1/2
0(-4)+2 = 2	1(-2)-1 = -3
1(-4)+2 = -2	4(-2)-1 = -9
2(-4)+2 = -6	16(-2)-1 = -33

Asymptote: y=0(-2)-1

$\boxed{y=-1}$

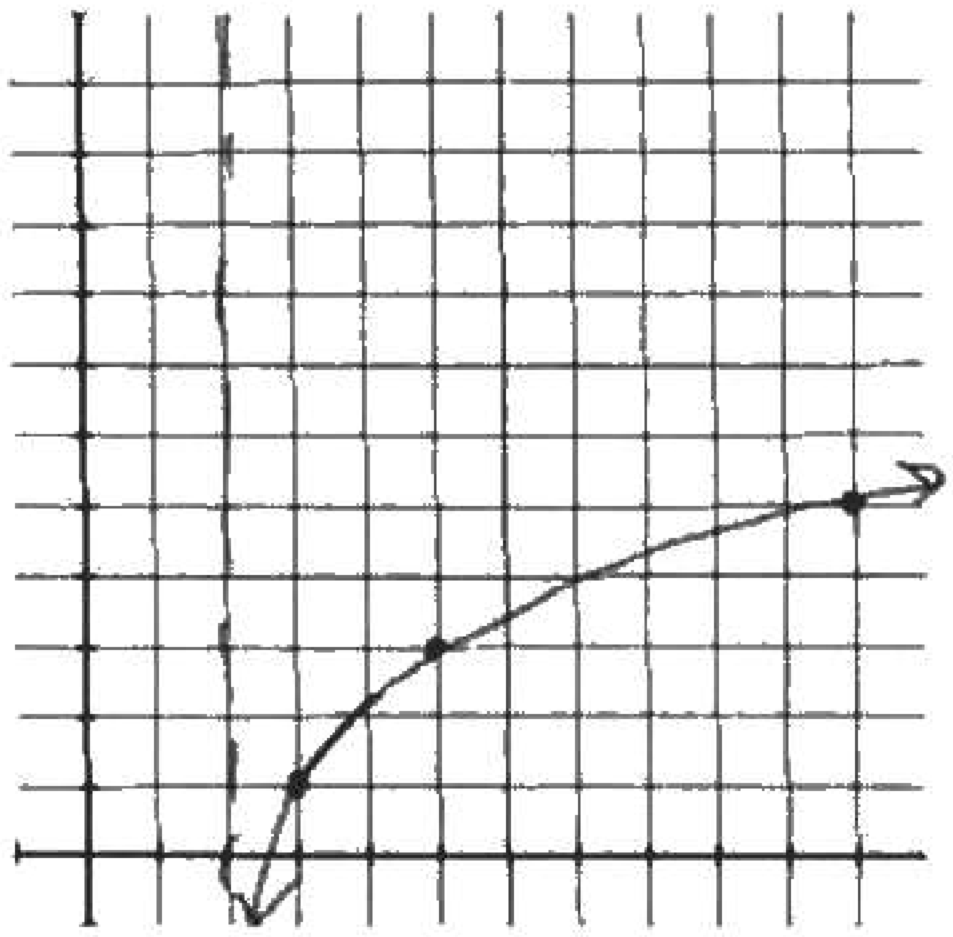


(c) $f(x) = 2\log_3\left[\frac{1}{9}(x-2)\right] + 5$

X	Y
1/9(9)+2 = 3	-2(2)+5 = 1
1/3(9)+2 = 5	-1(2)+5 = 3
1(9)+2 = 11	0(2)+5 = 5
3(9)+2 = 29	1(2)+5 = 7
9(9)+2 = 83	2(2)+5 = 9

Asymptote: x=0(9)+2

$\boxed{x=2}$

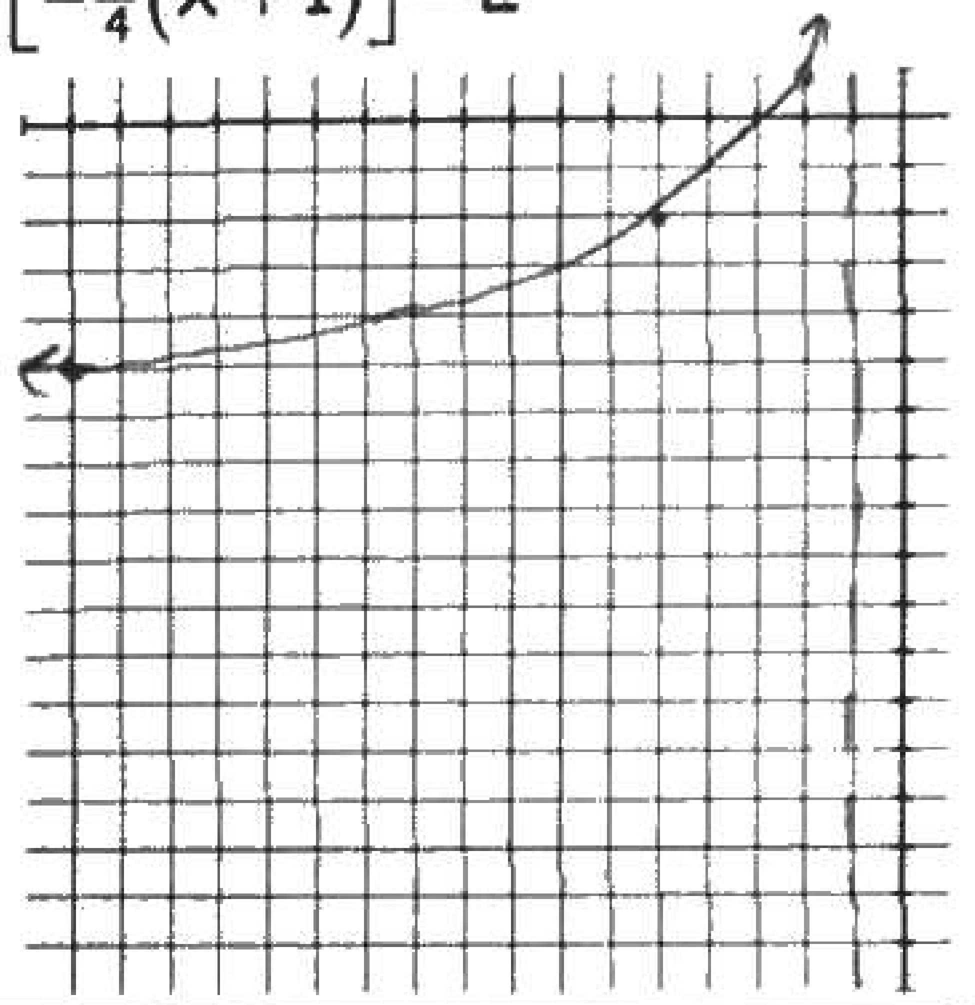


(d) $f(x) = -3\log_4\left[-\frac{1}{4}(x+1)\right] - 2$

X	Y
1/16(-4)-1 = -1 1/4	-2(-3)-2 = 4
1/4(-4)-1 = -2	-1(-3)-2 = 1
1(-4)-1 = -5	0(-3)-2 = -2
4(-4)-1 = -17	1(-3)-2 = -5
16(-4)-1 = -65	2(-3)-2 = -8

Asymptote: x=0(-4)-1

$\boxed{x=-1}$



13. Perform the following divisions (Non-Calculator):

(a) $\dfrac{4x^5 + 7x^4 - 20x^3 + 5x^2 - 13x + 28}{x^2 + 2x - 5}$

$4x^3 - x^2 + 2x - 4$

$x^2+2x-5 \overline{)\,4x^5 + 7x^4 - 20x^3 + 5x^2 - 13x + 28}$

$-4x^5 - 8x^4 + 20x^3$

$-x^4 + 5x^2 - 13x + 28$

$+x^4 + 2x^3 - 5x^2$

$2x^3 - 13x + 28$

$-2x^3 - 4x^2 + 10x$

$-4x^2 - 3x + 28$

$+4x^2 + 8x - 20$

$5x + 8$

$\boxed{4x^3 - x^2 + 2x - 4 + \frac{5x+8}{x^2+2x-5}}$

(b) $\dfrac{6x^4 - 23x^3 + 27x^2 - 19x + 8}{3x^2 - 4x - 1}$

$2x^2 - 5x + 3$

$3x^2 - 4x - 1 \overline{)\,6x^4 - 23x^3 + 27x^2 - 19x + 8}$

$-6x^4 + 8x^3 + 2x^2$

$-15x^3 + 29x^2 - 19x + 8$

$-15x^3 - 20x^2 - 5x$

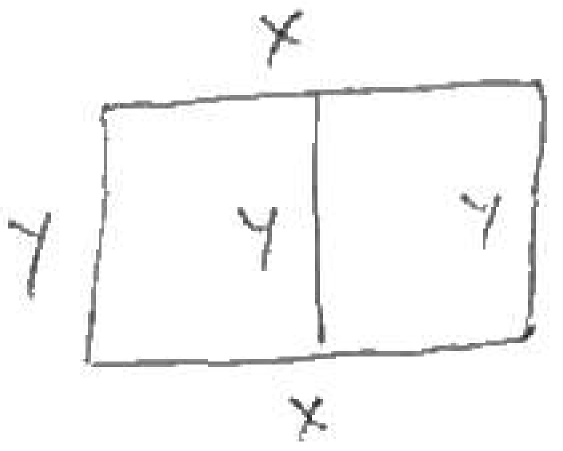
$9x^2 - 24x + 8$

$-9x^2 + 12x + 3$

$-12x + 11$

$\boxed{2x^2 - 5x + 3 + \frac{-12x+11}{3x^2-4x-1}}$

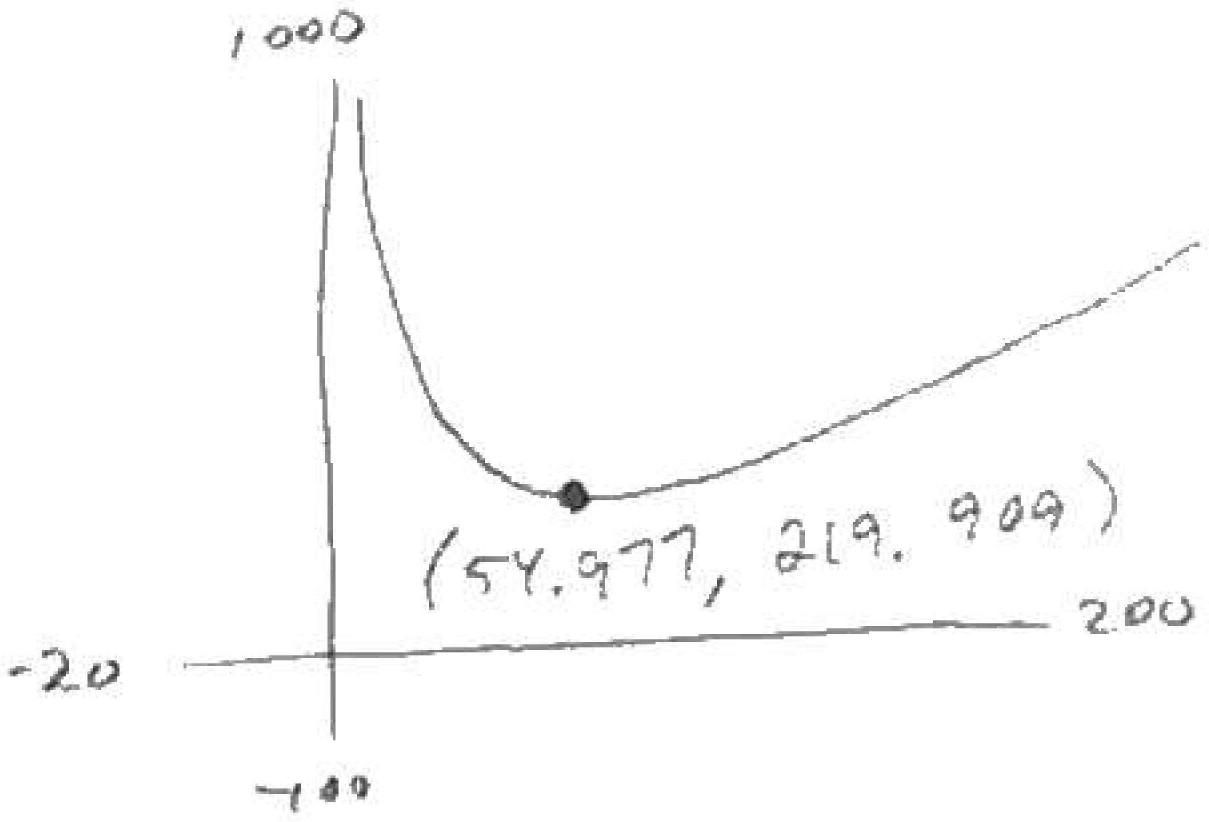
14. A 2015 m² rectangular patch of ground is to be enclosed by a fence and used as a pen for sheep and goats. To keep them separate, another length of fence, parallel to one of the sides, divides the patch into two parts. Find the minimum amount of fence needed to enclose the total area and give a **complete** explanation of how to achieve this minimum (Calculator).



$\text{Area} = 2015$

$xy = 2015$

$y = \frac{2015}{x}$



$F = 2x + 3y$

$F = 2x + 3\left(\frac{2015}{x}\right)$

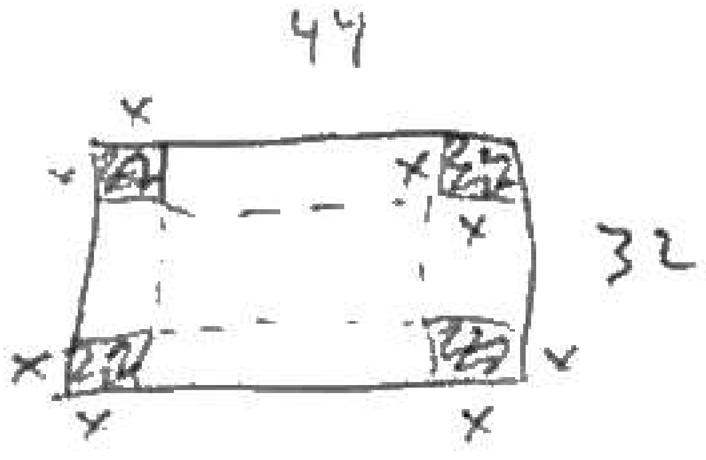
$F = 2x + \frac{6045}{x}$

$y = \frac{2015}{54.977}$

$y = 36.652$

A pen of width 36.652 meters and length 54.977 meters will have an area of 2015 m^2 and use the minimum amount of fence, which is 219.909 meters.

15. Squares with sides of length (x) are cut from the corners of a rectangular piece of sheet metal with dimensions of 32 inches and 44 inches. The metal is then folded to make an open-top box. Find the maximum volume of this box and give a **complete** explanation of how to achieve this maximum (Calculator).



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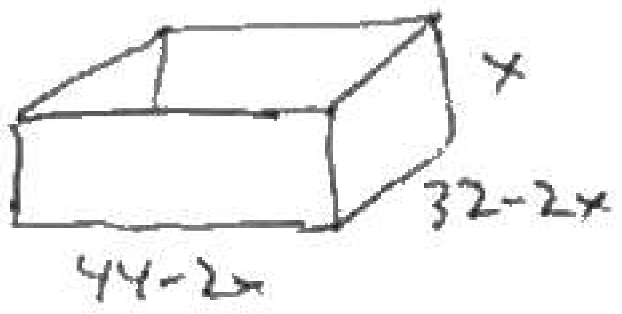
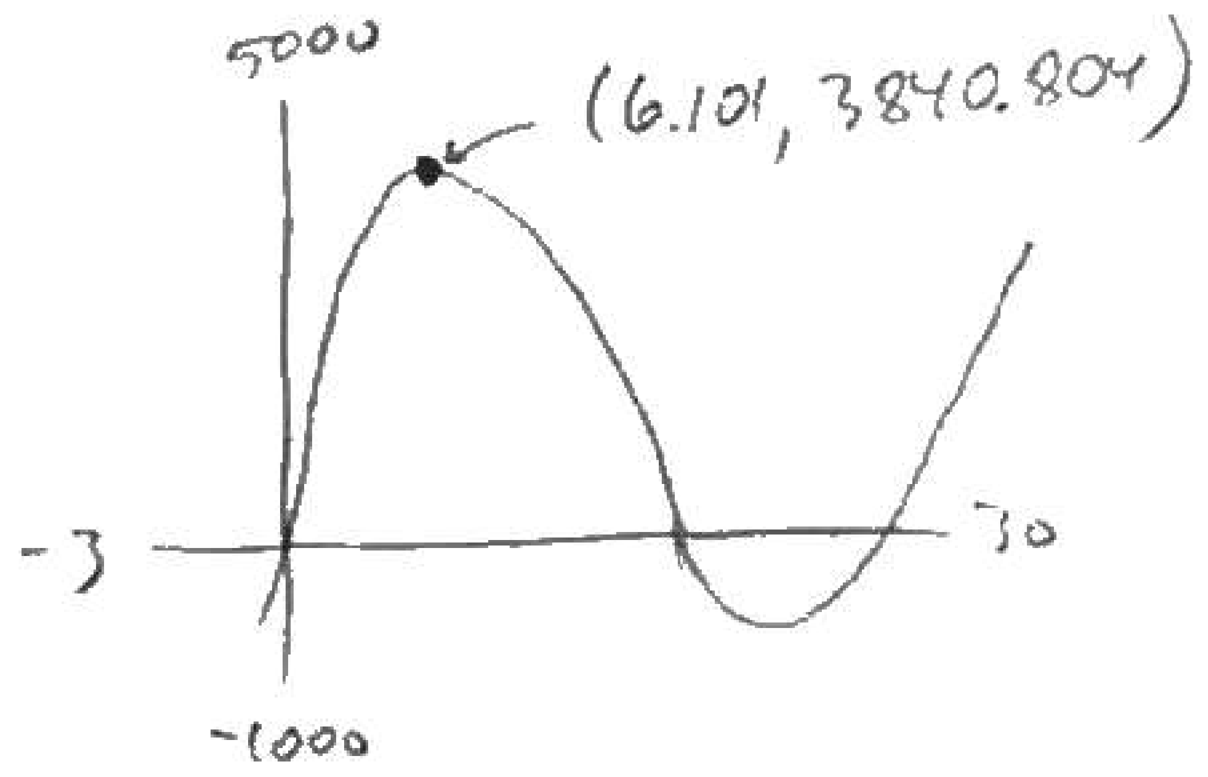




$V = l \cdot w \cdot h$

$V = (44-2x)(32-2x)(x)$

$l = 44-2(6.101) = 31.798$

$w = 32-2(6.101) = 19.798$

Cut 6.101" by 6.101" squares out of the corners to get a box that is 31.798" by 19.798" by 6.101" and has a maximum volume of 3840.804 in^3.

16. Determine the degree of the polynomial that has the given roots. Then find the polynomial in expanded form, assuming "a" is 1 (Calculator).

(a) Three roots are: -2, -1, 3*i*

$f(x) = (x+2)(x+1)(x-3i)(x+3i)$

$f(x) = (x^2+3x+2)(x^2+9)$

$\boxed{f(x) = x^4+3x^3+11x^2+27x+18}$

	x^2	$+3x$	$+2$
x^2	x^4	$3x^3$	$2x^2$
$+9$	$9x^2$	$27x$	18

(b) Two roots are: *i*, 3*i*

$f(x) = (x-i)(x+i)(x-3i)(x+3i)$

$f(x) = (x^2+1)(x^2+9)$

$\boxed{f(x) = x^4+10x^2+9}$

(c) Two roots are: 3, 4 - *i*

$f(x) = (x-3)(x-4+i)(x-4-i)$

$f(x) = (x-3)(x^2-8x+17)$

$\boxed{f(x) = x^3-11x^2+41x-51}$

	x	-4	$-i$
x	x^2	$-4x$	$\cancel{-ix}$
-4	$-4x$	16	$\cancel{4i}$
$+i$	$\cancel{ix}$	$\cancel{-4i}$	$-i^2$

$x^2-8x+16+1$

	x^2	$-8x$	$+17$
x	x^3	$-8x^2$	$17x$
-3	$-3x^2$	$24x$	-51

(d) Four roots are: ½ , -1, 1, 2 - 5*i*

$f(x) = (2x-1)(x+1)(x-1)(x-2+5i)(x-2-5i)$

$f(x) = (2x-1)(x^2-1)(x^2-4x+29)$

$f(x) = (2x^3-x^2-2x+1)(x^2-4x+29)$

$\boxed{f(x) = 2x^5-9x^4+60x^3-20x^2-62x+29}$

	x	-2	$-5i$
x	x^2	$-2x$	$\cancel{-5ix}$
-2	$-2x$	4	$\cancel{10i}$
$+5i$	$\cancel{5ix}$	$\cancel{-10i}$	$-25i^2$

$x^2-4x+4+25$

	x^2	$-4x$	$+29$
$2x^3$	$2x^5$	$-8x^4$	$58x^3$
$-x^2$	$-x^4$	$4x^3$	$-29x^2$
$-2x$	$-2x^3$	$8x^2$	$-58x$
$+1$	x^2	$-4x$	29

17. Find the equation of the polynomial (in expanded form) that has the given zeros and passes through the specified point (Calculator).

(a) Zeros: -4, 0, 2 Point: (1, 10)

$f(x) = a(x+4)(x)(x-2)$

$10 = a(1+4)(1)(1-2)$

$10 = -5a$

$a = -2$

$f(x) = -2x(x+4)(x-2)$

$f(x) = -2x(x^2+2x-8)$

$\boxed{f(x) = -2x^3-4x^2+16x}$

(b) Zeros: -4, -1, 2, 3 Point: (5, 6)

$f(x) = a(x+4)(x+1)(x-2)(x-3)$

$6 = a(5+4)(5+1)(5-2)(5-3)$

$6 = 324a$

$a = 1/54$

$f(x) = \frac{1}{54}(x+4)(x+1)(x-2)(x-3)$

$f(x) = \frac{1}{54}(x^2+5x+4)(x^2-5x+6)$

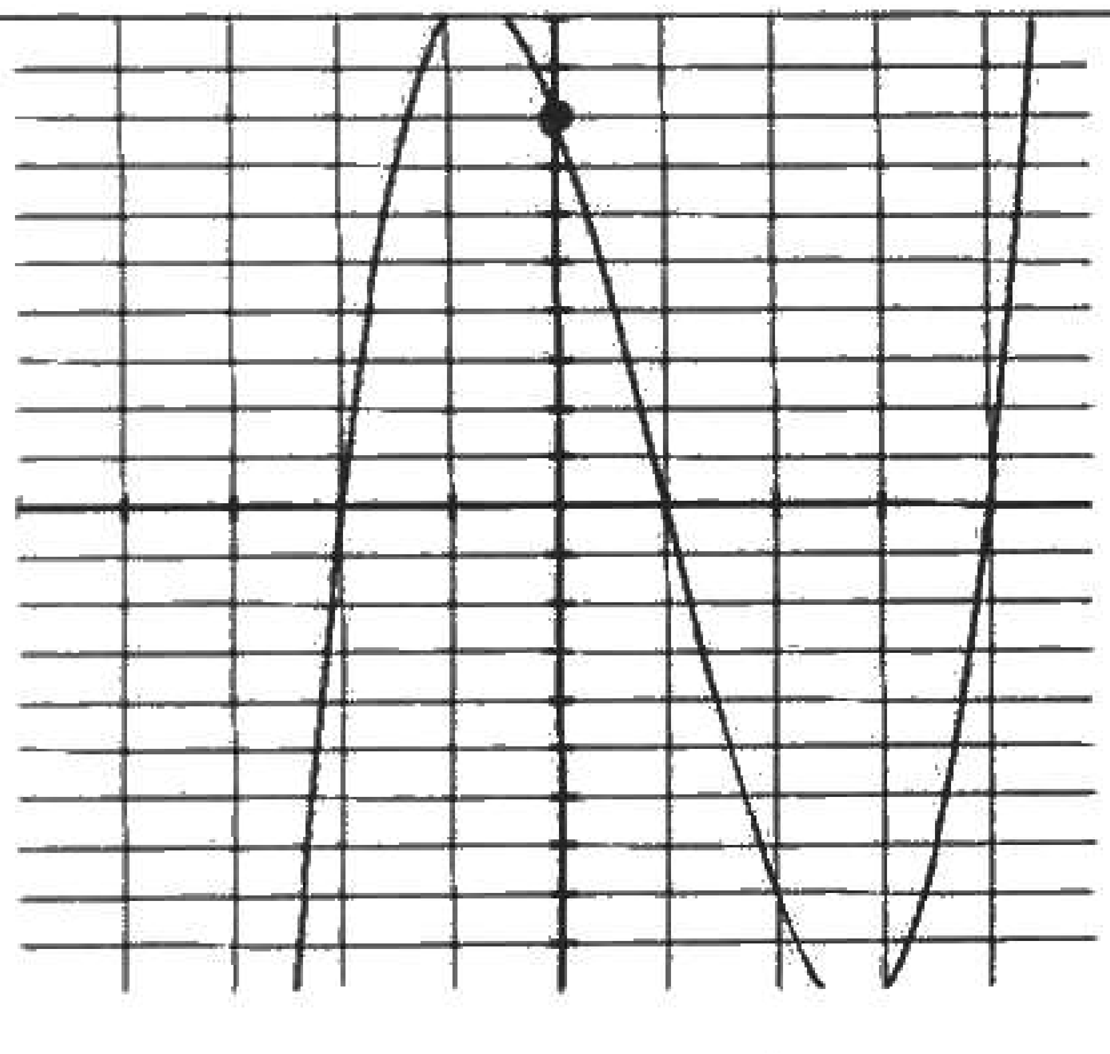
$f(x) = \frac{1}{54}(x^4-15x^2+10x+24)$

$\boxed{f(x) = \frac{1}{54}x^4 - \frac{5}{8}x^2 + \frac{5}{27}x + \frac{4}{9}}$

	x^2	$-5x$	$+6$
x^2	x^4	$-5x^3$	$6x^2$
$+5x$	$5x^3$	$-25x^2$	$30x$
$+4$	$4x^2$	$-20x$	24

18. Write the equation (in expanded form) of the polynomial shown in each graph (Calculator).

(a)



$f(x) = a(x+2)(x-1)(x-4)$

$8 = a(0+2)(0-1)(0-4)$

$8 = 8a$

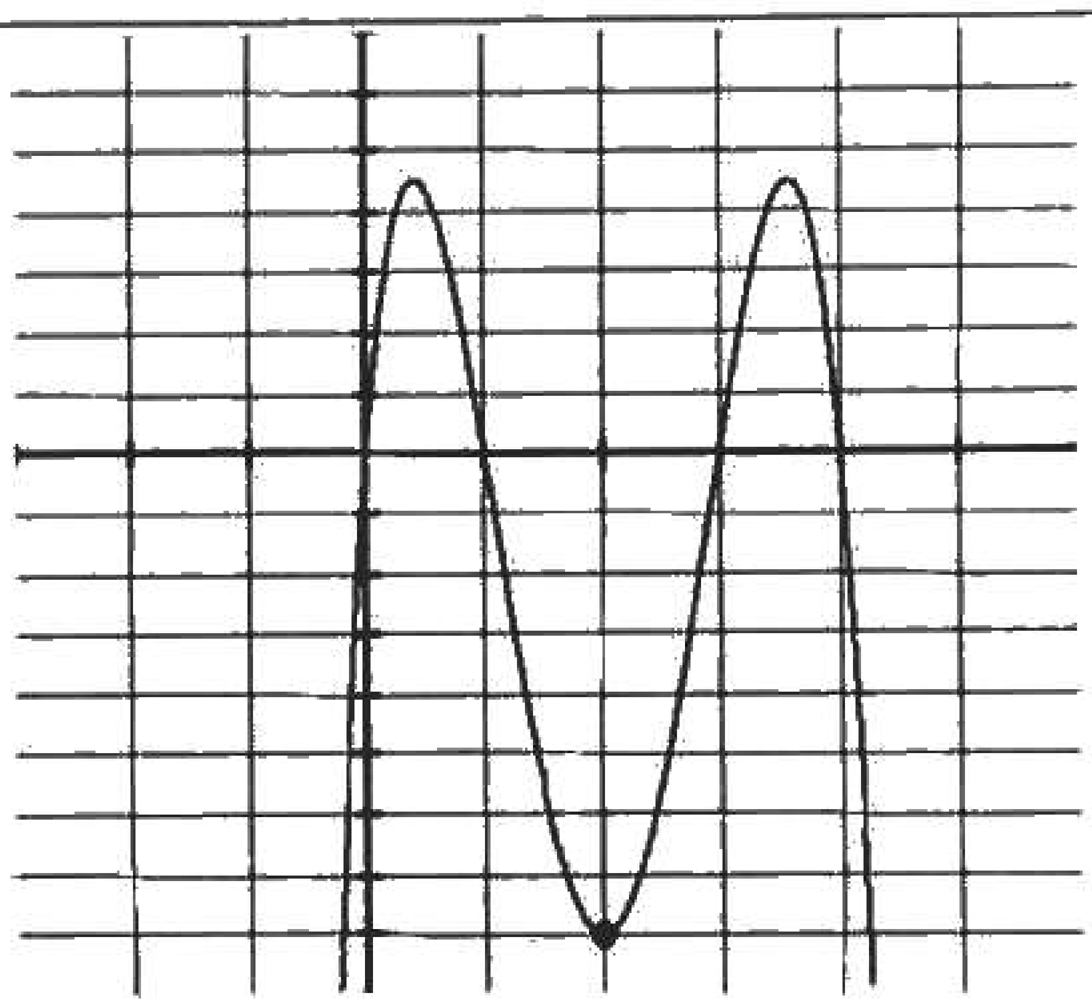
$a = 1$

$f(x) = (x+2)(x^2-5x+4)$

$\boxed{f(x) = x^3-3x^2-6x+8}$

	x^2	$-5x$	$+4$
x	x^3	$-5x^2$	$4x$
$+2$	$2x^2$	$-10x$	8

(b)



$f(x) = a(x)(x-1)(x-3)(x-4)$

$-8 = a(2)(2-1)(2-3)(2-4)$

$-8 = 4a$

$a = -2$

$f(x) = -2x(x-1)(x-3)(x-4)$

$f(x) = (-2x^2+2x)(x^2-7x+12)$

$\boxed{f(x) = -2x^4+16x^3-38x^2+24x}$

	x^2	$-7x$	$+12$
$-2x^2$	$-2x^4$	$14x^3$	$-24x^2$
$+2x$	$2x^3$	$-14x^2$	$24x$

19. The number of bacteria in a culture grows according to the law of exponential growth. If 100 bacteria are in the initial culture, and there are 300 bacteria after 5 hours, how long did it take for the number of bacteria to double? How many bacteria will there be after 30 minutes (Calculator)?

$Y = 100e^{kt}$

$300 = 100e^{k(5)}$

$3 = e^{5k}$

$\ln 3 = 5k$

$k = 0.220$

$200 = 100e^{0.220t}$

$2 = e^{0.220t}$

$\ln 2 = 0.220t$

$\boxed{t = 3.155 \text{ hours}}$

$y = 100e^{0.220(0.5)}$

$y = 111.612$

$\boxed{\text{about } 112 \text{ bacteria}}$

20. The value of a painting (in millions) grows exponentially. If the painting cost $10 million in 1990, and the same painting was sold for $65 million in 2004, predict the value of the painting in 2010. What was the value of the painting in 1985 (Calculator)?

1990 is $t=0$

$y = 10e^{kt}$

$65 = 10e^{k(14)}$

$6.5 = e^{14k}$

$\ln 6.5 = 14k$

$K = 0.134$

2010:

$y = 10e^{0.134(20)}$

$y = 144.9788983$

$\boxed{\$144,978,898.30}$

1985:

$y = 10e^{0.134(-5)}$

$y = 5.12476318$

$\boxed{\$5,124,763.18}$

21. The half-life of radioactive cobalt is 5.27 years. If 100 grams is present now, how much will be present in 20 years? In 40 years? How long will it take for only 5% of the cobalt to remain (Calculator)?

$y = 100e^{kt}$

$50 = 100e^{k(5.27)}$

$0.5 = e^{5.27k}$

$\ln(0.5) = 5.27k$

$k = -0.132$

$y = 100e^{-0.132(20)}$

$\boxed{y = 7.234 \text{ grams}}$

$y = 100e^{-0.132(40)}$

$\boxed{y = 0.519 \text{ grams}}$

$5 = 100e^{-0.132t}$

$0.05 = e^{-0.132t}$

$\ln 0.05 = -0.132t$

$\boxed{t = 22.777 \text{ years}}$

22. A 50-mg sample of a radioactive substance decays to 34-mg after 30 days. What is the half-life of this substance (Calculator)?

$y = 50e^{kt}$

$34 = 50e^{k(30)}$

$0.68 = e^{30k}$

$\ln 0.68 = 30k$

$k = -0.013$

$25 = 50e^{-0.013t}$

$0.5 = e^{-0.013t}$

$\ln 0.5 = -0.013t$

$\boxed{t = 53.919 \text{ days}}$

23. Use appropriate properties to solve the following equations (Calculator).

(a) $8^{6+3x} = 4$

$\cancel{\log_2 2}^{3(6+3x)} = \cancel{\log_2 2}^{2}$

$18+9x = 2$

$9x = -16$

$\boxed{x = -\frac{16}{9}}$

(b) $4^{x-x^2} = \frac{1}{2}$

$\cancel{\log_2 2}^{2(x-x^2)} = \cancel{\log_2 2}^{-1}$

$2x-2x^2 = -1$

$0 = 2x^2-2x-1$

$x = \frac{2 \pm \sqrt{4-4(2)(-1)}}{2(2)} = \frac{2 \pm \sqrt{12}}{4} = \frac{2 \pm 2\sqrt{3}}{4}$

$\boxed{\frac{1 \pm \sqrt{3}}{2}}$

(c) $\underset{\cancel{x}}{\cancel{\log_x}} 64 = \underset{x}{-3}$

$64 = x^{-3}$

$\frac{1}{64} = x^{3}$

$x = \sqrt[3]{\frac{1}{64}}$

$\boxed{x = \frac{1}{4}}$

(d) $5^x = 3^{x+2}$

$\ln 5^x = \ln 3^{x+2}$

$x \ln 5 = (x+2) \ln 3$

$x\ln 5 = x\ln 3 + 2\ln 3$

$x\ln 5 - x\ln 3 = 2\ln 3$

$x(\ln 5 - \ln 3) = 2\ln 3$

$\boxed{x = \frac{2\ln 3}{\ln 5 - \ln 3}}$

(e) $25^{2x} = 5^{x^2-12}$

$\cancel{\log_5 5}^{2(2x)} = \cancel{\log_5 5}^{x^2-12}$

$4x = x^2-12$

$0 = x^2-4x-12$

$0 = (x-6)(x+2)$

$\boxed{x = 6, -2}$

(f) $\log_6(x+3) + \log_6(x+4) = 1$

$\underset{\cancel{6}}{\cancel{\log_6}} (x+3)(x+4) = \underset{6}{1}$

$x^2+7x+12 = 6$

$x^2+7x+6 = 0$

$(x+6)(x+1) = 0$

$x = -6, -1$

$\boxed{x = -1}$

(g) $\log(7x-12) = 2\log x$

$\underset{\cancel{10}}{\cancel{\log}}(7x-12) = \underset{\cancel{10}}{\cancel{\log}} x^2$

$7x-12 = x^2$

$0 = x^2-7x+12$

$0 = (x-4)(x-3)$

$\boxed{x = 4, 3}$

(h) $e^{1-x} = 5$

$1-x = \ln 5$

$-x = -1+\ln 5$

$\boxed{x = 1-\ln 5}$

24. Use the appropriate regression to answer the following problems (Calculator):

(a) A retail store has various sizes of television sets made by the same manufacturer. The price for a set is a function of the screen size, measured in inches along the diagonal. The following table gives the price, p(x) in dollars as a function of diagonal screen length, x, in inches.

x	p(x)
5	166.50
10	199.00
15	181.50
20	189.00
25	296.50
30	579.00
35	1111.50

Find the particular equation for this situation.

$p(x) = 0.1x^3 - 4x^2 + 49x + 9$

How much would a 55 inch television cost?

$7,241.50

How big would a television be that cost $535?

29.422 in

(b) When the Pilgrims arrived in America, they brought along seeds from which to grow crops. Suppose that the number of bean plants, b(x), as a function of weeks since planting, x, is given by this table:

x	b(x)
3	34
4	83
5	138
6	205
7	290
8	399

Find the particular equation for this situation.

$b(x) = x^3 - 9x^2 + 75x - 137$

How many bean plants would there be 12 weeks after planting?

1195 bean plants

How long will it take for there to be 1000 bean plants?

11.284 weeks

32. Find the roots of the following functions (Calculator):

(a) $y = 5x^3 + 52x^2 + 150x + 52$

$$\begin{array}{r|rrrr} -\frac{2}{5} & 5 & 52 & 150 & 52 \\ & & -2 & -20 & -52 \\ \hline & 5 & 50 & 130 & \vert\ 0 \\ & 1 & 10 & 26 & \end{array}$$

$y = (5x+2)(x^2+10x+26)$

$$x = \frac{-10 \pm \sqrt{100-4(1)(26)}}{2(1)} = \frac{-10 \pm \sqrt{-4}}{2} = \frac{-10 \pm 2i}{2}$$

roots.

$(-\frac{2}{5}, 0)$

$(-5 \pm i, 0)$

(b) $y = x^4 + 16x^2 - 225$

$$\begin{array}{r|rrrrr} 3 & 1 & 0 & 16 & 0 & -225 \\ & & 3 & 9 & 75 & 225 \\ \hline -3 & 1 & 3 & 25 & 75 & \vert\ 0 \\ & & -3 & 0 & -75 & \\ \hline & 1 & 0 & 25 & \vert\ 0 & \end{array}$$

$y = (x-3)(x+3)(x^2+25)$

$x^2 + 25 = 0$

$x^2 = -25$

$x = \pm 5i$

roots:

$(\pm 3, 0)$

$(\pm 5i, 0)$

33. Find an equation for a_n for each of the following sequences, then find the 10th partial sum:

a) 5, 3, 1, −1, … (Non-Calculator)

Arithmetic, $d=-2$

$\boxed{a_n = 5-2(n-1)}$

$a_0 = 5-2(10-1)$

$= 5-2(9)$

$= 5-18$

$= -13$

$S_{10} = \frac{10}{2}(5-13)$

$= 5(-8)$

$= \boxed{-40}$

b) $\frac{1}{3}, -\frac{2}{3}, \frac{4}{3}, -\frac{8}{3}, \ldots$ (Calculator)

geometric, $r=-2$

$\boxed{a_n = \frac{1}{3}(-2)^{n-1}}$

$S_{10} = \frac{1}{3}\left[\frac{1-(-2)^{10}}{1-(-2)}\right]$

$= \boxed{\frac{-341}{3}}$

c) 54, 18, 6, 2, … (Calculator)

geometric, $r=\frac{1}{3}$

$\boxed{a_n = 54\left(\frac{1}{3}\right)^{n-1}}$

$S_{10} = 54\left[\frac{1-\left(\frac{1}{3}\right)^{10}}{1-\frac{1}{3}}\right]$

$= \boxed{80.999}$

d) $\frac{1}{2}, 1, \frac{3}{2}, 2, \ldots$ (Non-Calculator)

Arithmetic, $d=\frac{1}{2}$

$\boxed{a_n = \frac{1}{2}+\frac{1}{2}(n-1)}$

$a_{10} = \frac{1}{2}+\frac{1}{2}(10-1)$

$= \frac{1}{2}+\frac{9}{2}$

$= 5$

$S_{10} = \frac{10}{2}\left(\frac{1}{2}+5\right)$

$= 5\left(\frac{11}{2}\right)$

$= \boxed{\frac{55}{2}}$

34. Find each sum (Calculator).

a) $\sum_{k=1}^{5}(5k+3)$

$[5(1)+3]+[5(2)+3]+[5(3)+3]+[5(4)+3]+[5(5)+3]$

$8+13+18+23+28$

$\boxed{90}$

b) $\sum_{k=1}^{4}(k^2+4)$

$(1^2+4)+(2^2+4)+(3^2+4)+(4^2+4) =$

$5+8+13+20 =$

$\boxed{46}$

35. In 1993, the average cost of a ticket on a privately-owned airline was $107. This amount has increased by approximately $75 yearly. How much should you expect to pay for a ticket on this airline in 2010 (Calculator)?

$n=1$ is 1994, so $a_1 = 182$

$n=17$ is 2010

$a_n = 182+75(n-1)$

$a_{10} = 182+75(17-1)$

$= \boxed{\$1382}$

36. A woman made $35,000 during the first year of her new job at city hall. Each year she received a 10% raise. Find her total earnings during the first eight years on the job (Calculator). Geometric, $r = 1 + 0.1 = 1.1$

$$S_8 = 35000\left(\frac{1-1.1^8}{1-1.1}\right) = \boxed{\$400,256.08}$$

37. An amphitheater contains 20 rows. The top row can seat 70 people, and each row farther down can seat 2 fewer people. How many total people can the amphitheater seat (Calculator)?

Arithmetic, $d = -2$

$a_n = 70 - 2(n-1)$

$a_{20} = 70 - 2(20-1)$

$a_{20} = 32$

$S_{20} = \frac{20}{2}(70+32)$

$= \boxed{1020 \text{ people}}$

38. In a financial deal, you are promised $700 the first day and each day after that you will receive 65% of the previous day's amount. When one day's amount drops below $1, you stop getting paid from that day on. What day is the first day you would receive no payment (Calculator)? Geometric, $r = 0.65$

$a_n = 700(0.65)^{n-1}$

$1 = 700(0.65)^{n-1}$

$\frac{1}{700} = 0.65^{n-1}$

$\ln\left(\frac{1}{700}\right) = \ln 0.65^{n-1}$

$\ln\left(\frac{1}{700}\right) = (n-1)\ln 0.65$

$15.207 = n-1$

$n = 16.207$

$\boxed{17^{th} \text{ day}}$

39. Determine if each of the geometric series converges or diverges. If it converges, find the infinite sum (Non-Calculator):

a) $1 + \frac{7}{8} + \frac{49}{64} + \ldots$ Geometric, $r = \frac{7}{8}$ Converges since $\lvert r \rvert < 1$ $S = \frac{1}{1-7/8} = \frac{1}{1/8} = 1\left(\frac{8}{1}\right) = \boxed{8}$	b) $5 + 10 + 20 + 40 + \ldots$ Geometric, $r = 2$ Diverges since $\lvert r \rvert > 1$
c) $18 + 12 + 6 + \ldots$ Arithmetic, $d = -6$ Diverges	d) $18 + 12 + 8 + \ldots$ Geometric, $r = \frac{12}{18} = \frac{8}{12} = \frac{2}{3}$ Converges since $\lvert r \rvert < 1$ $S = \frac{18}{1-\frac{2}{3}} = \frac{18}{\frac{1}{3}} = 18\left(\frac{3}{1}\right) = \boxed{54}$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

40. Expand using the binomial expansion theorem (Calculator):

a) $(2x-3)^5$ =

$1(2x)^5(-3)^0+5(2x)^4(-3)^1+10(2x)^3(-3)^2+10(2x)^2(-3)^3+5(2x)^1(-3)^4+1(2x)^0(-3)^5=$

$\boxed{32x^5-240x^4+720x^3-1080x^2+810x-243}$

b) $(3x^2-4y)^4$ =

$1(3x^2)^4(-4y)^0+4(3x^2)^3(-4y)^1+6(3x^2)^2(-4y)^2+4(3x^2)^1(-4y)^3+1(3x^2)^0(-4y)^4=$

$\boxed{81x^8-432x^6y+864x^4y^2-768x^2y^3+256y^4}$

41. Use the binomial expansion theorem to evaluate the requested term for each binomial (Calculator).

a) The x^3 term in the expansion of $(2x+3)^{12}$.

$220(2x)^3(3)^9=$

$C=\frac{12!}{3!9!}$

$\boxed{34{,}642{,}080x^3}$

$C=220$

b) The y^8 term in the expansion of $(4x-y^2)^7$.

$35(4x)^3(-y^2)^4=$

$C=\frac{7!}{3!4!}$

$\boxed{2240x^3y^8}$

$C=35$

42. Transform the following conic sections into parametric form (Non-Calculator):

a) $16x^2+25y^2-64x+200y+64=0$

$16(x^2-4x+4)+25(y^2+8y+16)=-64+16(4)+25(16)$

$\frac{16(x-2)^2}{400}+\frac{25(y+4)^2}{400}=\frac{400}{400}$

$\frac{(x-2)^2}{25}+\frac{(y+4)^2}{16}=1$

$x=2+5\cos t$

$y=-4+4\sin t$

b) $16x^2-36y^2+320x-216y+700=0$

$16(x^2+20x+100)-36(y^2+6y+9)=-700+16(100)-36(9)$

$\frac{16(x+10)^2}{576}-\frac{36(y+3)^2}{576}=\frac{576}{576}$

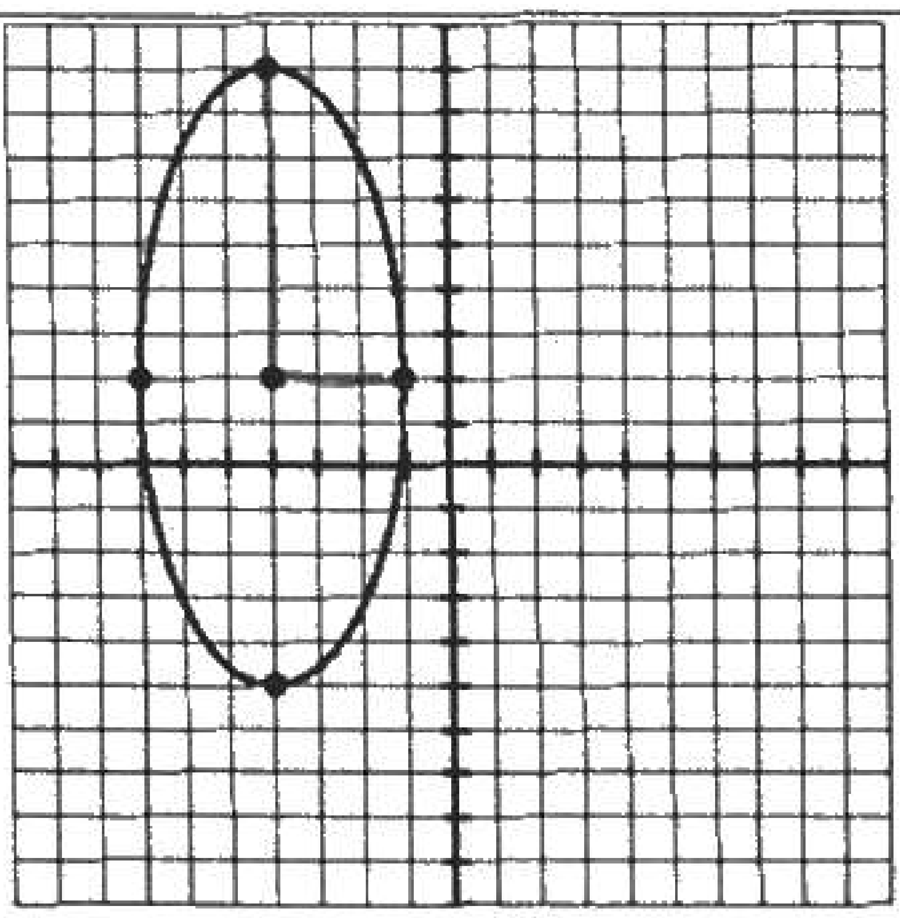
$\frac{(x+10)^2}{36}-\frac{(y+3)^2}{16}=1$

$x=-10+6\sec t$

$y=-3+4\tan t$

43 Write the equations in both parametric and Cartesian form for the following graphs (Non-Calculator):

a)

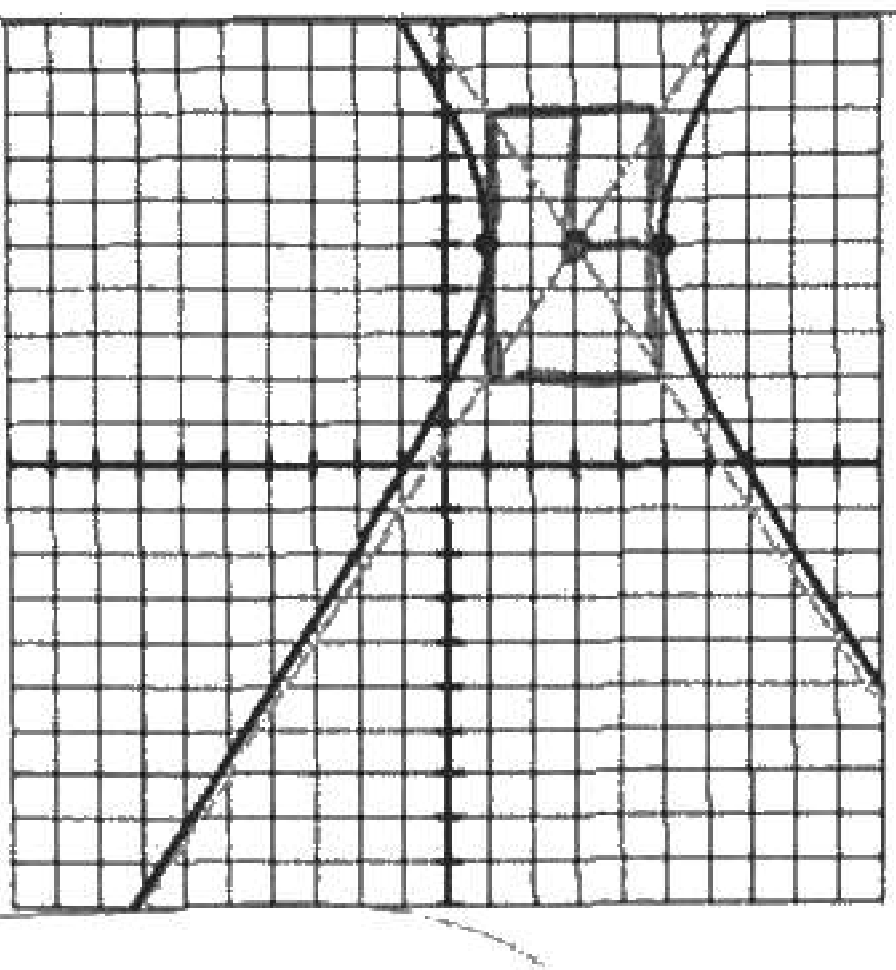


$x = -4 + 3\cos t$

$y = 2 + 7\sin t$

$$\frac{(x+4)^2}{9} + \frac{(y-2)^2}{49} = 1$$

b)

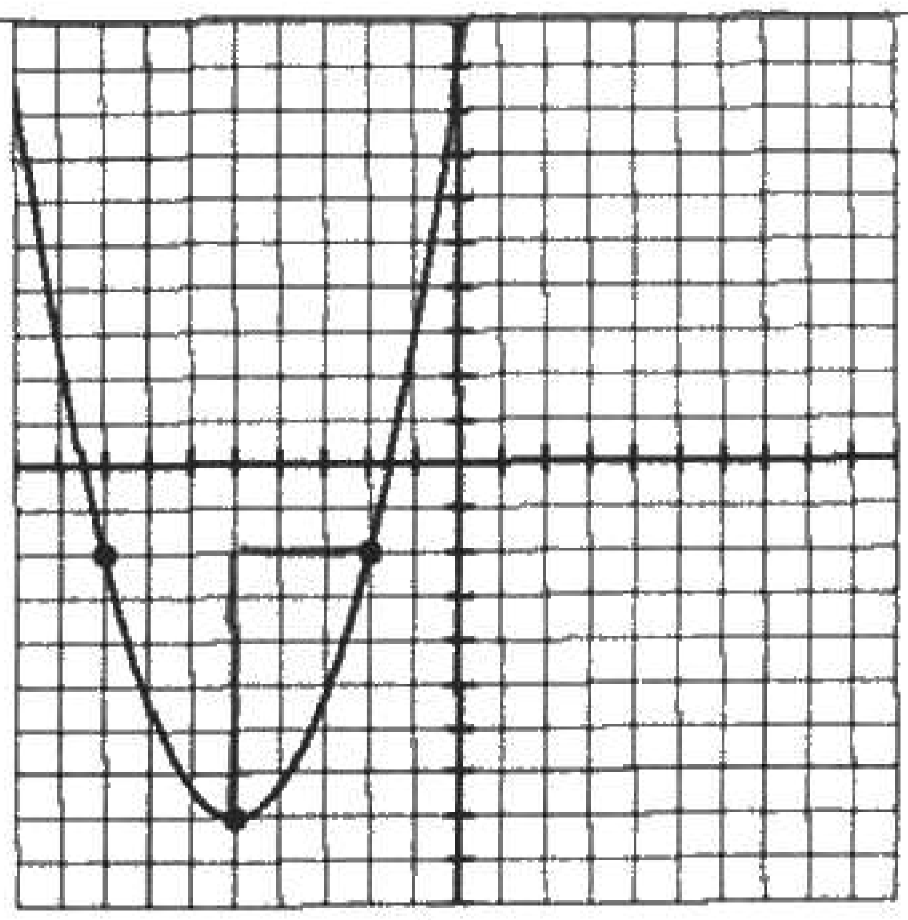


$x = 3 + 2\sec t$

$y = 5 + 3\tan t$

$$\frac{(x-3)^2}{4} - \frac{(y-5)^2}{9} = 1$$

c)



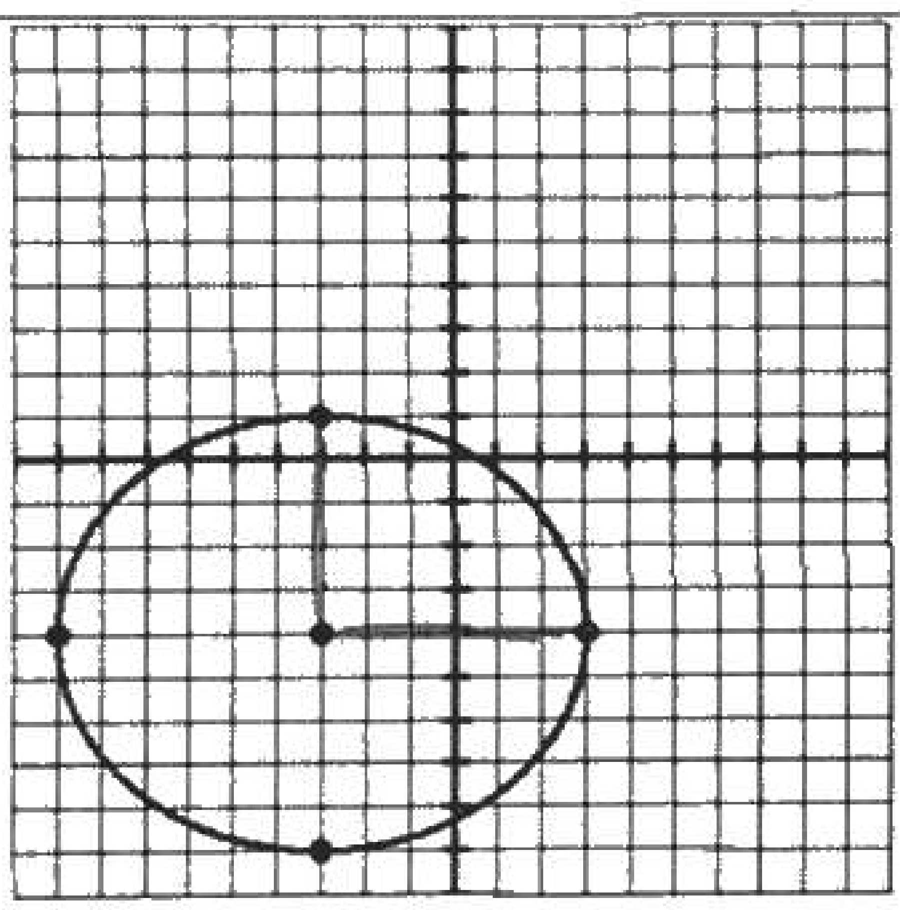
$a = \frac{6}{3^2} = \frac{6}{9} = \frac{2}{3}$

$x = -5 + t$

$y = -8 + \frac{2}{3}t^2$

$$y = \frac{2}{3}(x+5)^2 - 8$$

d)

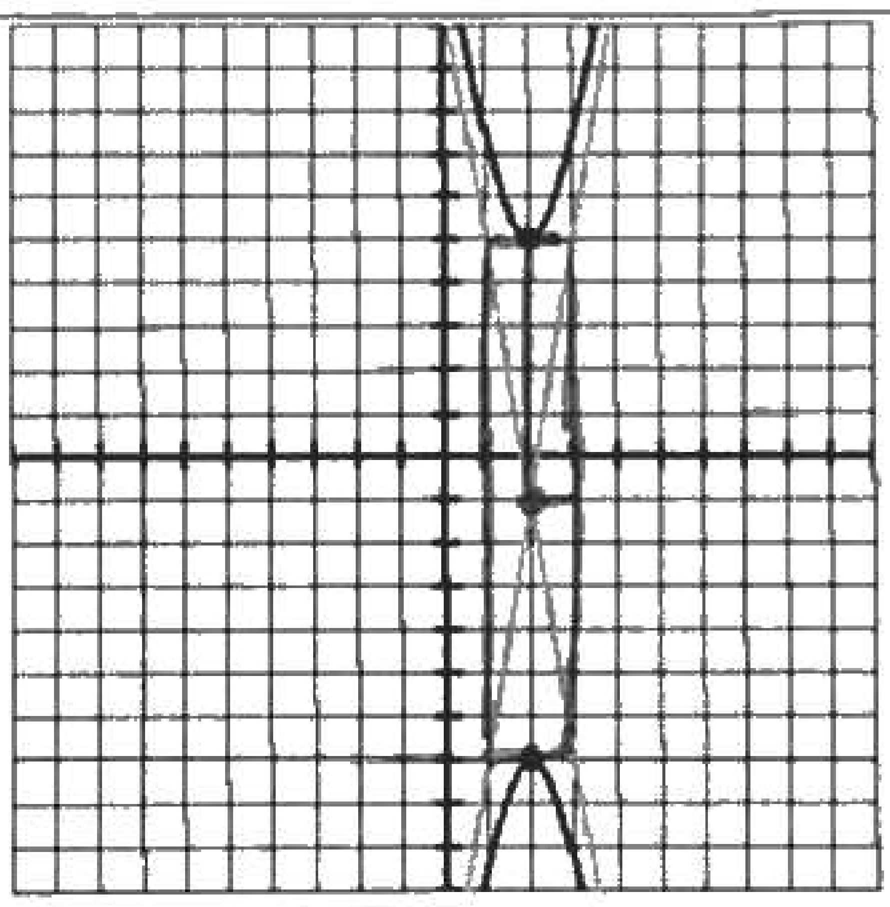


$x = -3 + 6\cos t$

$y = -4 + 5\sin t$

$$\frac{(x+3)^2}{36} + \frac{(y+4)^2}{25} = 1$$

e)

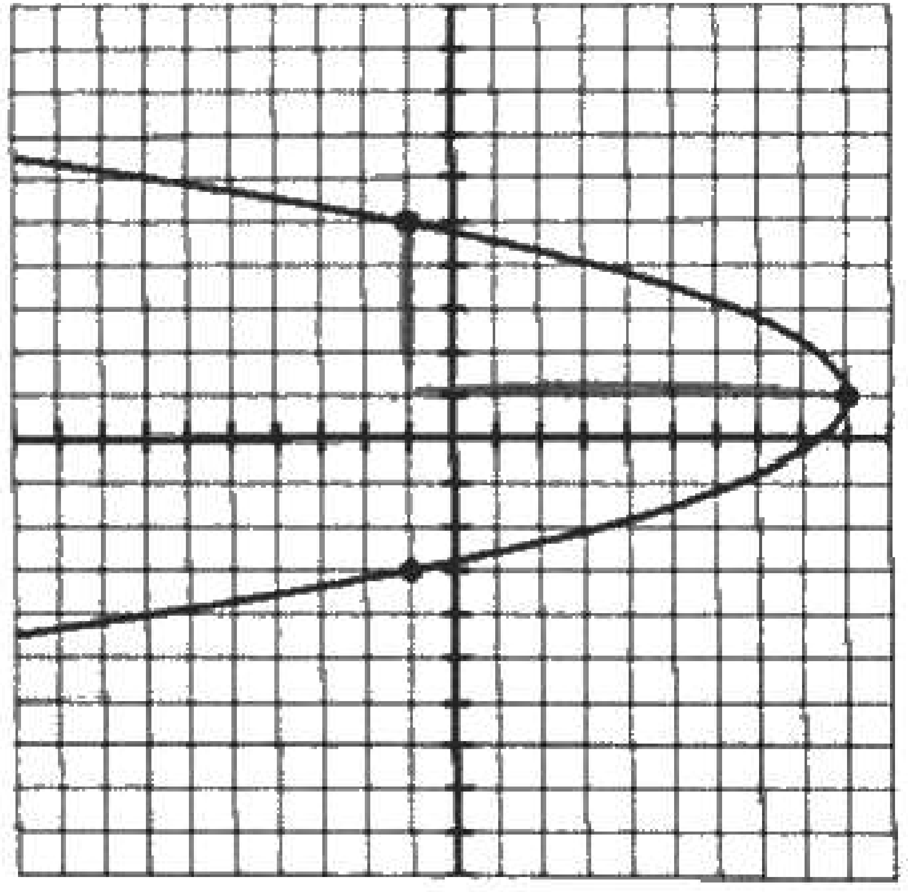


$x = 2 + \tan t$

$y = -1 + 6\sec t$

$$\frac{(y+1)^2}{36} - (x-2)^2 = 1$$

f)



$a = \frac{-10}{4^2} = \frac{-10}{16} = -\frac{5}{8}$

$x = 9 - \frac{5}{8}t^2$

$y = 1 + t$

$$x = -\frac{5}{8}(y-1)^2 + 9$$

44. Mercury's aphelion (the farthest it gets from the sun) is 69,816,900 km and its perihelion (the closest it gets to the sun) is 46,001,200 km. Find the eccentricity of Mercury's orbit as well as the equation in both parametric and Cartesian form. Assume the orbit is horizontal and the sun is at the left focus (Calculator).

$a = \frac{69,816,900 + 46,001,200}{2}$

$a = 57,909,050$

$c = 57,909,050 - 46,001,200$

$c = 11,907,850$

$57909050^2 = b^2 + 11907850^2$

$b = 56671,520.01$

$e = \frac{c}{a} = \frac{11,907,850}{57,909,050}$

$\boxed{e = 0.206}$

$x = 11,907,850 + 57,909,050 \cos t$

$y = 56,671,520.01 \sin t$

$$\boxed{\frac{(x - 11,907,850)^2}{57,909,050^2} + \frac{y^2}{56,671,520.01^2} = 1}$$

45. The closest a comet gets to the sun is 150 million miles. At a point perpendicular to that perihelion, the comet is 400 million miles away from the sun. Find the eccentricity of the comet's orbit as well as the equation in both parametric and Cartesian form. Assume the orbit is horizontal and the sun is at the left focus (Calculator).

$e = 1 + \frac{400 - 2(150)}{150}$

$\boxed{e = 1.667}$ hyperbola

$a = \frac{150^2}{400 - 2(150)}$

$a = 225$

$c = 225 + 150$

$c = 375$

$375^2 = 225^2 + b^2$

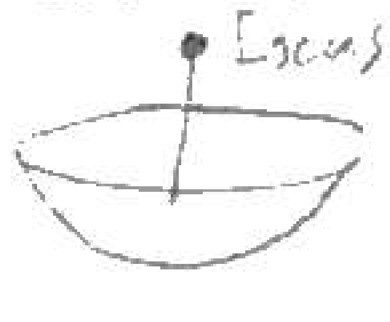
$b = 300$

$x = 375 + 225 \sec t$

$y = 300 \tan t$

$$\boxed{\frac{(x - 375)^2}{50,625} - \frac{y^2}{90,000} = 1}$$

46. A paraboloid for a spy microphone is formed by rotating the function $y = \frac{1}{81}x^2$ about the y-axis. How far from the vertex should the receiving microphone be placed (Calculator)?



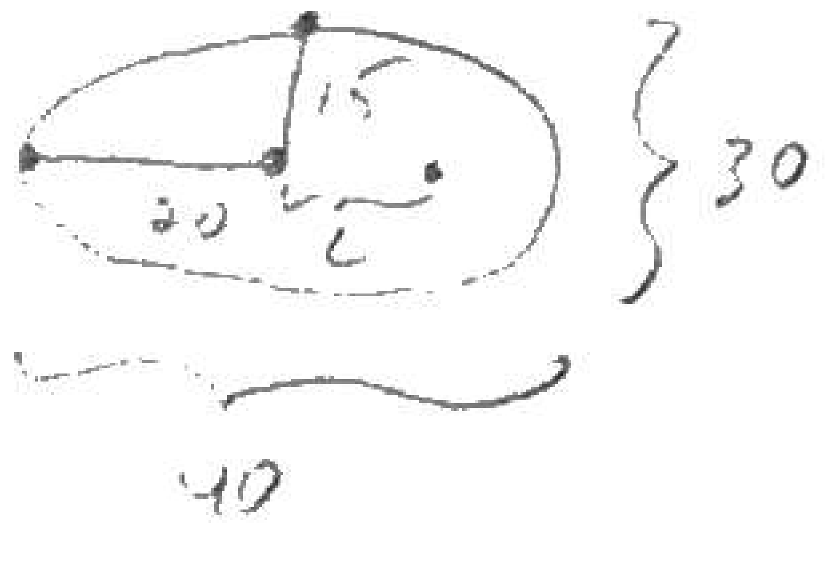
δ

$\frac{1}{81} = \frac{1}{4\delta}$

$4\delta = 81$

$\boxed{\delta = 20.25}$

47. An elliptical room is 40 feet long and 30 feet wide, and the center of the room is (0, 0). At what coordinates should chairs be placed so that the people sitting in them can most clearly hear the whispers of the person in the other chair (Calculator)?



$a = 20$

$b = 15$

$20^2 = 15^2 + c^2$

$c = 13.229$

center: $(0, 0)$

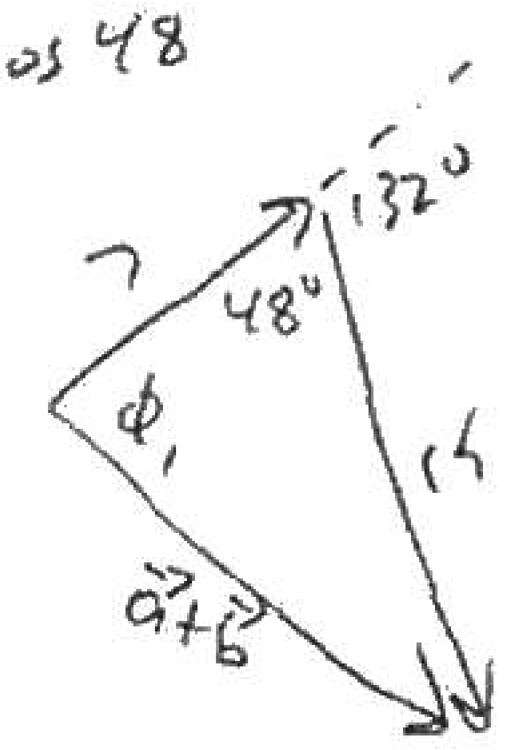
foci $= \boxed{(-13.229, 0) \text{ and } (13.229, 0)}$

48. Given the magnitude of two vectors and the angle between them, θ, find $\vec a + \vec b$ and $\vec a - \vec b$ as well as the angle each makes with $\vec a$ (Calculator).

a) $|\vec a| = 7$, $|\vec b| = 15$, $\theta = 132°$

$$|\vec a+\vec b|^2 = 7^2+15^2-2(7)(15)\cos 48$$

$$\boxed{|\vec a+\vec b| = 11.553}$$



$$\cos\phi_1 = \frac{7^2+11.553^2-15^2}{2(7)(11.553)}$$

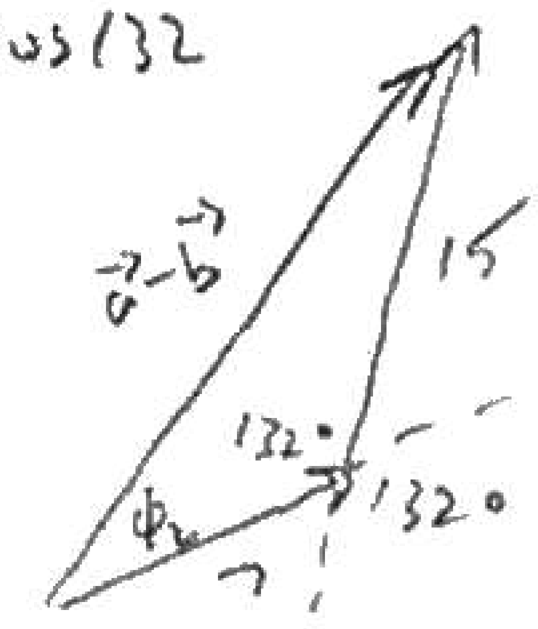
$$\boxed{\phi_1 = 105.240°}$$

$$|\vec a-\vec b|^2 = 7^2+15^2-2(7)(15)\cos 132$$

$$\boxed{|\vec a-\vec b| = 20.360}$$

$$\cos\phi_2 = \frac{7^2+20.360^2-15^2}{2(7)(20.360)}$$

$$\boxed{\phi_2 = 33.196°}$$



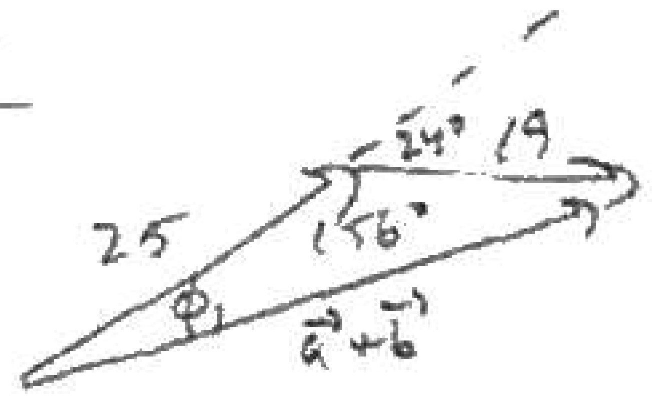
b) $|\vec a| = 25$, $|\vec b| = 19$, $\theta = 24°$

$$|\vec a+\vec b|^2 = 25^2+19^2-2(25)(19)\cos 156$$

$$\boxed{|\vec a+\vec b| = 43.057}$$

$$\cos\phi_1 = \frac{25^2+43.057^2-19^2}{2(25)(43.057)}$$

$$\boxed{\phi_1 = 10.340°}$$

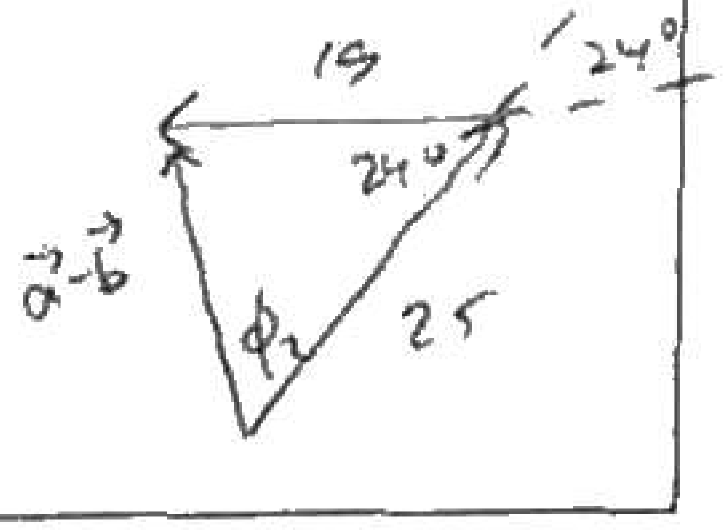


$$|\vec a-\vec b|^2 = 25^2+19^2-2(25)(19)\cos 24$$

$$\boxed{|\vec a-\vec b| = 10.869}$$

$$\cos\phi_2 = \frac{25^2+10.869^2-19^2}{2(25)(10.869)}$$

$$\boxed{\phi_2 = 45.318°}$$



49. Given two vectors and their directions as standard angles, θ, find the resultant vector as a direction (standard angle θ) and magnitude (Calculator).

a) $|\vec a| = 23$ at $\theta = 134°$ and $|\vec b| = 31$ at $\theta = 213°$

$$\vec a = 23\cos 134\,\vec i + 23\sin 134\,\vec j$$

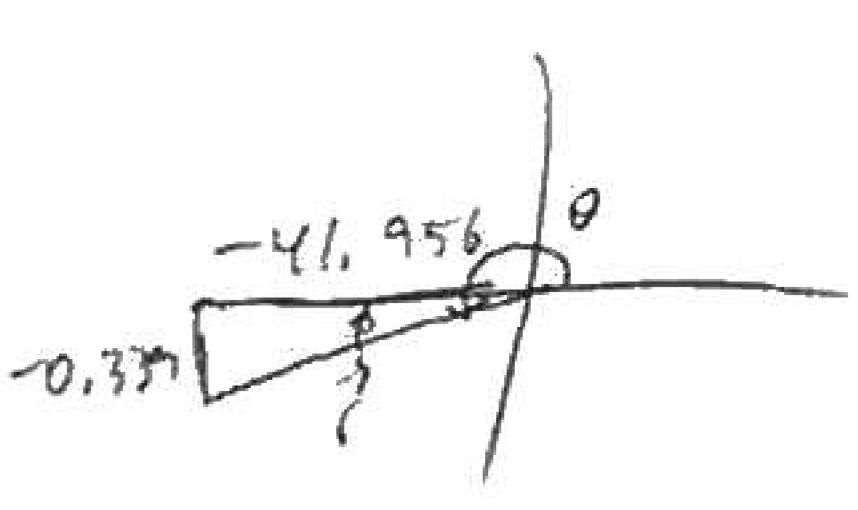
$$\vec b = 31\cos 213\,\vec i + 31\sin 213\,\vec j$$

$$\vec r = -41.956\,\vec i - 0.339\,\vec j$$

$$|\vec r|^2 = 41.956^2 + 0.339^2$$

$$\boxed{|\vec r| = 41.977}$$

$$\tan\phi = \frac{0.339}{41.956}$$



$$\phi = 0.463°$$

$$\theta = 180 + 0.463°$$

$$\boxed{\theta = 180.463°}$$

b) $|\vec a| = 42$ at $\theta = 26°$ and $|\vec b| = 68$ at $\theta = 301°$

$$\vec a = 42\cos 26\,\vec i + 42\sin 26\,\vec j$$

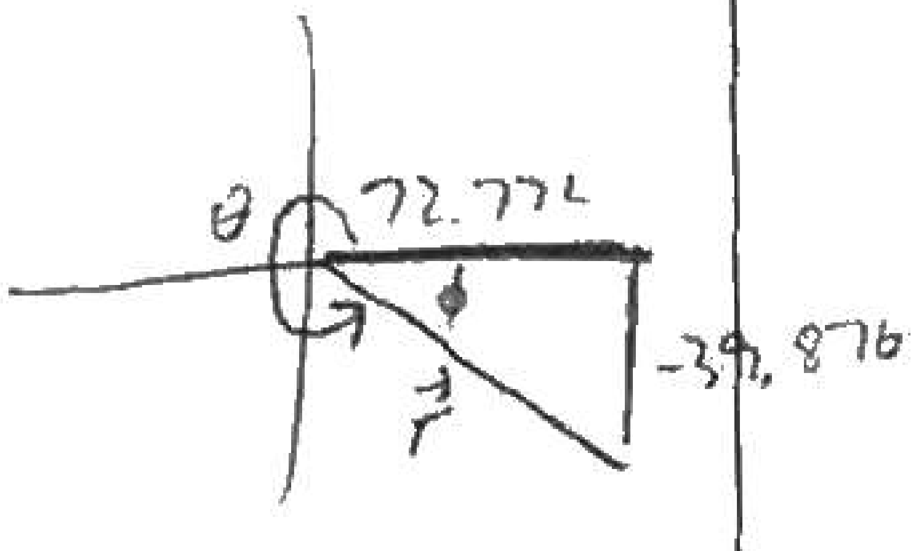
$$\vec b = 68\cos 301\,\vec i + 68\sin 301\,\vec j$$

$$\vec r = 72.772\,\vec i - 39.876\,\vec j$$

$$|\vec r|^2 = 72.772^2 + 39.876^2$$

$$\boxed{|\vec r| = 82.981}$$

$$\tan\phi = \frac{39.876}{72.772}$$



$$\phi = 28.721°$$

$$\theta = 360 - 28.721$$

$$\boxed{\theta = 331.279°}$$